White Paper on Computing Tools for Promoting Sound Investment Decisions

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SUMMARY

Numerous academic studies document that retail investors make systematic "mistakes" in assessing the risk and return characteristics of their portfolios. In this paper, I introduce a computing tool which can help individuals better assess these characteristics by displaying simulated portfolio returns. This paper features a demo version of this calculator for the federal government's Thrift Savings Plan. The output shows that riskier investments can lead to higher wealth outcomes than less risky investments although this outperformance is sensitive to assumptions about the mean return, particularly at long horizons. In addition, the risk of such investments does not decrease but rather increases with time horizon. I conclude by proposing ways in which to study the impact this tool can have on investors' behavior.

[1] I thank Jaime Harris and Gregory Barbieri for invaluable assistance with this project as well as seminar participants at the U.S. Securities and Exchange Commission.

I. Background

There are numerous academic studies which document systematic "mistakes" made by households in their investment portfolios.[3] In particular, many retail investors appear to incorrectly assess the risk and return characteristics of investment alternatives. As a result, they systematically deviate from the standard rational model of investment decision making, i.e., minimizing risk or maximizing expected return all else equal. In this paper, we offer remedies designed to correct mistakes that investors make in assessing the risk and return of their portfolio.

The first relevant finding in this regard is that many investors do not fully diversify their portfolios to minimize risk. For example, investors tend to overinvest in their company stock. A study by Mitchell and Utkus (2004) estimates that over 5 million Americans had over 60% of their retirement portfolios invested in their company's stock at the end of 2000.[4] A second relevant finding is the lack of participation in stock markets by a significant proportion of households. For example, Campbell (JF 2006) documents that roughly 50% of households with median wealth did not own any equities in the 2001 Survey of Consumer Finances.[5] This finding seems to indicate a lack of familiarity with the stock market and its historical returns. A final relevant stylized fact is that many financial professionals (and the investors they advise) believe that the risk of stock investment decreases over time, a belief known as "time diversification". They argue that variations in stock market returns decrease when averaged over time, just as variations in individual stock returns decrease in a well-diversified portfolio. On this basis, financial advisors commonly recommend that young investors allocate the preponderance of their savings to the stock market. Several academic studies, however, have concluded this reasoning to be flawed, ignoring the prospect of large losses over long time horizons.[6]

A lack of familiarity with the stock market on the part of some households and a reliance on time diversification on the part of others may lead to distorted investment decisions, which provide unideal exposure to risk and return. These forces may lead to extreme equity allocations across households of either 0 or 100% as observed in retirement account data.[7] These investors may be subsequently disappointed if their excessively risky investments lose money or their insufficiently risky investments earn meager yields. These mistakes are costly particularly in light of the fact that households are becoming increasingly responsible for managing their retirement savings in defined contribution plans,

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[3] See, e.g., Barber and Odean (WP 2011), Benartzi and Thaler (JEP 2007), and Daniel, Hirshleifer, and Teoh (JME 2002).
[4] Benartzi (JF 2001) and Poterba (AER 2003) also document overinvestment in company stock by retirement plan investors. Investors have also been documented to overinvest in local stocks and in their domestic equity market (see Lewis, JEL 1999 and Huberman, RFS 2001).
[5] A 2013 Gallup survey yielded similar findings, e.g., that 50% of middle-income households did not own stocks. See: http://money.cnn.com/2014/01/05/pf/bull-market/index.html?iid=HP_LN.
[6] See Bodie (FAJ 1995).
[7] Copeland (2014) finds that around 60% of IRA account holders had an extreme equity allocation in 2012.

whose assets grew from $1.7 trillion in 1995 to $5.1 trillion in 2012.[8] Unfortunately, financial education may not be sufficient in helping individuals make more sound investment decisions. For example, Choi, Laibson, Madrian, and Metrick (2002) find that seminars have, at best, a limited effect on employee participation in tax-advantaged savings plans. Similarly, professional financial advice also does not necessarily help households in their decision making. A recent study by Mullainathan, Nöth, and Schoar (2010) indicates that financial advisors increase rather than decrease their clients' propensity for making investment mistakes.

In this white paper, I introduce a risk calculator tool which can help people properly assess the risk and return of their portfolios. With this tool, investors can determine the distribution of returns for different portfolios. Investors can then choose among these portfolios to optimize their exposure to risk and return. Otherwise, retail investors may have to rely on faulty judgments, which can lead to the errors discussed. In this paper, I focus on portfolios in retirement accounts as this investment decision is arguably the most important for lifetime financial well-being. A principal goal of this project is to encourage hosting this proposed tool online by organizations such as not-for-profits, self-regulatory organizations, financial media outlets, and retirement plan fiduciaries.

There is a number of existing free online investment calculators. Some focus on the amount that individuals should save, incorporating varying degrees of real world complication into the calculation.[9] None of these calculators characterize risk other than possibly at a single point in the distribution. In contrast, my calculator more fully characterizes the risk of a portfolio at multiple points of the distribution. Therefore, this calculator provides more information to the user about the probabilities of different outcomes. The individual can potentially make more informed decisions based on this larger set of information. Another set of online calculators report the investor's optimal asset allocations based on unspecified algorithms.[10] These calculators choose an optimal or benchmark portfolio for investors based on an assessment of risk-aversion that is either ill-defined or absent.[11] The proposed calculator, in contrast, allows investors to freely choose their optimal portfolio based on an accurate assessment of risk and return. This calculator has the advantage of not making any problematic or implicit assumptions about risk aversion. Calculators similar to the one described in this paper are

[8] http://www.icifactbook.org/fb_ch7.html. A study by Poterba, Venti, and Wise (2008) found that the fraction of US private-sector pension contributions directed to defined contribution plans increased from 40% in the early 1980's to 90% two decades later.

