Economics Note: The Distribution of Leveraged ETF Returns

Division of Economic and Risk Analysis[1] ***November 2019***

I. Introduction

Leveraged exchange-traded funds ("LETFs") seek to generate returns that are equal to a multiple, inverse, or inverse multiple of the return on a particular index or benchmark over a short period of time, typically one trading day.[2] These funds allow investors to obtain levered (or inverse) exposure to an underlying asset class without using brokerage margin accounts or engaging in more complex trading strategies using futures or options. Therefore, LETFs can offer a cost-effective, easily accessible, and generally liquid tool to increase or decrease such exposure. At the same time, LETFs employ dynamic trading strategies to achieve their objectives and, when held over longer periods, can have returns with complex properties, similar to those of options. As a result, investor protection concerns regarding these funds have been raised.[3]

Investors who hold LETFs over longer holding periods can experience returns that deviate significantly from an LETF's target multiple due to compounding.[4] We analyze the performance of LETFs by: (a) deriving the theoretical distribution of the investment returns for a buy-and-hold strategy in an LETF; and (b) estimating the empirical distribution of investment returns for a hypothetical LETF by repeatedly sampling historical S&P 500 Index returns. While the theoretical approach allows us to study the entire return distribution of an LETF and the effect of different values for leverage and investment horizon on that distribution, such an analysis is only possible under a set of simplifying assumptions about the underlying index return distribution. Conversely, while the empirical analysis allows us to rely on the historical distribution of the underlying index returns, and thus can capture more complex properties such as "fat

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[1] The Staff of the Division of Economic and Risk Analysis of the U.S. Securities and Exchange Commission composed this note. The Commission has expressed no view regarding the analysis, findings, or conclusions contained herein. Nor has the Commission approved or disapproved its content.

[2] For example, an ETF employing leverage to double the daily return of the S&P 500 Index would aim to return an investor 2% on a day the S&P 500 increases by 1%. Similarly, an inverse ETF employing leverage to return twice the inverse of the S&P 500 would aim to return to an investor 2% on a day when the S&P 500 *declines* by 1%.

[3] See, e.g., SEC Chairman Jay Clayton's public statement on "Taking Significant Steps to Modernize our Regulatory Framework" (Sept. 26, 2019), available at https://www.sec.gov/news/public-statement/clayton-2019-09-26-three-rulemakings.

[4] Because of potential confusion among investors about the performance objectives of LETFs and inverse LETFs, the SEC and FINRA issued an "Investor Alert" advising investors that "because leveraged and inverse ETFs reset each day, their performance can quickly diverge from the performance of the underlying index or benchmark. In other words, it is possible that you could suffer significant losses even if the long-term performance of the index showed a gain." See "Leveraged and Inverse ETFs: Specialized Products with Extra Risks for Buy-and-Hold Investors." (Aug. 1, 2009) Available at: https://www.sec.gov/investor/pubs/leveragedetfs-alert.htm.

tails", it does not allow us to analyze how the properties of LETF returns vary with an LETF's leverage and the investment horizon as precisely.

Our analysis of the theoretical distribution of long-term LETF returns shows that, under certain simplifying assumptions, the likelihood of experiencing losses from a long-term investment in an LETF increases with leverage, while the magnitude of potential gains, when they do occur, also increases with leverage. The returns to holding an option have similar characteristics. Our subsequent empirical analysis using randomly sampled S&P 500 Index returns suggests that the derived theoretical distribution provides a good approximation to levered ETF returns on the S&P 500 Index.

