Banks and Settlement

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Abstract

We present a two village economy in which entrepreneurs face transportation costs when they take investment to the next village to buy inputs for production. Banks can issue chits or bank claims that can be used in lieu of legal tender. Bank money is superior to specie because it is costless to transport. In the absence of uncertainty, we characterize real output and capital flows in the decentralized economy and in economy in which there is a bank in each village. If the productive opportunities and wealth in both villages is the same, we illustrate that in spite of an advantage in making payments, banking equilibria are characterized by lower real activity and positive bank profits. In order to reduce settlement fees, banks have an incentive to match investment. If the villages are asymmetric, so that the wealthy village has worse production possibilities, we show that banks do fulfill their role of aiding efficient capital flows but the coordination inefficiency that arises from the settlement fees still exists.

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1 Introduction

A bank can be viewed as an accounts based payments system. In the wake of recent problems with banks' assets, this vital economic function has not received much attention. However, the global economy is supported by a smooth payments and settlement system. The banking sector is an important part of this; by allowing agents to transfer value without resorting to legal tender, they permit swift and efficient settlement of retail and wholesale obligations. For this reason, monetary policy is frequently implemented through the banking sector. What are the real effects of having a bank that can both settle claims and provide loans?

We present a stylized economy with two villages, two banks and two entrepreneurs. Each entrepreneur has to travel to the next village in order to acquire inputs that are essential to his production. He could pay for his inputs with legal tender, but transporting it is costly and so there is a role for a bank who provides credible claims. We consider two cases, one in which both villages are identical so that the only benefit a bank could have would be to reduce the cost of transferring specie. Second, we consider a case in which there is both a wealth and productivity difference in the two villages: Specifically the one with the better project has less wealth. In both cases, we characterize the equilibrium settlement fees banks charge and the equilibrium level of investment they induce.

We find that even though banks have an advantage in settlement services (in that they can issue "promises to pay" that are costless to transport), they may induce less than the efficient level of investment. The intuition for this result is as follows: For the entrepreneur or anyone else operating in a specific village, the opportunity cost of inputs is simply the gold coin less the transportation costs. For a bank, as they can issue paper money, the cost of issuing gold becomes positive if they issue paper claims in excess of their counter party bank. Therefore, there can be equilibria in which banks choose lower lending amounts because they want to avoid being a net debtor in the inter-bank market.

There is no uncertainty in our model, and so our effects arise solely through banks' optimal actions. We also do not have moral hazard on the part of the entrepreneur, so our model is absent the notion that bank capital is special and that bank loans add value through monitoring the recipient. To some extent, the idea that banks provide settlement serves and so have more information about economic agents is a natural precursor to the assumption that they have private information or can monitor more effectively than other investors.

The history of early banking is focuses on intermediaries' role in transferring value and providing a payments system. (See Kohn (1999) and Speight et. al (2006) for a description of early banking and settlement systems.) Both mention the extensive Medieval fair at Champagne in which trading was divided into the early sale of cloth and the later sale of spices. Merchants from Flanders sold cloth which was purchased by the Italians, whereas the spice importers were the Italians who sold to the merchants from Flanders. Payments were effectuated by transfers of credits through banks, and were not necessarily backed by gold coins. In effect, the banks facilitated a complicated barter arrangement between differing pairs of traders. This arrangement mirrors a net settlement system.

Kahn and Roberds (2009) provide an introduction to the economics of payment and settlement systems in the modern economy. They distinguish between wholesale or large-value settlement between two banks and retail or small-value settlement between a bank and a household. As they point out, both sets of payments have been growing rapidly over time—in 2004, in the U.S. approximately $75 in payments were recorded for every dollar of GDP.

As our banks operate across two markets (the local and foreign market), the structure of the economy is similar to a two-sided market as surveyed in Rochet and Tirole (2006). Competition for deposits and the non-existence of two sided Bertrand equilibria are presented in Yanelle (1997).

2 Model

Consider an economy with two goods: specie or coin and input goods. The entire population is divided into two villages that we denote A, B. Each village $i = A, B$ has a mass λ_i of investors, each of which has one gold coin. In addition, in each village i there is a penniless entrepreneur who has access to an investment technology described by a production function $\theta_i f(k)$, where k represents the input good. Here, $f(k)$ is strictly increasing and concave and common to both villages. In contrast, the productivity parameter θ_i, $i = A, B$, is village specific. The technology directly produces gold. Gold is consumable and provides utility. The input good can be used by an entrepreneur in production, but is not directly consumed.

The basic friction in the model is that villages are some distance apart and that transporting gold is costly. Each piece of gold costs $1 - \alpha$ to ship. This is a friction because the entrepreneur in village A requires the input good provided in village B and vice versa. As each is unknown in the foreign village, he cannot raise funds there. We normalize the prices of the input goods to 1. The economy is depicted in Figure 1 below.