[9] Savings calculators are provided by FINRA (http://apps.finra.org/investor_information/calculators/2/retirementcalc.aspx#Instructions), T. Rowe Price (https://www3.troweprice.com/ric/ricweb/public/ric.do), and E$Planner (http://basic.esplanner.com/) among others.

[10] A sophisticated asset allocation calculator is provided by Smart Money (http://sm.marketwatch.com/calculator/investing/managing-asset-allocation-1304479164310/).

[11] The Smart Money calculator prompts users to input their risk tolerance, which can range from "low" to "high". Methods for assessing risk-aversion through hypothetical lottery choices, frequently used by financial advisors, may also be problematic. For example, Nosic and Weber (2007) find in an experimental setting that hypothetical lottery choices are statistically unrelated to investment choices.

provided by Financial Engines and Ibbotson Associates. However, neither of these firms provides their tools for free to the general public.

II. Proposed Computing Tools

The risk calculator tool I propose has a few general features. First, investors will input data on their retirement account holdings and contributions as well as relevant demographic data. The distribution of the portfolio's returns will then be estimated using an historical sample of returns for the assets in the portfolio. The calculator will then report this distribution in terms of terminal income assuming the individual purchases a lifetime fixed annuity contract. Reporting annuitized income is desirable for at least two reasons. First, it can help investors understand outcomes in terms of their standard of living. Second, there have been numerous academic studies, which indicate that most households should annuitize at least part of their retirement wealth.[12] Specifically, annuitizing wealth can help insure against overconsuming or underconsuming one's assets after retirement. Alternatively, the calculator can report the distribution of the terminal wealth of the portfolio.

For demonstration purposes, I created a version of this calculator specific to the Thrift Savings Plan (TSP), the defined contribution plan for federal government employees. The investment options of the TSP are five underlying funds and five lifecycle funds. The underlying funds of the TSP are the C, S, I, F, and G funds. The first four funds track the S&P 500, Dow Jones Completion, MSCI EAFE, and Lehman Brothers Aggregate Bond Indexes, respectively, The G fund is the default investment for the TSP and holds non-marketable short-term treasury securities whose yield is based on long-term treasury yields. The five lifecycle funds hold these underlying funds as investments in a manner that decreases the equity allocation and, thereby, the risk of the fund over time. The planned allocation of a lifecycle fund over time is known as its "glidepath". The first four lifecycle funds are named by the approximate target retirement date of the investor as the L 2050, L 2040, L 2030, and L 2020 funds. The glidepaths for these four funds are identical as a function of the time until their target dates.[13] The final lifecycle fund, the L Income fund, holds the fixed terminal allocation, to which the other lifecycle funds converge on their target dates. I obtained quarterly glidepath allocation data for these lifecycle funds over a 40 year time span as shown in figure 1. These funds begin with a total equity allocation of 90% and end with a total equity allocation of 20% at the target date.

I also obtained monthly return data for the five underlying TSP funds (or their underlying indexes) from January 1988 until December 2012.[14] Summary statistics for the real (inflation-adjusted) returns of

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[12] See Benartzi, Previtero, and Thaler (JEP 2011) for a survey of academic papers related to annuitization.

[13] For example, the planned allocations of the L 2050, L 2040, L 2030, and L 2020 funds are identical at April 2045, 2035, 2025, and 2015, respectively.

[14] Monthly return data for this time period was available for the C, F, and G funds. For the S and I funds, however, monthly return data was only available from January 2001 until December 2012. We backfilled returns for these

these funds over this time period is shown in table 1. Of the five underlying funds, the S fund had the highest average annualized real return over this time period of 8.81% while the G fund had the lowest average real return of 2.70%. Of equity funds, the C fund had the highest annualized real mean to volatility ratio of 0.50=7.43%/14.8% versus 0.48=8.81%/18.4% for the S fund and 0.20=3.57%/17.8% for the I fund. Not surprisingly, the estimated correlation coefficients among equity funds are positive as their benchmark indexes tend to move together. They are also statistically significant at the 1% level. In other words, there is a less than 1% chance of observing correlation estimates of this magnitude if the true correlation is zero.

In order to simulate annuitization of portfolio wealth, I also obtained semiannual annuity rate data for males and females aged 60, 65, 70, and 75 from immediateannuities.com starting May 1988 and ending November 2012. Average annuity rates for these gender and age categories are shown in table 2. The rates for females and younger annuity recipients are lower since these groups have a longer remaining expected lifespan. I backfilled missing months in this data to match the monthly frequency of the TSP return data. In order to backfill, I obtained benchmark interest rate data in the form of the Moody's AAA yield on a monthly frequency. Changes in this interest rate have a positive and significant (at the 1% level) correlation with changes in annuity rates as seen in table 2. The backfilling procedure involved first estimating α and β in the following linear regression relationship:

$$(\log r_{annuity,t} - \log r_{annuity,t-6}) = \alpha + \beta \cdot (\log r_{AAA,t} - \log r_{AAA,t-6}) + \varepsilon_t \quad (1)$$