II. Summary of the Exchange-Traded Funds Industry

The ETF industry has experienced extensive growth since the first U.S. ETF began trading in 1993. Since 2007, the average growth rate in the number of ETFs has been 10% annually and total net assets of ETFs have increased 15% annually; as of September 2019 there were 1,910 ETFs with total net assets of $3,081 billion. LETFs started trading in 2003 and, as of September 2019, there were 164 LETFs with $33.9 billion in total net assets (comprising approximately 1% of all ETF AUM).[5]

III. Theoretical and Empirical Return Distributions for LETFs

Theoretical Distribution of LETF Returns

Using simplifying assumptions, we derive the return from a buy-and-hold investment in an LETF as a function of the leverage multiple and holding period.[6] These simplifying assumptions allow us to understand the first-order effects of investing in an LETF that is rebalanced to achieve a constant daily leverage multiple. We assume daily log-returns r_{t+1} of the underlying index between period *t* and period *t+1* are independent and identically distributed normal variables:

$$r_{t+1} \sim \mathcal{N}(\mu, \sigma^2)$$

[5] Form N-CEN data.

[6] Our assumptions are that (i) daily returns for an underlying index are independent and identically distributed log-normal (there are no jumps and no serial correlation) and (ii) that there are no transaction costs that would impact the performance of an LETF because of daily rebalancing. Furthermore, we use a standard approximation to get the log return of the levered portfolio. This approximation holds perfectly in continuous time and reasonably well over short time horizons. While we could analyze how LETF returns vary with underlying asset volatility, varying the volatility of the underlying asset also typically changes its expected return (e.g., bonds are less volatile than stocks but earn lower expected returns). Therefore, we focus on how the leverage multiple and holding period affect LETF returns because these factors are independent of the underlying asset being studied.

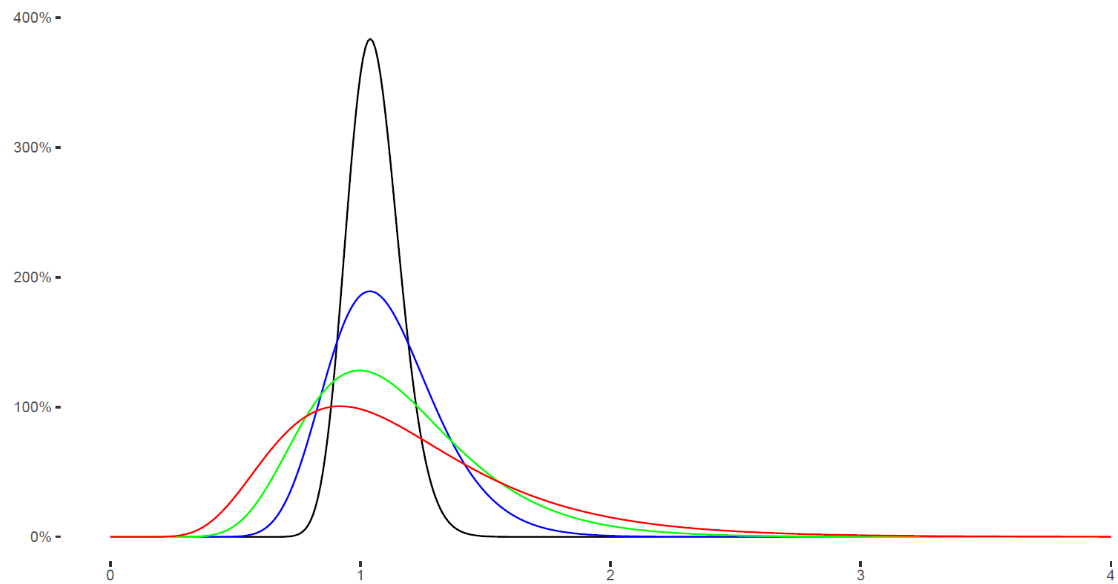

Figure 1: Theoretical distributions (density functions) of gross returns (the payoff from a $1 initial investment) over a 6-month investment horizon for the underlying index *(Black)* and LETFs with leverage multiples of plus two (*Blue*), plus three (*Green*), and plus four (*Red*), respectively.