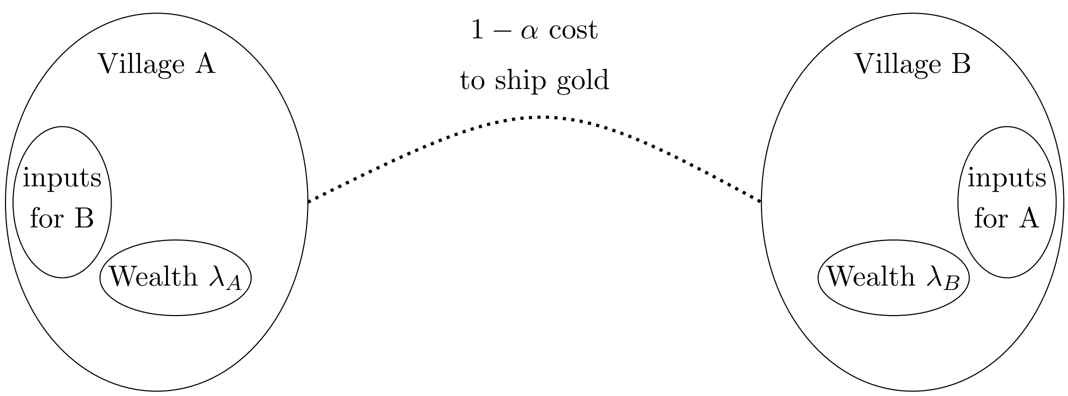

Figure 1: **The Village economy**

In the absence of any financial intermediaries, the sequence of events is as follows. In the morning, all village burghers meet in their respective central square. The entrepreneur makes a take-it-or-leave it offer to the village burghers. In return for gold, he promises to pay them from his production. As villagers are identical, they are either all willing to accept the offer or all reject the offer. In the former case, the entrepreneur chooses how much gold to acquire. Each investor is promised a payment at the end of the day, so we call the aggregate amount acquired by the entrepreneur a loan.

After the morning session is concluded, the entrepreneur takes his specie and goes to the next village. Transporting gold is costly: For each piece of gold carried from one village to the other, an amount $1 - \alpha$ is consumed in transportation costs. When he arrives at the next village, he uses the remaining gold to buy inputs for his project and returns home. The cost of transporting inputs back to the entrepreneur's village is normalized to zero. After returning home, the entrepreneur executes his project. The output is measured in gold coins.

In the evening, after the project is completed, the entrepreneur returns to the village square and shares his proceeds, at which point all residents consume their share. Their utility is equivalent to their evening meal (i.e., utility is equivalent to wealth). We refer to this entire scenario as the decentralized outcome.

While entrepreneur A is away in village B, entrepreneur B comes to village A to purchase his inputs. As a result of that transaction, suppliers in A obtain gold from entrepreneur B. This gold adds to their stock of gold that is consumed at the end of the day.

Finally, any of the investors can found a financial intermediary, which we call a "bank." Gold may be deposited in the bank, which issues paper chits that constitute proof of claims. There are benefits to such claims. First, they are costless to transport between villages. By reducing transportation costs, banks effectively issue liquid claims. Second, they can be verified (i.e., bank statements can be issued.)

A bank in village i can raise deposits from the citizens of village i in the morning. Note that deposits cannot be raised in the afternoon. The bank offers to repay d on each gold coin deposited. The opportunity cost of deposits for the citizens is 1 (the alternative is to eat the coin at the end of the day). Therefore, in equilibrium, if there is no uncertainty on repayment, the deposit rate with a monopolist bank is $d = 1$. Observe that in the decentralized outcome, the entrepreneur makes the same offer to investors. The entrepreneur is a monopolist on his own project, and offers to repay each investor exactly one coin in the evening.

If banks are in operation, at the end of a day banks across the two villages net out their claims. Let $\gamma = k_a - k_b$ be the net gold owed by bank A to bank B (here, γ can be positive or negative). An amount γ must be delivered by bank A to bank B if it is positive, and by B to A if it is negative. Transporting gold for banks is costly and $1 - \alpha$ is lost.[1] For now, we do not allow either banks or entrepreneurs to default on their obligations.

Figure 2 illustrates some of the flows of chits and gold in the economy. First, a bank provides the entrepreneur in its own village with chits. The amount of the chit is denoted as k_i for entrepreneur i. Entrepreneur i takes the chit to the bank in the next village (j). Bank j charges him a settlement fee $1-\alpha$ per unit, and issues him with local (i.e., village j) chits in the amount of αk_i. The entrepreneur uses this to buy inputs for his technology which he produces in his home village. The vendors or input suppliers in village j return the chits to the bank at the end of the day and receive an aggregate payout of αk_i in bullion. The same sequence of events occurs in village j.

The contract between the entrepreneur in village i and bank i involves the bank issuing a chit to the entrepreneur in the morning, and the entrepreneur promising to deliver a quantity of gold in the evening. Since there is a single repayment and there is no randomness, we continue to describe the transaction as a loan.

2.1 The Planner's Problem

Consider a central planner maximizing total welfare across the two villages. He can marshal the resources in each village to recompense the input suppliers of that village. By doing so, he can mitigate transport costs of gold. He only incurs the latter if he wishes to commandeer the inputs in a village

[1] The modern equivalent of this settlement fee is the federal funds rate.