The dependent variable on the left hand side of the equation is the change in the natural logarithm of the annuity rate for each gender and age category at month t over the prior six-month period. The independent variable on the right hand side is the change in the natural logarithm of the Moody's yield at month t over the prior six-month period. Regression results are shown in table 2. One can see that the regression coefficients (β) are positive and significant at the 1% level for all age and gender categories. Missing annuity rate data at month t were backfilled by using the prior actual or estimated natural logarithm of the annuity rate at month t-1 and adding the change in the natural logarithm of the Moody's yield between months t and t-1 scaled by the estimated β plus a random noise term (ε_t) as in the following formula:

$$\log r_{annuity,t} = \log r_{annuity,t-1} + \beta \cdot (\log r_{AAA,t} - \log r_{AAA,t-1}) + \varepsilon_t$$

The constant, α, is omitted from this formula since it was insignificant in all regressions. In other words, this estimate was statistically indistinguishable from zero. The random noise term was bootstrapped from the sample of residuals produced from the regression in equation (1).

I created the demo TSP calculator in MATLAB although a full working version of this tool should ideally have a user-friendly interface most likely hosted on an internet webpage. The demo calculator begins by prompting users to input their current age, projected retirement age, gender, current account

two funds from January 1988 until December 2000 by taking the benchmark index returns (including dividends) and deducting the current TSP expense ratio of 0.27 basis points.

balance and allocation, and current contribution amount and allocation. Users are then asked whether they have a plan for changing their allocation over time and their projected allocation at retirement, if so. A linear glidepath is assumed for users who answer yes. The input screens are shown in a MATLAB window in figures 2 and 3.

I simulate possible returns for the portfolio over its holding period from the current date until retirement. I use two different approaches to serial correlation between past and future returns using either 1 or 12 month return blocks. With 1 month blocks, returns at each month of the portfolio are simulated by selecting a random past month from our historical sample and using the returns for the five underlying funds from that past month. At the terminal month of the portfolio, a random month is again drawn to determine the final month of returns as well as the annuity rate, which determines post-retirement income. With 12 month blocks, past annual returns are drawn to simulate portfolio returns each year.[15]

In principle, simulations with 1 and 12 month blocks should yield similar results if returns follow a random walk, which implies that there is zero serial correlation between past and future monthly returns. However, higher serial correlation in monthly returns results in higher volatility for 12-month returns just as higher correlation among assets in a portfolio increases the overall volatility of the portfolio. In the presence of positive serial correlation, the volatility of portfolios from the 12 month block simulation should exceed that from the 1 month block simulation. The ratio between the variance of 12 month log return blocks and the annualized variance of 1 month log returns (i.e., the 1 month variance times 12) are reported in table 1 for the five underlying funds. These variance ratios should be equal to 1 if there is no serial correlation in monthly returns. I find the magnitude of these variance ratios to be greater than one, reflecting possible positive serial correlation in stock indexes as documented in Campbell, Lo, and Mackinlay (1997). However, none of these variance ratios is statistically distinguishable from one except for that of the G fund.[16]

Several academic studies indicate that expected returns in equity markets are lower than average equity returns in historical data. For example, Fama and French (JF 2002) estimate the premium (i.e., the expected return minus the risk-free rate) on the S&P 500 Index using the earnings or dividends growth rates in place of stock price growth rates. Namely, stock returns are equal to returns from dividends (or dividend yield) plus stock price growth (or capital gains). Fama and French (JF 2002) conjecture that the ratio between stock prices and dividends or earnings should be stable statistically and go to neither zero nor infinity. Therefore, the growth rate for dividends or earnings should be the same as the growth rate for stock prices, which would otherwise imply unstable ratios. Under this

[15] The past annual return periods are overlapping. For example, one annual return of the portfolio may come from the historical return from January 1991 – December 1991 while another may come from the historical return from July 1991 – June 1992. In addition, all simulations draw past returns with replacement. In other words, the past return month February 1998 may be drawn twice for a single portfolio simulation with 1 month return blocks.

[16] Under certain assumptions, market efficiency theory implies that asset prices follow a random walk with zero serial correlation. See Samuelson (1973). In the case of the G fund, market forces do not act to push its NAV toward a random walk because this value is based on non-market interest rates.

assumption, these alternative estimates for the expected return have less error since both dividend and earnings growth are less volatile than stock price growth.

I use two different approaches with respect to the expected return of the equity funds. The first is to simply draw unaltered returns from the sample under the assumption that the future expected return is equal to the mean in the sample. The second is to shift the returns of each equity fund by a constant amount so that the average return is equal to the Fama-French estimate. They find the expected log equity premium for the S&P 500 from 1950-2000 to be 2.9%-4.7% lower based on dividend or earnings growth than for the historical average equity premium. Therefore, we deduct 3.5% from the average log annual post-war real return for the S&P 500 reported in Ahmed, Barber, and Odean (2013) of 6%. Transforming to real returns from log real returns then deducting the current TSP expense ratio yields an annualized return of 3.99% compared to the annualized average real return of 7.43%, 8.81%, and 3.57% for the C, S, and I funds, respectively. Details of this calculation are available upon request. I reserve more sophisticated approaches to the estimation of expected returns for each equity fund separately for future work.[17]

III. Simulation Output

In this paper, I generate terminal annuitized income for a female retiring at age 65 over three time horizons: 5 years, 15 years, and 30 years. All simulations assume an initial account balance of $100,000 and a biweekly contribution of $800.[18] I generate simulation output for 5000 runs of five different portfolios: the C fund, equal-weighted equity funds (C, S, and I funds), equal-weighted risky funds (C, S, I, and F funds), the lifecycle fund (with target date closest to the retirement date of the investor), and the G fund. I select an equal-weighting scheme for two of these portfolios because of a paper by DeMiguel, Garlappi, and Uppal (RFS 2009). These authors find that equal-weighted portfolios generally outperform portfolios which explicitly optimize based on estimated mean and variance. These estimates contain statistical errors which diminishes the performance of these optimized portfolios.