This assumption implies that the log *k*-period return from period *t* to period *t+k* on an *l*-times levered asset is also approximately normal:

$$r^l_{t,t+k} = l\sum_{i=1}^{k} r_{t+i} + \frac{k}{2}l(1-l)\sigma^2 = lr_{t,t+k} + \frac{k}{2}l(1-l)\sigma^2 \sim \mathcal{N}(kl\mu + \frac{k}{2}l(1-l)\sigma^2, kl^2\sigma^2)$$

Because many LETFs use the S&P 500 Index as an underlying index, we calibrate the parameters of our theoretical model to match the annual mean and standard deviation of S&P 500 Index returns. This allows us to analyze the effects of different values for the leverage (*l*) and holding period (*k*) parameters on the distribution of LETF payoffs.[7] Figures 1 and 2 graph the distribution of gross returns—the payoff from a one-dollar initial investment—for various levered and inverse-levered ETF structures over a six-month horizon.[8] As leverage increases, the probability mass of the return distribution shifts to the left. At the same time, the mass allocated to the right tail of the distribution also increases. These changes reflect

[7] Based on S&P 500 index historical returns, we assume an average annual return of 6% and an annual volatility of 15% for the underlying index.

[8] More precisely, the figures show probability density functions that can be used to evaluate the probability that an outcome (*x* axis) falls within a particular range of values. This probability is given by the integral of the density (*y* axis) over that range. When the *y* axis has values larger than 100%, this does not mean the associated outcome has a probability of more than 100% of happening. Rather, the area under the curve indicates the probability associated with a set of outcomes, and the total area under a given curve is always equal to 1.

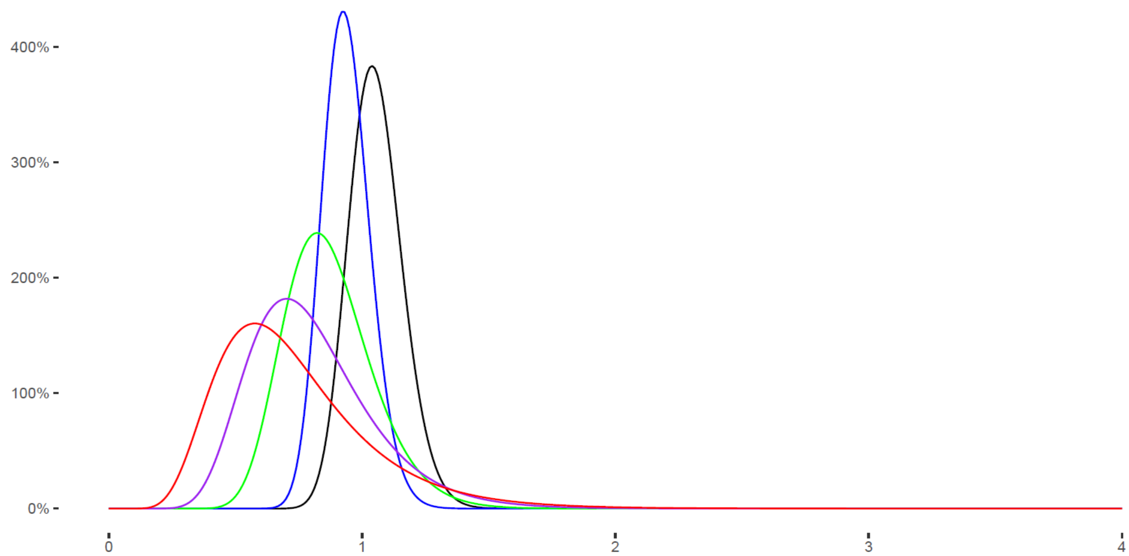

Figure 2: Theoretical distributions (density functions) of gross returns (the payoff from a $1 initial investment) over a 6-month investment horizon for the underlying index *(Black)* and inverse LETFs with leverage multiples of minus one (*Blue*), minus two (*Green*), minus three (*Purple*), and minus four (*Red*), respectively.

the increased skewness[9] of the return distribution: negative returns become more likely, while positive returns, when they do occur, tend to be larger in magnitude. While we show results for a fixed investment horizon and various leverage multiples, fixing leverage and varying the investment horizon has a similar effect: as the investment horizon increases, so does the skewness of LETF returns.