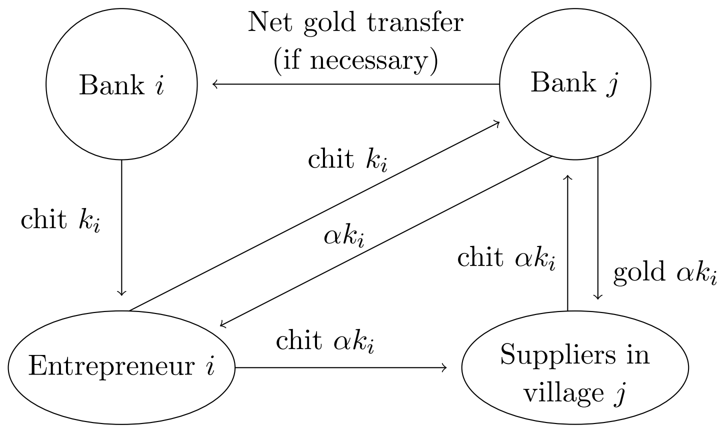

Figure 2: **Chit and Gold flows**

beyond the total value of its wealth, in which case he would have to move gold from the other village. Our planner can therefore be interpreted as a benevolent "multi-branch" bank or alternatively as a regulated banking system.

The objective function of the planner is represented as:

$$\Psi = \sum_{i=a,b} [\theta_i f(k_i) - k_i] - \frac{1-\alpha}{\alpha}[(k_a - \lambda_b)^+ + (k_b - \lambda_a)^+], \quad (1)$$

where $X^+ = \max\{X, 0\}$. His aggregate resource constraint is:

$$\sum_{i=1}^{2} k_i + \frac{1-\alpha}{\alpha}[(k_a - \lambda_b)^+ + (k_b - \lambda_a)^+] \leq \sum_{i=1}^{2} \lambda_i \quad (2)$$

Here, k_i is the amount invested in village i. If $k_i < \lambda_j$, the planner has to transport gold from village i. A total of $k_i - \lambda_j$ must be available to invest after the transportation cost has been paid. Therefore, a total of $\frac{k_i - \lambda_j}{\alpha}$ must be transported from village i to village j, incurring a cost $\frac{1-\alpha}{\alpha}(k_i - \lambda_i)$.

2.2 The Banking Problem

In contrast to the central planner, a bank in village i cannot command resources in the adjacent village. It can, however, issue a chit to entrepreneur i that can be used in lieu of gold that is recognized by the bank in the other village j. In addition, if approached by entrepreneur j, it can issue him local chits that he can use to purchase inputs from the local merchants in village i. The merchants can then deposit the chits and recover gold.

First, the bank in village i contracts with the entrepreneur in the same village. As the latter has the outside option of going directly to the local investors, the bank has to ensure that he receives at least this reservation payment. After this, the bank effectively owns the project and becomes the residual claimant. It issues a chit to the entrepreneur who travels to the next village. On presentation of his chit to the foreign bank in village j, he will receive village j "bank money" which he can use to buy the necessary inputs.

The bank's payoff is essentially the amount of gold it has remaining in its vault at the end of the day. This payoff consists of up to four components. First, bank i owns the project of entrepreneur i. If $\tilde{k}$ is the amount invested by entrepreneur i, at the end of the day the project yields (in gold) an amount $\theta f(\tilde{k})$. Bank i writes a chit for k_i to entrepreneur i. When he cashes in the chit at bank j, the latter imposes a settlement fee $1 - s_j$ per unit. He is able to purchase inputs in the amount $s_j k_i$. Therefore, at the end of the day, bank i obtains $\theta_i f(s_j k_i)$ from its project.

Second, when entrepreneur j presents his chit to bank i, the latter charges a total settlement fee $(1 - s_i)k_j$. It issues entrepreneur j local chits in an amount $s_i k_j$ that are used to buy inputs in village i. At the end of the day, local suppliers in village i turn these chits in to bank i, which must pay out in gold. This component of payoff is therefore equal to $-s_i k_j$.

Third, if $k_i > k_j$, bank i has to transport enough gold to village j by the evening to ensure that the difference is available to bank j to pay out to its own claimants. That is, $k_i - k_j$ units of gold must reach village j in the evening. Bank i therefore has to send an amount $\frac{k_i - k_j}{\alpha}$ to bank j.

Conversely, if $k_i < k_j$, bank i receives an amount $k_j - k_i$ from bank j.

Finally, from the gold that bank i has at the of the day, it must turn over to the entrepreneur just enough to ensure that the entrepreneur is willing to accept a loan from the bank rather than directly raise money from the village population. That is, the entrepreneur must receive the same amount that he obtains in the decentralized outcome. Denote this amount by ϕ^d.

The bank's payoff function may then be written in terms of these four components:

$$\begin{aligned} \pi_i(k_i, s_i) &= \theta_i f(s_j k_i) - s_i k_j + 1_{\{k_i < k_j\}}(k_j - k_i) - 1_{\{k_i > k_j\}} \frac{k_i - k_j}{\alpha} - \phi^d \\ &= \begin{cases} \theta_i f(\alpha k_i) - k_i + (1 - s_i) k_j - \phi^d & \text{if } k_i < k_j \\ \theta_i f(\alpha k_i) - \frac{k_i}{\alpha} + (\frac{1}{\alpha} - s_i) k_j - \phi^d & \text{if } k_i > k_j. \end{cases} \end{aligned} \tag{3}$$

The bank chooses k_i and s_i. Observe that $\frac{\partial \pi_i}{\partial s_i} = -k_j < 0$. Therefore, it is optimal to set s_i as low as feasible; i.e., to increase the settlement fee as much as possible. The maximal value of the settlement fee is $1 - \alpha$ (if s_i is any lower, the entrepreneur will directly raise money and transport gold to village j). The optimal value of s_i when the bank is a monopolist is therefore α, so that the settlement fee is equal to the transportation cost of gold.[2]