This demo calculator can generate simulation output in either graphical or textual format. The graphical output is a histogram of the probability density function (pdf) of terminal portfolio outcomes, i.e., a bar chart of the probabilities of different outcome ranges. The textual output is a list of percentile breakpoints for the distribution of terminal portfolio outcomes. I report the 5th, 50th, and 95th percentile breakpoints in this white paper. Bateman, et al. (2013) and Vlaev, Chater, and Stewart (2009) find that this type of breakpoint information conveys risk effectively according to a number of measures.

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[17] One possible approach is to estimate expected returns based on the dividend or earnings growth rate (data permitting) of each benchmark index separately.

[18] An $800 biweekly contribution roughly represents the maximum tax-deferred amount that someone earning $75,000 could accrue counting agency contributions (but not counting catch-up contributions) as of 2014.

Figures 4 and 5 show the pdf of terminal incomes for the C fund and G fund portfolios, respectively, over a 15 year horizon. The G fund portfolio has a median terminal income of $40,800. Not surprisingly, the C fund portfolio has a higher median terminal income of $59,600 with a higher variability. For the remainder of this paper, I focus on distributional breakpoints to facilitate quantitative comparisons between different portfolios and horizons.

The breakpoint results are reported in tables 3, 4, and 5 for 5, 15, and 30 year horizons, respectively. This output has a number of noteworthy features. First, it is clear that time diversification does not hold in these simulation results. Namely, the dollar and percent variability of terminal income increases with the time horizon of the portfolio. Therefore, the output from this calculator can help correct an individual's faulty beliefs in time diversification. Second, equity fund portfolio outcomes are sensitive to the choice of mean returns, especially at long horizons. The dollar outcomes at the listed breakpoints are roughly 50-100% higher with sample means than Fama-French means for the C fund and equal-weighted equity portfolios at a 30-year horizon. Third, there appears to be no benefit to diversifying equity holdings beyond the C fund. The C fund dominates the equal-weighted equity portfolio at the listed breakpoints for sample means. As noted earlier, the C fund has the highest real mean to volatility ratio of the three equity funds over our sample period. The C fund also has lower variability in terms of the range between high and low outcomes than the equal-weighted equity portfolio for Fama-French means. Fourth, portfolio outcomes can be improved by at least partially investing in equities. For example, the L fund portfolios dominate the G fund at the 5th and 50th percentiles while performing within 15% of the G fund portfolio at the 95th percentile. This type of information can benefit investors by conveying the returns associated with stock investment. Therefore, it may encourage stock market participation by those avoiding this market as a result of unfamiliarity.

Finally, it may be worth noting that the C fund exhibits more volatility with a blocksize of 12 months than 1 month. This outcome is not surprising given that the variance ratio for the C fund of 1.35 is highest among equity funds as seen in table 1. However, it may be surprising that the variability of terminal income for the G fund does not increase with block size in spite of its statistically significant variance ratio of 1.95. The reason is that this variability in terminal income derives mainly from the variability in the terminal annuity rate and not from the variability in terminal portfolio wealth.

IV. Further Work

This paper represents a preliminary step in a larger endeavor to create tools which help individuals assess the risk and return of their investment portfolios. There are several possible next steps to improve the proposed calculator and its utility to investors. First, there are several ways to refine the statistical methods employed in the portfolio simulation. For example, the simulation methodology implicitly assumes that fund expected returns (whether based on sample means or alternative estimators) are known with certainty. However, there is an extensive academic literature which accounts for uncertainty in the estimation of such parameters. Such uncertainty would generally increase the volatility of outcomes relative to the case of certainty in estimates. It would be desirable to incorporate parameter uncertainty into the simulation algorithm. Second, a complete analysis of this type should incorporate a household's total exposure to financial risks through labor income, real estate investments, debt, etc. Future versions of this calculator would ideally incorporate these other sources of risk into the simulation of outcomes.

Third, it is important to study how this tool affects actual investment decisions with the goal of improving them. Several studies have shown that the framing of information can have a significant impact on behavior.[19] With the proposed calculator, an emphasis on the left side versus the right side of the distribution might lead to different decisions. For example, the statement, "a 95% chance of income less than $51,000", might generate different behavior than the equivalent statement, "a 5% chance of income greater than $51,000". It would also be useful to explore how this tool might help both new investors and existing investors in retirement accounts. The latter have been documented to exhibit significant inertia in their choices.[20] For example, a significant proportion of individuals do not change their retirement plan allocations or contribution rates over time. Therefore, one challenge of this project is to ascertain how to break investors away from pre-existing decisions that are unideal. For example, this tool might offer blind comparisons to optimized contribution rates and allocations.