Empirical Distribution of LETF Returns

The theoretical analysis above relies on simplifying assumptions to derive the return distribution of an investment in an LETF over longer holding periods. To relax these assumptions, we simulate LETF returns based on randomly sampled historical S&P 500 Index returns and analyze their distribution for various leverage multiples.

Specifically, using daily return data for the S&P 500 Index from January 1964 to July 2017, we simulate 100,000 daily index return price paths over investment horizons ranging from one month to one year.[10] Then, for each simulated return price path, we compute the holding period return of the index and the implied holding period return for daily-rebalanced LETFs with leverage ratios from minus four to plus four.

Figures 3 and 4 below are the empirical analogs of Figures 1 and 2 above, and are qualitatively similar to the theoretical results in the previous section: as the magnitude of the leverage ratio increases,

[9] Skewness is a measure of asymmetry in a statistical distribution reflecting the degree to which the distribution curve is skewed to the left or right of the mean of a variable.
[10] We generate price paths by randomly sampling daily S&P 500 Index returns with replacement. This method captures certain features of the historical return distribution (e.g., non-normality), but does not capture any time-series correlation in returns.

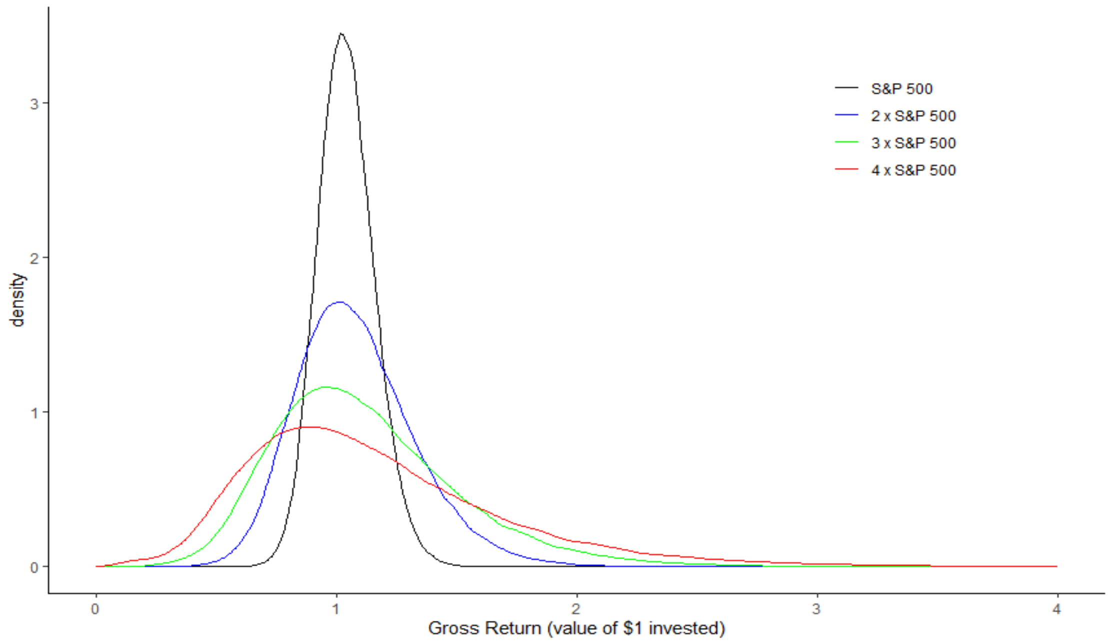

Figure 3: Empirical distributions (density functions) of gross returns (the payoff from a $1 initial investment) over a 6-month investment horizon for the underlying index *(Black)* and LETFs with leverage multiples of plus two (*Blue*), plus three (*Green*), and plus four (*Red*), respectively.

negative returns become more likely, while positive returns, when they do occur, tend to be larger in magnitude. In Figure 5, we fix the leverage ratio to plus four and show the effect of holding an LETF over holding periods ranging from one month to a year. These empirical distributions show that increasing the amount of time an LETF is held has an effect that is similar to increasing the magnitude of the leverage ratio in Figures 3 and 4, consistent with our theoretical results.