Setting $s_i = s_j = \alpha$, the first derivative of π_i with respect to k_i is then:

$$\frac{\partial \pi_i}{\partial k_i} = \begin{cases} \theta_i \alpha f'(\alpha k_i) - 1 & \text{if } k_i < k_j \\ \theta_i \alpha f'(\alpha k_i) - \frac{1}{\alpha} & \text{if } k_i > k_j. \end{cases} \tag{4}$$

At $k_i = k_j$, the payoff function is kinked and therefore not differentiable. However, the marginal impact on payoff of increasing k_i is equal to $\theta_i \alpha f'(\alpha k_i) - \frac{1}{\alpha}$, and that of decreasing k_i is $\theta_i \alpha f'(\alpha k_i) - 1$.

In addition, each bank has a participation constraint: its overall payoff must be at least zero. That is, $\pi_i(k_i, s_i) \geq 0$ for each i. If this constraint is violated, bank i will refuse to participate.

[2] If there is competition in the banking sector in a village or if a regulator prevents such a high settlement fee, a bank may charge a settlement fee smaller than $1 - \alpha$ per unit. For now, we ignore these possibilities.

A banking equilibrium with positive lending is then defined by lending levels and net settlement quantities $(k_a^e, k_b^e, s_a^e, s_b^e)$such that (i) for each $i = a, b$ and $j \neq i$, the loan $k_i^e > 0$ and net settlement quantity s_i^e constitute a best response given k_j^e and s_j^e and (ii) $\pi_i(k_i, s_i) \geq 0$ for each i. As the optimal settlement fee for each bank is $1 - \alpha$ per unit, in what follows we fix $s_i^e = \alpha$ for each i, and focus on the lending levels.

3 Symmetric Villages

The simplest case to consider is the one in which the villages are perfectly symmetric. That is, the productivitty of the project is the same in both villages, so $\theta_A = \theta_B = \theta$. In addition, the wealth level is the same across both villages so that $\lambda_A = \lambda_B = \lambda$. In this case, the only benefit of a bank relative to the decentralized outcome is its ability to produce paper claims, both within its own village and across the village with some help from the next bank.

To provide a benchmark, we first characterize the decentralized outcome. Define $h(x) = f'^{-1}(x)$, so $h(x)$ recovers the capital level that corresponds to a particular marginal product of capital. Higher marginal products correspond to lower capital levels and so $h(\cdot)$ is a decreasing function.

Let $\hat{k}^d$ be the level of debt at which $\theta\alpha f'(\alpha k) = 1$, so that $\hat{k}^d = \frac{1}{\alpha} h\left(\frac{1}{\alpha\theta}\right)$. Then, $\hat{k}^d$ represents the unconstrained (or techonologically optimal) amount of the loan an entrepreneur will take out in the decentralized outcome. Let $k^d = \min\{\hat{k}^d, \lambda\}$, to incorporate the constraint that an entrepreneur cannot raise capital of more than λ in his own village. Recall that some of the acquired capital will be lost in transportation, so that the resulting investment level will be αk^d.

Lemma 1 *In the decentralized outcome with two symmetric villages,*

(i) Investors in each village i provide loans at a gross interest rate of 1.

(ii) The entrepreneur takes out a loan in the amount k^d.

(iii) The entrepreneur receives a payoff of $\phi_i^d = \theta f(\alpha k^d) - k^d$.

The utility that the entrepreneur gets from the decentralized outcome, ϕ^d, is the minimum payoff that bank i has to provide him in a banking equilibrium. If the bank fails to do so, he will reject the bank's contract and raise money directly from the investors in his village.

Another natural benchmark to consider is the solution to the planner's problem. Recall that we consider a planner who can commandeer resources in a village but has to move gold if the amount that he uses is larger than the reserves in the village. Let the superscript ^{p} denote the planning solution.

Lemma 2 *If the two villages are symmetric, then*

(i) The planner sets $k_a^* = k_b^* = k^p$*, where* $k^p = \min\{\lambda, h(1/\theta)\}$.

(ii) The planning surplus is $\Psi^* = 2\theta f(k^p) - k^p$.

As the villages are symmetric, it is intuitive that the planner sets the production in each village to be the same. The planner thereby avoids any transaction cost of transporting gold across villages. Note also that whereas in the decentralized outcome, the investment level is strictly less than the loan granted, in the planner's case these two amounts are the same.

3.1 Symmetric Banking Equilibrium

Now, consider the case in which two banks are formed, one in each village. We want to consider the effect of inside bank money on the real economy, and so we assume there is no moral hazard on the part of the entrepreneur. Specifically, we assume entrepreneur i must invest the entire remaining amount of bank loan i after he has paid the settlement fee to bank j. Recall that the optimal settlement fee for each bank is $1-\alpha$, so the entrepreneur invests αk_i.

Let $\hat{k}^\alpha$ be the loan level at which

$$\theta \alpha f'(\alpha k) = \frac{1}{\alpha}, \tag{5}$$

so that $\hat{k}^\alpha = \frac{1}{\alpha} h\left(\frac{1}{\theta\alpha^2}\right)$. Recall that when λ is high, the decentralized lending level is $\hat{k}^d$, which satisfies $\theta\alpha f'(\alpha k^d) = 1$. As $h(\cdot)$ is decreasing and $\frac{1}{\alpha} > 1$, it follows that $\hat{k}^\alpha < \hat{k}^d$.