V. Conclusion

In this white paper, I have introduced a tool which is designed to help retail investors better understand the risk and return characteristics of their retirement portfolios. Greater familiarity with the distribution of outcomes from various investment choices can potentially steer individuals toward superior allocations and contribution rates. Consequently, this tool may help prevent the aforementioned common investment "mistakes" as well as increasing savings rates generally. Within the federal government's Thrift Savings Plan, my output has shown that riskier investments can lead to higher wealth outcomes than less risky options. Therefore, individuals, who may avoid the stock market as a result of unfamiliarity, may be encouraged to participate in it. However, this outperformance is sensitive to assumptions about expected equity returns, particularly at long horizons. In addition, the risk of such investments increases with time horizon, in contradiction to the fallacy of time diversification. As mentioned previously, the long-term goal of this project is to encourage hosting the proposed tool online by various relevant organizations. This calculator offers a number of advantages over existing online calculators. One important advantage is that it more fully characterizes risk and return, providing more information on which to make investment decisions.

[19] One example is in the domain of health-related choices in emphasizing the probability of success versus failure. See, e.g., Block and Anand Keller (1995).

[20] See, e.g., Madrian and Shea (QJE 2001).

VI. Technical Glossary

Equity premium: the expected return on a stock market index minus the riskfree rate, generally measured by the treasury bill rate.

Probability density function: a function that describes the relative likelihood for a random variable to take on a given value. The probability of the random variable falling within a particular range of values is equal to the area under the density function in that range.

Serial correlation: correlation between observations of a random variable at different times.

Statistical significance: the probability that an estimate with a certain magnitude is observed by chance alone. An estimate, which could be observed by chance alone with sufficiently low (high) probability, is called *statistically significant (insignificant)*.

Variance ratio: the variance of the change in a random process over a number of periods, *n*, divided by the variance of the change in the process over a single period times *n*. This ratio should be equal to one if there is zero serial correlation in changes over time. It should be greater (less) than one if there is positive (negative) serial correlation.

Table 1: Return Statistics for TSP Funds

This table shows real return statistics for the five underlying TSP funds (C, S, I, F, and G funds) based on monthly data from January 1988 – December 2012. The mean and volatility reported below are annualized by multiplying the monthly mean and variance estimate by 12 (i.e., the monthly volatility estimate is multiplied by $12^{½}$). The variance ratio below is the variance of overlapping annual log returns divided by 12 times the variance of monthly log returns. Estimates for variance ratios and correlation coefficients, which are significant at the 5% level, are shown in bold.

Panel A:	Univariate Statistics				
	C	S	I	F	G
Mean	7.43%	8.81%	3.57%	4.09%	2.70%
Volatility	14.8%	18.4%	17.8%	4.03%	1.18%
12-month Variance Ratio	1.35 (0.175)	1.12 (0.655)	1.19 (0.475)	1.08 (0.731)	**1.95** (0.0027)

Panel B:	Correlation				
	C	S	I	F	G
C	**1** (0.000)				
S	**0.853** (0.000)	**1** (0.000)			
I	**0.729** (0.000)	**0.682** (0.000)	**1** (0.000)		
F	**0.172** (0.0028)	0.080 (0.167)	0.109 (0.0599)	**1** (0.000)	
G	0.107 (0.0635)	0.060 (0.299)	0.065 (0.264)	**0.430** (0.000)	**1** (0.000)

Table 2: Annuity Rate Analysis

The annuity rates used below are semiannual data for males and females for four age categories from May 1988 – November 2012 taken from immediateannuities.com. The Pearson correlation estimates below are for the change in annuity rate versus the concurrent change in Moody's AAA yield. The regression estimates below are for the change in the natural logarithm of the annuity rate versus the concurrent change in the natural logarithm of Moody's AAA yield. Correlation coefficients and regression estimates, which are significant at the 5% level, are shown in bold.

	Female Annuity Rates			
Age	60	65	70	75
Mean	7.46%	8.14%	9.10%	10.5%
Moody's AAA Correlation	**0.675** (0.000)	**0.691** (0.000)	**0.686** (0.000)	**0.654** (0.000)
Coefficient	**3.295** (0.000)	**3.031** (0.000)	**2.693** (0.000)	**2.586** (0.000)
Constant	-0.071 (0.234)	-0.074 (0.166)	-0.080 (0.106)	-0.087 (0.104)
	Male Annuity Rates			
Age	60	65	70	75
Mean	7.99%	8.85%	10.0%	11.7%
Moody's AAA Correlation	**0.656** (0.000)	**0.678** (0.000)	**0.651** (0.000)	**0.598** (0.000)
Coefficient	**2.976** (0.000)	**2.890** (0.000)	**2.511** (0.000)	**2.410** (0.000)
Constant	-0.082 (0.154)	-0.083 (0.126)	-0.094 (0.077)	-0.105 (0.084)

Table 3: Terminal Income Breakpoints for TSP Portfolios – 5-Year Horizon

This table below shows the 5th, 50th, and 95th percentile breakpoints of terminal income for five TSP portfolios for a 5-year horizon. The "Eq-Wt Equity" portfolio has equal weights on the C, S, and I funds while the "Eq-Wt Risky" portfolio has equal weights on the C, S, I, and F funds. These statistics are based on 5000 simulations of a portfolio held by a female with a $100,000 initial account balance and an $800 bi-weekly contribution between the ages of 60 and 65.