Investor Preferences for LETFs

Risk-averse investors generally prefer higher positive skewness in returns and higher expected returns, while having an aversion to volatility and other even moments, such as variance and kurtosis. However, since the leverage multiple and the investment horizon both increase skewness while simultaneously decreasing the Sharpe ratio of an LETF investment, it is not *a priori* clear that investors ought to disfavor LETFs only because they do not deliver the leveraged multiple of a correspondingly levered but non-rebalanced investment.[11] In other words, investors might prefer the higher skewness provided by holding an LETF over longer horizons even if it means they expect lower risk-adjusted returns and do not necessarily receive the LETF's daily leverage multiple.

[11] A mathematical analysis of how these moments vary with an LETF's leverage multiple and the investment horizon is available in a separate technical appendix.

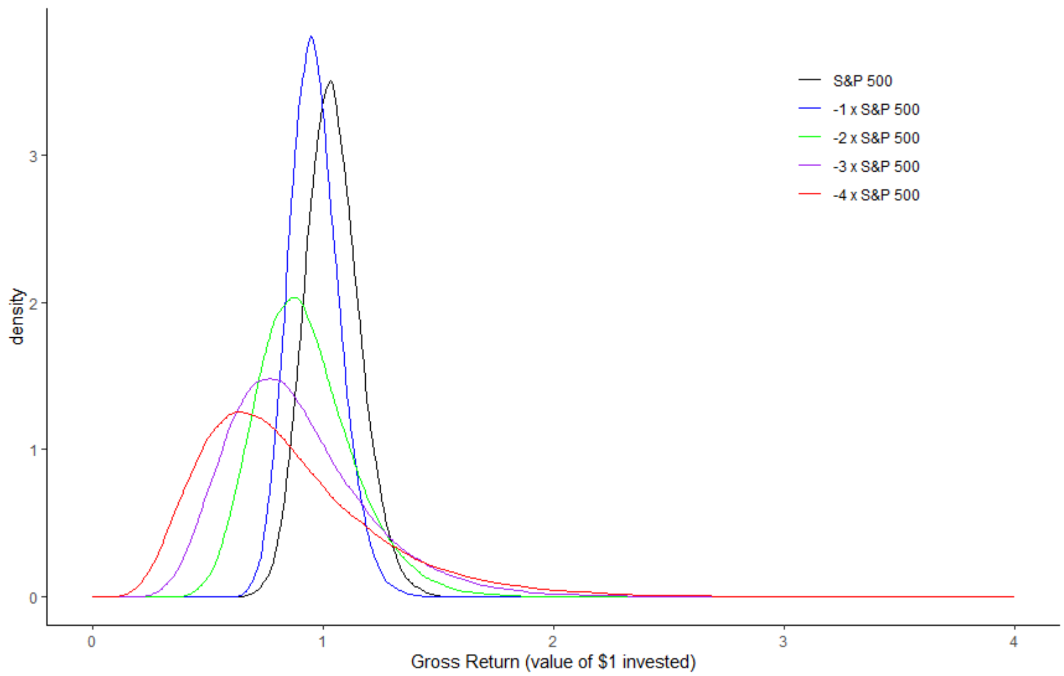

Figure 4: Empirical distributions (density functions) of gross returns (the payoff from a $1 initial investment) over a 6-month investment horizon for the underlying index *(Black)* and inverse LETFs with leverage multiples of minus one (*Blue*), minus two (*Green*), minus three (*Purple*), and minus four (*Red*), respectively.

IV. Similarities Between LETFs and Options

As discussed above, both the leverage and the holding period of an LETF increase the skewness of the payoff distribution while decreasing the expected payoff per unit of volatility. Similarly, just as a long-term investment in an LETF becomes more likely to pay off only a fraction of the initial investment as the daily leverage multiple increases, the probability of an option paying off nothing at maturity increases with the degree to which the option is out-of-the-money.