We assume that each village has abundant resources. Specifically,

Assumption 1 *(i) λ is sufficiently high so that $\hat{k}^d \leq \lambda$.*

(ii) $\theta f(\alpha\hat{k}^\alpha) - \alpha\hat{k}^\alpha \geq \phi^d$.

Part (a) of Assumption 1 ensures that there is enough financial capital in the economy to achieve a decentralized lending level corresponding to $\hat{k}^d$. Part (b) ensures that a bank earns a non-negative payoff if both banks lend $\hat{k}^\alpha$.

Consider the best response of bank i, given that bank j lends an amount k_j.

Lemma 3 *If the villages are symmetric, then the best response of bank i, given the chosen funding level of bank j is given by*

$$k_i^*(k_j) = \begin{cases} \hat{k}^\alpha & \text{if } k_j \leq \hat{k}^\alpha \\ k_j & \text{if } k_j \in (\hat{k}^\alpha, \hat{k}^d) \\ \hat{k}^d & \text{if } k_j \geq \hat{k}^d. \end{cases} \tag{6}$$

Observe that bank lending exhibits strategic complementarity—the best response of bank i is weakly increasing in the loan of bank j. The complementarity is directly traceable to the transportation cost of gold, which generates a wedge in the marginal benefit of increasing and decreasing the loan size when $k_i = k_j$. There is a benefit to each bank to issuing the same number of claims as the bank in the next village: It avoids the cost of transporting gold between the villages. As a result, over some range, it is optimal to set $k_i = k_j$.

However, if the other bank chooses an extremely low capital level, bank i is willing to incur the cost of transportation because the marginal productivity of capital is sufficiently high. Indeed, it will lend and incur the transportation cost up to the point $\hat{k}^\alpha$, at which point the increase in production generated by an extra unit of capital is exactly offset by the increase in transport costs of that unit. Above the value $\hat{k}^{\alpha}$, the production benefit from increasing k is smaller than the transport costs they might incur and so banks perfectly match their counterpart in the next village. Finally, at some point (the decentralized level $\hat{k}^{d}$), the bank has no incentive to invest further even if the other bank increases their capital level. The best response of bank i is depicted in Figure 3.

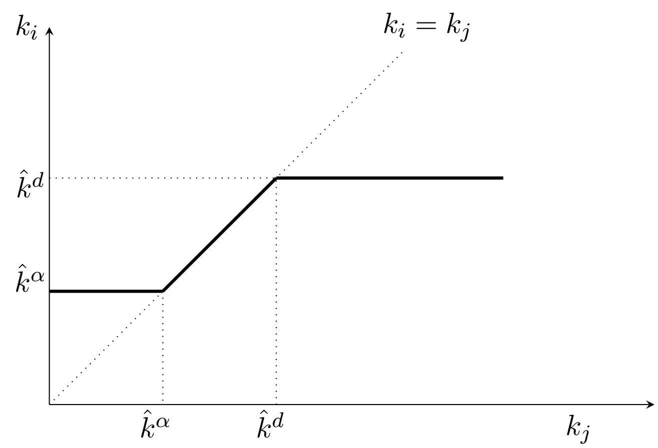

Figure 3: **Best Response of bank** i

Strategic complementarity in the best response functions of each bank in turn leads to multiple equilibria in the model. We focus on pure strategy equilibria. Each pure strategy equilibrium is symmetric, with $k_a^e = k_b^e$. From Figure 3, it follows that there is a range of funding levels over which banks wish to coordinate, leading to a continuum of equilibria.

Proposition 1 *For every* $k \in [\hat{k}^{\alpha}, \hat{k}^{d}]$*, there is a banking equilibrium in which each bank lends* k *to its entrepreneur. The investment level in each village is* αk*, and the bank earns a payoff* $\theta f(\alpha k) - \alpha k - \phi^{d}$*.*

We remark on several features of the banking equilibrium. First, in a symmetric equilibrium, no gold is transported between the villages. As a settlement system, therefore, the banks work perfectly—the entire deadweight cost of transporting gold is saved.

Second, observe that the *maximal* amount of lending exactly equals the decentralized level. Therefore, the settlement benefits do not translate to any benefit to the real economy. With monopoly banking in each village, the bank captures the entire increase in surplus generated by its existence. Indeed, the system corresponds to a moneylender equilibrium in a village—the moneylender charges high rates on both investment loans and on settlement transactions.

Third, the equilibrium with decentralized lending is only one of a continuum. In all other equilibria, projects are under-funded, so that despite the presence of a banking system, the activity in the real sector is lower than with a decentralized capital market. Of course, under Assumption 1 part (b), it follows that the overall welfare in the economy (i.e., the sum of the payoffs of entrepreneur and bank) is greater under banking. However, the benefits of banking do not spill over to the real sector.

In our model, a bank plays the dual roles of providing settlement and funding investment. More broadly, we argue that the settlement and investment funding roles of a financial system are inter-twined. Frictions in settlement (represented in our model by the settlement fee) carry over to the real sector, resulting in under-investment. The transportation cost represents a potential friction in settlement that is never incurred in equilibrium, but leads to a co-ordination problem between the banks. As a result, in the model, the presence of a banking sector can even reduce investment below the benchmark level of the decentralized outcome.