	1-month blocks, sample means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2020	G fund
95th	$12,100	$11,500	$12,700	$13,600	$13,200
50th	$20,200	$19,200	$19,100	$18,500	$17,600
5th	$33,100	$32,400	$29,500	$24,800	$22,300
	12-month blocks, sample means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2020	G fund
95th	$11,400	$10,700	$12,200	$13,200	$13,000
50th	$20,300	$19,500	$19,200	$18,300	$17,500
5th	$33,300	$31,100	$28,500	$23,900	$21,800
	1-month blocks, Fama-French means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2020	
95th	$10,700	$10,500	$12,000	$13,000	
50th	$17,800	$17,700	$17,900	$17,800	
5th	$28,500	$29,600	$27,300	$23,800	

Table 4: Terminal Income Breakpoints for TSP Portfolios – 15-Year Horizon

This table below shows the 5th, 50th, and 95th percentile breakpoints of terminal income for five TSP portfolios for a 15-year horizon. The "Eq-Wt Equity" portfolio has equal weights on the C, S, and I funds while the "Eq-Wt Risky" portfolio has equal weights on the C, S, I, and F funds. These statistics are based on 5000 simulations of a portfolio held by a female with a $100,000 initial account balance and an $800 bi-weekly contribution between the ages of 50 and 65.

	1-month blocks, sample means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2030	G fund
95th	$29,200	$26,000	$29,900	$31,500	$30,900
50th	$59,600	$54,400	$53,200	$48,100	$40,800
5th	$130,000	$123,000	$106,000	$73,800	$52,100
	12-month blocks, sample means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2030	G fund
95th	$25,900	$22,900	$28,300	$30,100	$29,800
50th	$59,600	$53,900	$52,500	$47,900	$40,600
5th	$135,000	$123,000	$104,000	$73,700	$51,000
	1-month blocks, Fama-French means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2030	
95th	$21,200	$20,300	$25,500	$27,900	
50th	$42,500	$41,500	$43,000	$42,600	
5th	$89,100	$89,900	$79,400	$65,200	

Table 5: Terminal Income Breakpoints for TSP Portfolios – 30-Year Horizon

This table below shows the 5th, 50th, and 95th percentile breakpoints of terminal income for five TSP portfolios for a 30-year horizon. The "Eq-Wt Equity" portfolio has equal weights on the C, S, and I funds while the "Eq-Wt Risky" portfolio has equal weights on the C, S, I, and F funds. These statistics are based on 5000 simulations of a portfolio held by a female with a $100,000 initial account balance and an $800 bi-weekly contribution between the ages of 35 and 65.

	1-month blocks, sample means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2040	G fund
95th	$72,100	$58,300	$73,200	$72,900	$67,300
50th	$201,000	$170,000	$157,000	$124,000	$89,500
5th	$604,000	$553,000	$440,000	$217,000	$115,000
	12-month blocks, sample means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2040	G fund
95th	$60,400	$48,700	$66,700	$68,000	$65,600
50th	$197,000	$164,000	$156,000	$122,000	$88,900
5th	$645,000	$535,000	$437,000	$218,000	$113,000
	1-month blocks, Fama-French means				
	C fund	Eq-Wt Equity	Eq-Wt Risky	L 2040	
95th	$38,400	$36,600	$53,400	$58,900	
50th	$96,800	$93,500	$102,000	$97,500	
5th	$255,000	$275,000	$239,000	$168,000	