For example, a call option's payoff is similar to that of the underlying stock when the option is deep in the money. However, as the strike price increases, a call option investment becomes more levered, the mass of its return distribution shifts to the left, and the probability of rare large payoffs in the right tail of this distribution increases.[12] This is similar to the effect of increasing an LETF's leverage multiple or the period over which the LETF is held. As the option becomes more out of the money, it has a payoff that is zero most of the time, resulting in an investment return of -100%.[13] However, in the unlikely event of a

[12] These similarities are with respect to call options. An analysis with respect to put options would be identical, but the comparative statics are reversed.
[13] Note that some unlevered investments in equity or debt markets, such as distressed firm debt or unprofitable growth stocks, can also exhibit payoff distributions that are similar to out-of-the-money options.

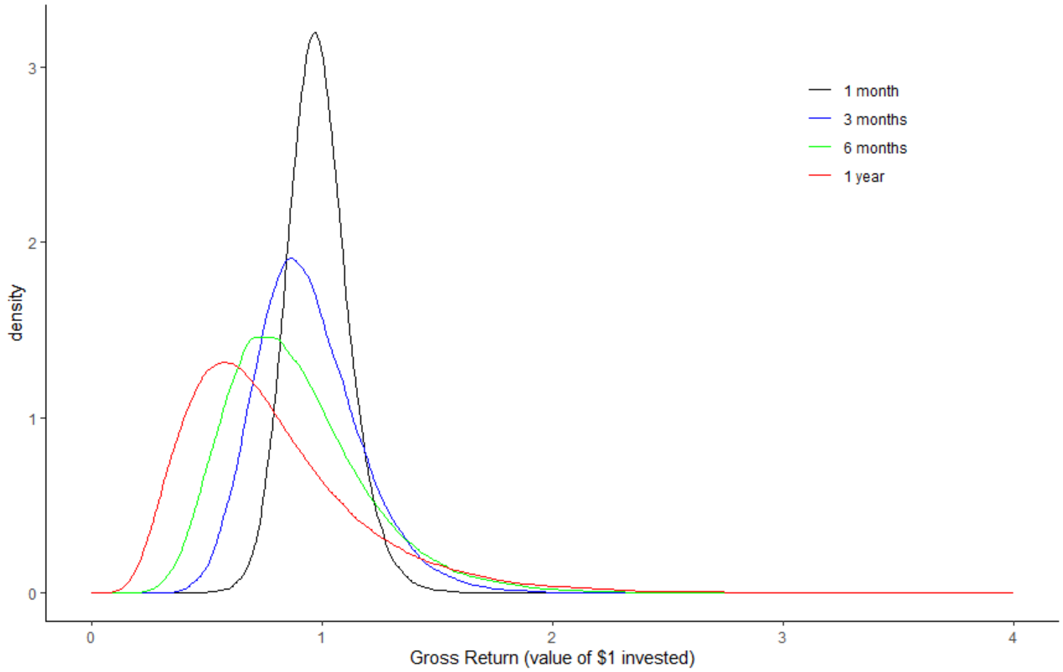

Figure 5: Empirical distributions (density function) of gross returns (the payoff from a $1 initial investment) for a four times levered LETF for investment horizons of one month (*Black*), three months (*Blue*), six months (*Green*), and one year (*Red*), respectively.

non-zero payoff, the investor is likely to receive a large positive return on the investor's initial investment. As discussed above, under certain assumptions, investors may value the positively skewed payoff characteristics of LETFs and may value these same characteristics in options, despite the fact they may require investors to accept a high likelihood of low payoffs.

Figure 6 shows how the empirical distribution of the holding period returns on an S&P 500 call option changes with the strike price of the call option. As with the LETF distributions in Figures 1 and 3, increasing a call option's leverage by increasing its strike price shifts probability mass to the left, increasing the likelihood of negative payoffs. At the same time, the probability of rare but extremely high payoffs increases with the call option's leverage. Comparing Figure 6 to Figures 1 and 3, the effect of leverage on call options manifests more suddenly, which is to be expected given the non-linearity in their payoffs. Finally, it is worth noting that an option contract can be replicated by a dynamic trading strategy in the underlying asset or index, while LETFs are also dynamic trading strategies in the underlying index or benchmark.