The equilibrium with the highest level of lending recovers the same level of activity in the real sector as achieved under decentralization. In this equilibrium, each bank lends $\hat{k}^d$, so the payoff to each bank is

$$\pi(\hat{k}^d, \alpha) \quad = \quad \theta f(\alpha \hat{k}^d) - \alpha \hat{k}^d - \phi^d = (1-\alpha)\hat{k}^d,$$

where the second equation follows from noting that $\phi^d = \theta f(\alpha \hat{k}^d) - \hat{k}^d$.

From Proposition 1, it is immediate that the equilibria are Pareto-ranked, with welfare increasing in k. The payoff of the bank strictly increases in k. In each equilibrium, investors make a zero payoff—they deposit one unit in the bank in the morning and recover it in the evening. The entrepreneur too is exactly indifferent across the equilibria, earning ϕ^d in each one.

Corollary 1.1 *When villages are symmetric, banking equilibria are Pareto-ranked: Higher loan amounts are preferred.*

In addition to a Pareto ranking, the equilibria differ in the amount of bank money in circulation. In this economy, there are two types of bank money in circulation; the claims that banks have on each other (similar to a Large Value Transfer System) and the claims that banks issue to an entrepreneur so that he can effect his purchases when he arrives at the input village. Recall, that entrepreneur i cashes his chits in with bank j and receives local currency sufficient to buy αk_i in inputs. The total bank money in circulation in the entire economy depends on the equilibrium that obtains.

Corollary 1.2 *The amount of bank money in circulation depends on the production capacity of the economy. For all $k \in [\hat{k}^{\alpha}, \hat{k}^{d}]$,*

i) There are $2k$ inter-bank obligations,

ii) There are $2\alpha k$ retail claims in the economy.

Therefore, real investment and claims outstanding are positively correlated. The relationship is, of course, not causal in the following sense. Suppose bank i increases its lending level beyond $\hat{k}^d$. A best response by bank j would be to increase its settlement fee above $1 - \alpha$ (in the decentralized outcome, the entrepreneur can only obtain inputs worth $\alpha \hat{k}^d$ in gold). That is, further lending will merely lead to inflation, with no additional real effect.

4 Asymmetric Villages

We now consider a case with asymmetric villages. In particular, we assume that $\lambda_a < \lambda_b$ but $\theta_a > \theta_b$. That is, village A is more productive but has less financial capital. One goal of the financial system is then to allow for capital to flow from the less productive to more productive village.

With asymmetry in the production technology, the thresholds $\hat{k}^\alpha$ and $\hat{k}^d$ are now village-specific (recall that each threshold depends on θ). For each i, define $\hat{k}_i^\alpha$ to be the lending level at which $\theta_i f(\alpha k) = \frac{1}{\alpha}$, so that $\hat{k}_i^\alpha = \frac{1}{\alpha}\, h\left(\frac{1}{\theta_i \alpha^2}\right)$. Further, let $\hat{k}_i^d$ be the lending level at which $\theta_i f(\alpha k) = 1$, so that $\hat{k}_i^d = \frac{1}{\alpha} h\left(\frac{1}{\theta_i \alpha}\right)$.

Observe that $\hat{k}_i^\alpha \leq \hat{k}_i^d$. Also, as we have assumed that entrepreneur A is more productive (i.e., $\theta_a > \theta_b$), it follows that $\hat{k}_a^\alpha > \hat{k}_b^\alpha$ and $\hat{k}_a^d > \hat{k}_b^d$. However, $\hat{k}_a^\alpha$ may be greater or less than $\hat{k}_b^d$, depending on the size of the difference $\theta_a - \theta_b$. That is, if θ_a is much larger than θ_b, we will have $\hat{k}_a^\alpha > \hat{k}_b^d$. Conversely, if θ_a is close to θ_b, it will be the case that $\hat{k}_a^\alpha < \hat{k}_b^d$.

Let $\hat{k}_i^s$ be the surplus-maximizing investment level in village i. That is, $\hat{k}_i^s$ satisfies $\theta_i f'(k_i^s) = 1$. We assume that village A is constrained in terms of financial resources, but village B is not. Specifically,

Assumption 2 *(i)* $\lambda_a \leq \hat{k}_a^\alpha$, *and* $\lambda_b \geq \hat{k}_b^d$. *That is, village A is capital-constrained but village B is not.*

(ii) $\theta f(\alpha \hat{k}_a^\alpha) - \hat{k}_a^\alpha + (1-\alpha)\hat{k}_b^d \geq \phi^d$. *That is, if bank A lends* $\hat{k}_a^\alpha$ *and bank B lends* $\hat{k}_b^d$, *bank A earns a non-negative payoff.*

(iii) $\sum_{i=a,b} \lambda_i \geq \sum_{i=a,b} \hat{k}_i^s$. *Collectively, there is enough capital in the two economies for the planner to fund each project to the first-best level.*

Let $k_i^\alpha = \min\{\lambda_i, \hat{k}_i^\alpha\}$ and $k_i^d = \min\{\lambda_i, \hat{k}_i^d\}$ for each i. The decentralized outcome is similar to the symmetric case: in village i, the entrepreneur raises capital k_i^d and invests αk_i^d. His payoff is $\phi_i^d = \theta_i f(\alpha k_i^d) - k_i^d$.