Figure 1: Glidepaths for TSP lifecycle funds

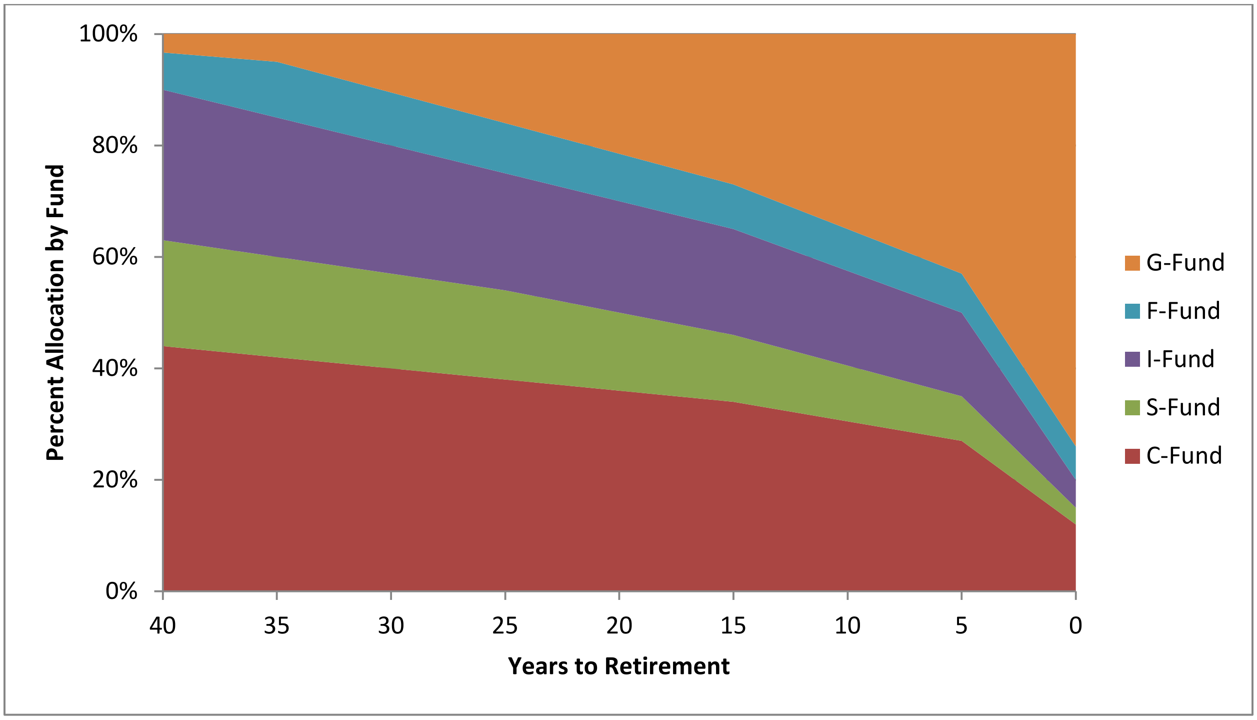


Figure 2: Demo calculator input screen 1

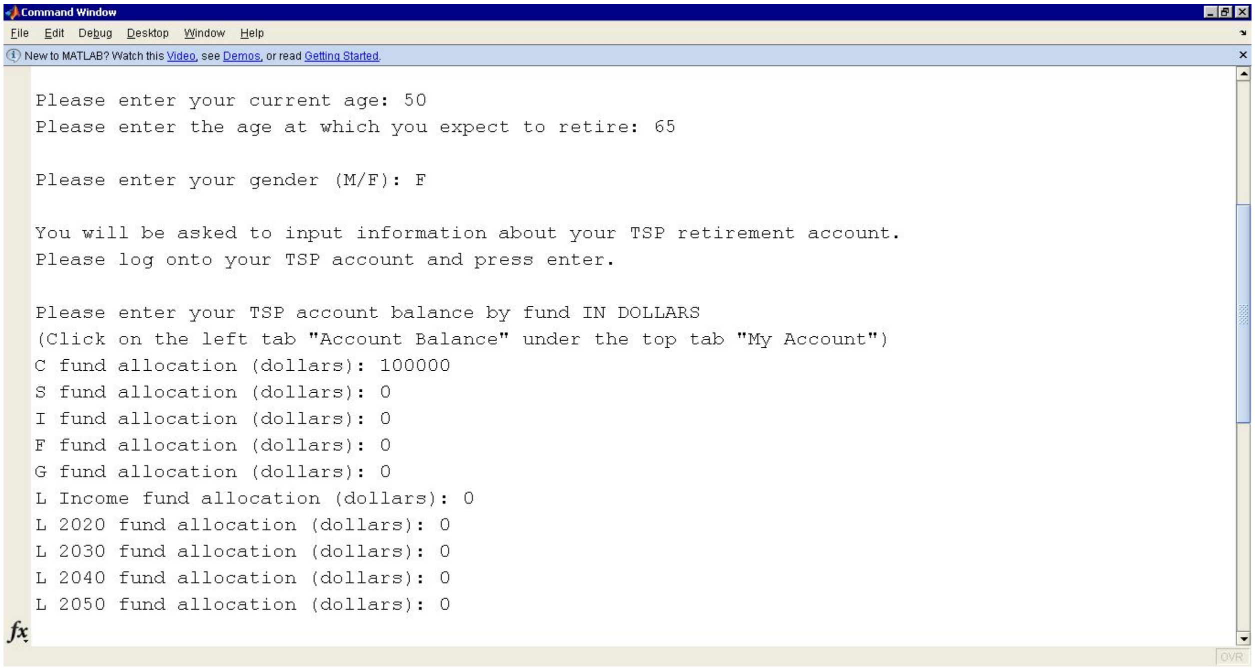


Figure 3: Demo calculator input screen 2

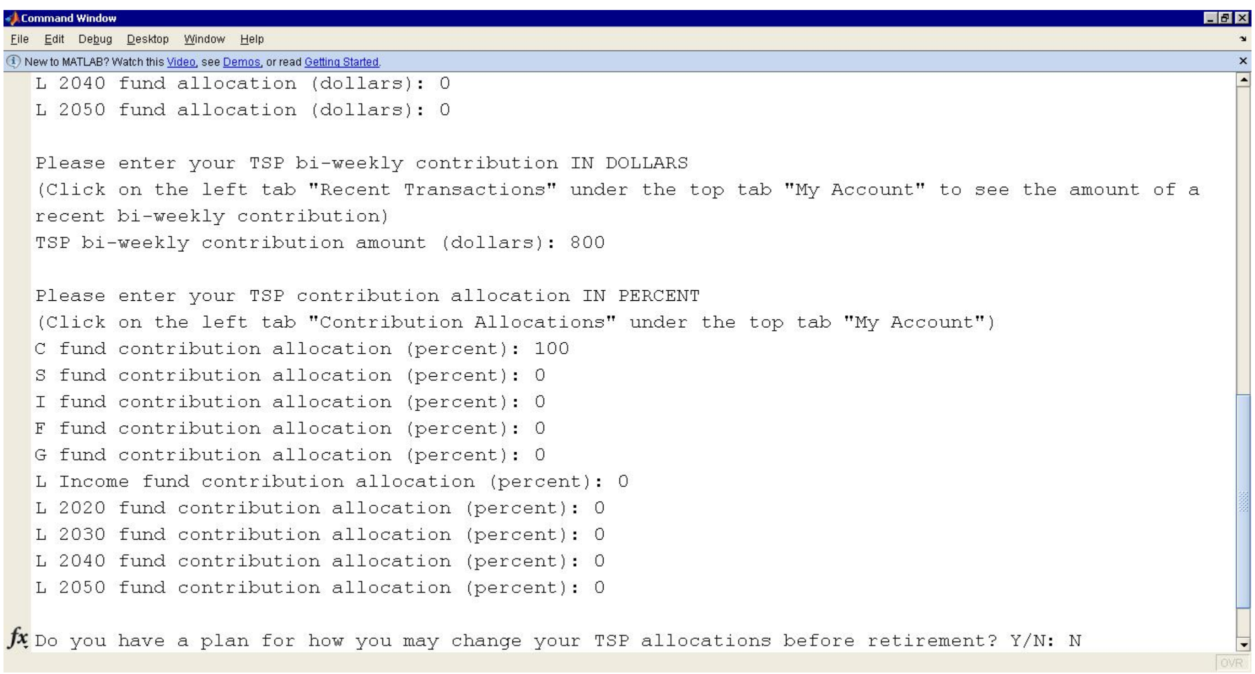


Figure 4: Probability density function histogram of terminal income for 15-year