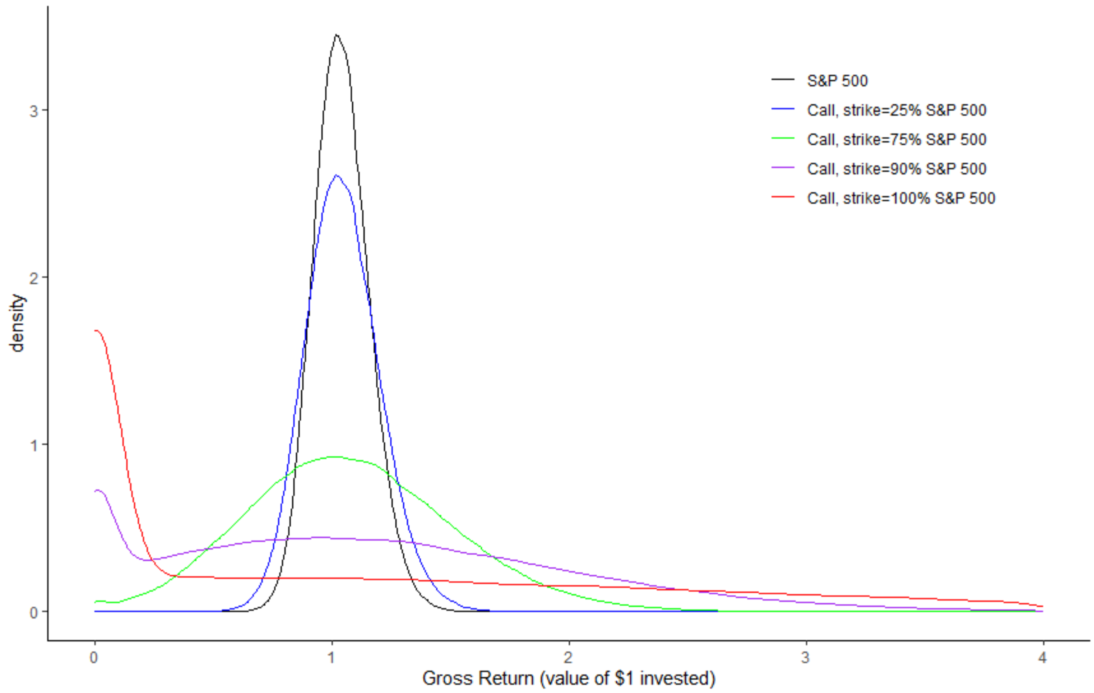

Figure 6: Empirical distributions (density function) of gross returns (the payoff from a $1 initial investment) for the underlying index *(Black)* and 6-month call options on the S&P 500 with varying strike prices. The strike prices correspond to 25% (*Blue*), 75% (*Green*), 90% (*Purple*), and 100% (*Red*) of the value of the S&P 500 at purchase. The initial call price is obtained using the Black-Scholes formula assuming a risk-free interest rate of 5% and an annual volatility of 15%.

V. Conclusion

The above analysis shows that the distribution of LETF returns becomes more skewed as their leverage multiple increases: the likelihood of experiencing losses from a long-term investment in an LETF increases, while the magnitude of potential gains, when they do occur, also increases. These features of LETF returns are similar to those of options, whose skewness increases with the extent to which in option is out of the money. Just as investors may need a higher level of sophistication to understand the return characteristics of options, they may also need a higher level of sophistication to understand the returns of LETFs over longer holding periods. While a broker-dealer accepting a customer's order for options is subject to FINRA account approval and due diligence requirements[14], similar requirements for transactions in LETFs currently do not exist.

[14] *See, e.g.*, FINRA rule 2360(b)(16), (17) (requiring for options accounts, firm approval, diligence and recordkeeping).