The planner funds investment in project i from the gold in village j. By assumption, there is enough capital in each village to achieve the surplus-maximizing investment level, so the planner sets $k_i^p = \hat{k}_i^s$ in each village i. The overall surplus in the first-best solution is then

$$\Psi = \sum_{i=a,b} [\theta_i f(\hat{k}_i^s) - \hat{k}_i^s]. \tag{7}$$

Consider the banking equilibrium. The best response of bank i given the loan of bank j is characterized in a similar fashion as in the symmetric case:

$$k_i^*(k_j) = \begin{cases} \hat{k}_i^\alpha & \text{if } k_j < \hat{k}_i^\alpha \\ k_j & \text{if } k_j \in [\hat{k}_i^\alpha, \hat{k}_i^d] \\ \hat{k}_i^d & \text{if } k_j > \hat{k}_i^d \end{cases} \tag{8}$$

In describing the banking equilibrium in the asymmetric case, we distinguish between two cases. If $\hat{k}_a^\alpha > \hat{k}_b^d$, there is a unique pure strategy equilibrium in which bank A lends $\hat{k}_a^\alpha$ and bank B lends $\hat{k}_b^d$. This equilibrium is characterized by a net transfer from bank A to bank B at the end of the day. Conversely, if $\hat{k}_a^\alpha < \hat{k}_b^d$, we recover the features of the symmetric equilibrium—any lending level $k \in [\hat{k}_a^\alpha, \hat{k}_b^d]$ is sustained as an equilibrium, with both banks lending k.

Proposition 2 *(i) Suppose $\hat{k}_a^\alpha > \hat{k}_b^d$. Then, there is a unique pure strategy equilibrium in which bank A lends $\hat{k}_a^\alpha$ and bank B lends $\hat{k}_b^d$. At the end of the day, bank A sends $\frac{\hat{k}_a^\alpha - \hat{k}_b^d}{\alpha}$ to bank B.*

(ii) Suppose $\hat{k}_a^\alpha \leq \hat{k}_b^d$. Then, for every $k \in [\hat{k}_a^\alpha, \hat{k}_b^d]$, there is a pure strategy equilibrium in which each bank lends k. There are no net transfers between the banks. Further, these equilibria are Pareto-ranked by k, with higher k being preferred.

As $\hat{k}_a^\alpha \geq \lambda_a$, the banking equilibrium in this case improves investment efficiency in project A. The improvement is most valuable in the case that θ_a is significantly higher than θ_b (corresponding to case (i) of Proposition 2) and λ_a is low. In this case, the decentralized economy obtains an investment of $\alpha\lambda_a$ in project A, whereas the banking equilibrium recovers an investment $\alpha\hat{k}_a^\alpha$. Further, the banking equilibrium leads to the same investment αk_b^d in project B. Overall investment efficiency is therefore improved compared to the decentralized case. In addition, this equilibrium features a net transfer from bank A to bank B at the end of the day.

Case (ii) of Proposition 2 corresponds to the case of θ_a being close to θ_b. In this case, the improved investment in project A comes possibly at the cost of reduced investment in project B (compared to the decentralized outcome). The economy therefore exhibits the same co-ordination friction as in the symmetric case.

5 Conclusion

Banks perform many functions in the economy. We argue that their role in the payments system is important both for policy implementation and to understand their incentives. We show that potential settlement costs affect banks' incentives to lend and may even lead to an inefficient investment level. Given this, it is natural to consider the effect of government regulation on the settlement fees. Clearly a central authority could ensure that banks do not charge such a large discount $(1-\alpha)$ to issue local currency. However, this haircut is a transfer between banks, or distant entrepreneurs. The key cost that affects bank's disincentive to increase their investment levels given another bank's level is the cost of interbank settlement (in our model, the cost of transporting gold). When this is large, banks will have an incentive to coordinate the size of their loans.

The implication of the coordination friction we illustrate is that central bank policy that affects banks reserve may induce a correlation in the type of projects that banks accept beyond a simple change in cost of reserves. The concave production function can easily be interpreted as a sequence of projects with differing marginal productivities. Banks may restrict lending, even if funding costs are low, simply because of the settlement friction.

Appendix

Proof of Lemma 1

(i) As the entrepreneur is making take-it-or-leave-it offers, investors receive their reservation payoff, which implies a gross rate of 1.

(ii) The entrepreneur chooses the investment level by maximizing:

$$\phi_i^d = \theta f(\alpha k) - k. \tag{9}$$

First, suppose that λ is high. The first-order condition is $\alpha\theta f'(\alpha k) = 1$, so that the optimal financing level is $k^* = \min\left\{\frac{1}{\alpha}h\left(\frac{1}{\alpha\theta}\right), \lambda\right\}$, where $h(x) = f'^{-1}(x)$. That is, $k^* = \hat{k}^d$. It follows immediately that if $\lambda < \hat{k}^d$, the optimal financing level is λ. Therefore, the entrepeneur takes out a loan in the amount $k^d = \min\{\lambda, \hat{k}^d\}$.