horizon C fund portfolio (sample means with 1 month blocksize)

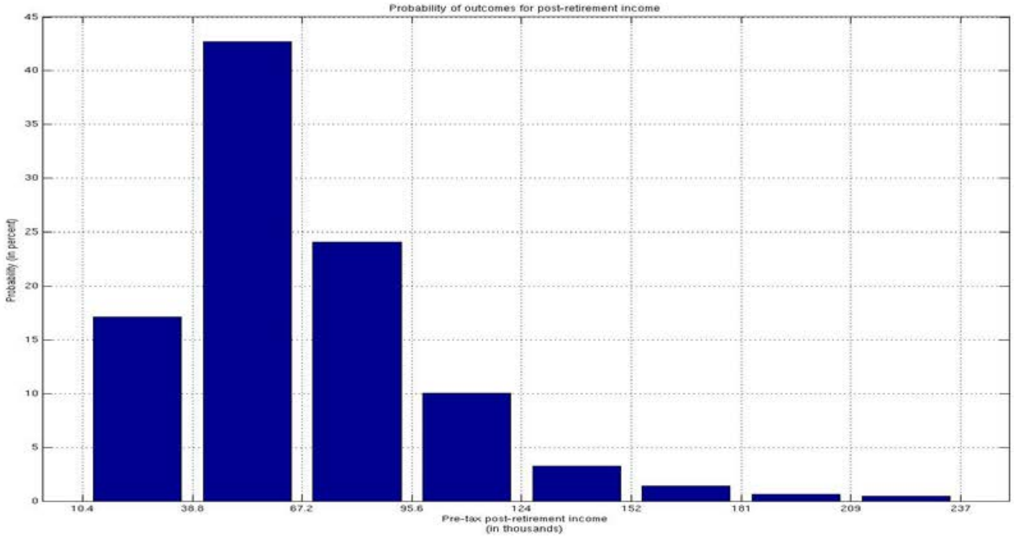
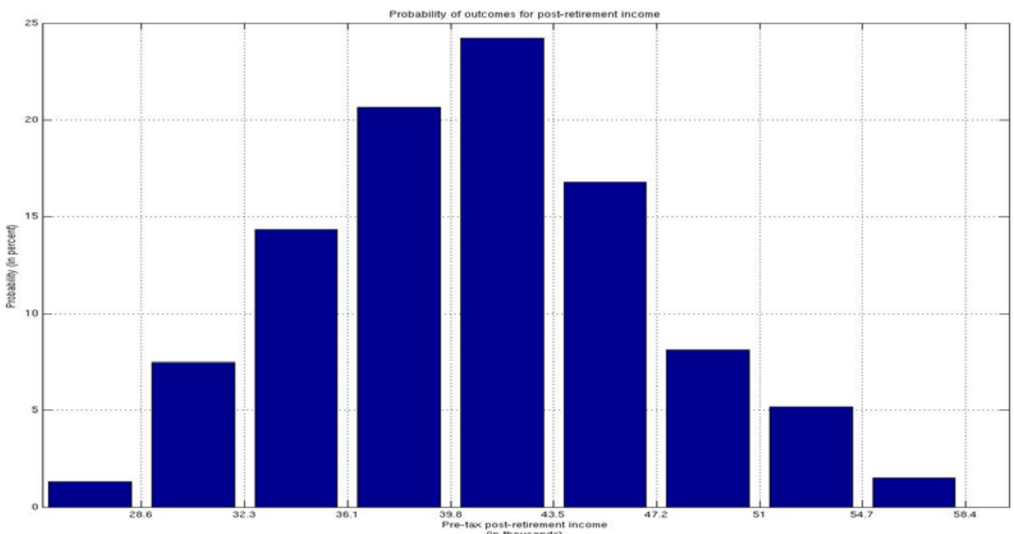


Figure 5: Probability density function histogram of terminal income for 15-year horizon G fund portfolio (sample means with 1 month blocksize)



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