(iii) The entrepreneur takes out a loan k^d. Of this, $(1-\alpha)k^d$ is lost in transportation, so that he purchases inputs worth αk^d. The output is therefore $\theta f(\alpha k^d)$. However, he owes k^d to his investors. Thus, his payoff is $\phi_i^d = \theta f(\alpha k^d) - k^d$. ■

Proof of Lemma 2

(i) The planner chooses k_a, k_b to maximize

$$\Psi = \sum_{i=a,b} [\theta_i f(k_i) - k_i] - \frac{1-\alpha}{\alpha}[(k_a - \lambda_b)^+ + (k_b - \lambda_a)^+], \tag{10}$$

where $X^+ = \max\{X, 0\}$.

The planner faces the aggregate resource constraint

$$\sum_{i=1}^{2} k_i + \frac{1-\alpha}{\alpha}[(k_a - \lambda_b)^+ + (k_b - \lambda_a)^+] \leq \sum_{i=1}^{2} \lambda_i \tag{11}$$

In the symmetric case, it is optimal to set $k_a^* = k_b^* = k^*$. Let the subscript ^{p} denote the planning solution. Then, $k^p = \min\{\lambda, h(1/\theta)\}$.

(ii) As no transportation costs are paid in the symmetric case, the planning surplus is $\Psi^* = 2\theta f(k^p) - k^p$. ■

Proof of Lemma 3

Fix k_j and consider the best response of bank i. There are three subcases here:

(i) Suppose $k_j < \hat{k}^\alpha$. If $k_i < k_j$, the derivative of payoff with respect to k_i is $\theta_i \alpha f'(\alpha k_i) - 1$, which is increasing until $k_i = k_j$. If $k_i \geq k_j$, a small increase in k_i results in a marginal payoff $\theta_i \alpha f'(\alpha k_i) - \frac{1}{\alpha}$, which is increasing until $k_i = \hat{k}^\alpha$. Therefore, the best response is $k_i^* = \hat{k}^\alpha$.

(ii) Suppose $k_j \in (\hat{k}^\alpha, \hat{k}^d)$. If $k_i < k_j$, the marginal payoff of bank i increases in k_i until $k_i = k_j$. Similarly, if $k_i > k_j$, the marginal payoff of bank i increases as k_i decreases, until $k_i = k_j$. Therefore, the best response is k_j.

(iii) Suppose $k_j \geq \hat{k}^d$. If $k_i < k_j$, the derivative of payoff with respect to k_i is $\theta_i \alpha f'(\alpha k_i) - 1$, which is increasing until $k_i = \hat{k}^d$. If bank i increases its loan beyond k_j, the marginal payoff falls to $\theta_i \alpha f'(\alpha k_i) - \frac{1}{\alpha}$. Therefore, the best response is $k_i^* = \hat{k}^d$.

The statement of the lemma follows. ∎

Proof of Proposition 1

Consider any $k \in [\hat{k}^\alpha, \hat{k}^d]$. From the best response function in Lemma 3, if bank j lends k, it is a best response for bank i to also lend k. Therefore, both banks lending k results in a banking equilibrium.

In this case, each entrepreneur i obtains a chit in the amount k from bank i. When he takes it to bank j, the latter imposes an aggregate settlement fee $(1-\alpha)k$. Therefore, the investment level is k.

Substituting in $k_i = k_j = k$ and $s_i = s_j = \alpha$ into the bank's payoff function in equatin (3), we obtain that each bank earns a payoff $\theta f(\alpha k) - \alpha k - \phi^d$. ∎

Proof of Proposition 2

(i) Suppose that $\hat{k}_a^\alpha > \hat{k}_b^d$. Assumption 2 (ii) directly implies that the payoff to bank A is non-negative. The payoff to bank B is $(1-\alpha)\hat{k}_a$, which is strictly greater than zero. Now, from the best response function in equation (8), it is clear that each bank is playing a best response. Bank A lends an amount greater than k_b^e, so its best response is $\hat{k}_a^\alpha$. Bank B lends an amount less than k_a^e, so its best response is $\hat{k}_b^d$. Therefore, the lending pair $(\hat{k}_a^\alpha, \hat{k}_b^d)$ is a Nash equilibrium. Clearly, at the end of the day, bank A sends $\frac{\hat{k}_a^\alpha - \hat{k}_b^d}{\alpha}$ to bank B. Of this, a proportion α is lost to the transportation cost, so that bank B receives $\hat{k}_a^\alpha - \hat{k}_b^d$.

Now, we argue that this is the unique equilibrium in pure strategies. Clearly it is the unique equilibrium in which $k_a^e > k_b^e$. Suppose there is another equilibrium in which $k_a^e = k_b^e = k$. Then, from the best response function in equation (8), it must be that $k \geq k_a^\alpha$ but $k \leq k_b^d$. However, since $k_a^\alpha > k_b^d$, this is impossible. Next, suppose there is another equilibrium in which $k_a^e < k_b^d$. Then, the best response of bank A is $\hat{k}_a^\alpha$, which is greater than k_b^d, a contradiction. Therefore, there is no other equilibrium in pure strategies.

(ii) Suppose $\hat{k}_a^\alpha \leq \hat{k}_b^d$. As $\hat{k}_b^\alpha < \hat{k}_a^\alpha$ and $\hat{k}_b^d < \hat{k}_a^d$, every $k \in [\hat{k}_a^\alpha, \hat{k}_b^d]$ satisfies the property that $k \in [\hat{k}_i^\alpha, \hat{k}_i^d]$ for each i. Therefore, every such k supports a banking equilibrium in which each bank lends k. It is immediate there are no net transfers between banks at the end of the day. ∎

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