# Portfolio Margining: Strategy vs Risk*

E. G. Coffman, Jr.† D. Matsypura‡ V. G. Timkovsky‡

June 8, 2009

**Abstract**

Introduced in the late eighties for margining certain accounts of brokers, the *risk-based* approach to margining portfolios with equity derivatives has yielded *substantially* lower margin requirements in comparison with the *strategy-based* approach that has been used for margining customer accounts for more than four decades. For this reason, after the approvals of the *pilot program* on July 14, 2005, and its extensions on December 12, 2006, and July 19, 2007, the final approval of using the risk-based approach to margining customer accounts by the SEC on July 29, 2008, at the time of the global financial crisis, appeared to be one of the most radical and puzzling steps in the history of margin regulations. This paper presents the results of a novel mathematical and experimental analysis of both approaches which support the thesis that the pilot program could have influenced or even triggered the equity market crash in October 2008. It also provides recommendations on ways to set appropriate margin requirements to help avoid such failures in the future.

## 1 Introduction

> "We still have a 1930s regulatory system in place. We've got to update our institutions, our regulatory frameworks, . . . ". . . the banking system has been "dealt a heavy blow," the result of "lax regulation, massive overleverage, huge systematic risks taken by unregulated institutions, as well as regulated institutions." – *Barack Obama*[1]

In the *margin accounts* of investors, i.e., *customers* of brokers, margin payments are based on established minimum *margin requirements* which depend on a large number of factors, such as security type, market price, expiry date, rating and other characteristics of securities held in the *positions* of the accounts. They are the subject of *margin regulations.* Margin rules exist for margining single positions, small groups of positions like those in trading *strategies* and entire *portfolios* of positions with a common *underlying instrument* or strongly correlated underlying instruments.

*The paper was prepared when the first coauthor was visiting Faculty of Economics and Business at The University of Sydney, Australia.

†Columbia University, NY 10027, USA, `coffman@cs.columbia.edu`.

‡University of Sydney, NSW 2006, Australia, {`d.matsypura,v.timkovsky`}`@econ.usyd.edu.au`.

[1] From "Obama, Brown call for global changes, say financial regulations need to be revamped" by Roger Runnigen and Robert Hutton, Bloomberg News, p. 4A · MARCH 4, 2009 · USA TODAY.

Deducting the minimum margin requirement for an account from its market value we obtain its *loan value*, which is the maximum portion of the account's market value that the broker can lend. The amount that is actually lent to a customer by the broker constitute the *margin credit* given for all customer's margin accounts. The total margin credit provided by all brokers in a market constitutes the market's *margin debt*.

With adjustments in minimum margin requirements (or the maximum credit requirements), margin regulators keep margin debt at a level consistent with the canons of a healthy economy. High margin requirements can reduce investors' activity, lead to underpricing of securities, and cause economic slowdowns. Low margin requirements, in turn, lead to overpricing of securities, high levels of speculation, cash deficits, market crashes, and, again, economic slowdowns. The challenge is to find a "golden mean" in an approach to margining that keeps the growth of margin debt within tolerable limits.

Historical records show that a fast growth of margin debt can be a sign of an approaching market crash. Such was the case in October 1929 and October 1987. Examining the two most recent examples, consider the assessments in [Fleckenstein and Sheehan, 2008] (p. 87), and the origins of the market crash of October 2000:

> "As of February 2000, total margin debt stood at $265 billion. It had grown 45 percent since the previous October and had more than tripled since the end of 1995. Relative to GDP, margin debt was the highest it had been since 1929, and over three times as high as it was in October 1987. It was an unmistakable sign of rampant speculation."

Bringing these assertions more up-to-date, we observe that, as of July 2007, total margin debt stood at $381 billion. It had grown 30 percent since the previous March and had almost tripled since the end of 2002. Margin debt was the highest it had been since 1929, and over four times as high as it was in October 1987. It was an unmistakable sign of rampant speculation, which presaged the market crash of October 2008.

We argue in this paper that the market crash of October 2008 has a direct link to an alternative approach to portfolio margining of customer accounts that appeared in margining practice three years earlier. A substantial margin reduction since then, especially for short sales, created a meaningful advantage for speculators playing bear. To better understand the challenges of portfolio margining, we need to track key regulatory updates of the last five years and the current state of margin calculation technology.

## 1.1 The Regulatory Breakthrough of 2005: Strategy

The *strategy-based approach* to margining customer accounts with equity derivatives such as stock or index options, futures, warrants, convertible bonds or preferred stocks have been used in the brokerage industry for more than four decades. By the end of the last century, it was commonly recognized that this approach yields excessively high margin requirements. This can be partially explained by the fact that margin rules by that time permitted the use of offsets with only few *legs*.[2] However, the more legs an offset has the more margin reduction it gives. Thus, the reduction of minimum margin requirements can be achieved by designing new offsets with larger number of legs.

[2] This brokerage term stands for a position in options with the same exercise price.

Offsets with two, three and four legs, imitating options trading strategies, such as calendar spreads, straddles, butterfly spreads, condor spreads, collars and box spreads were known and permitted in margin regulations since the mid seventies. Offsets with more than four legs, a very efficient means of achieving adequate margin reductions, did not appear until 30 years later in 2005.

The first endeavor to break through this regulatory quad happened, nevertheless, in August 2003 when the CBOE proposed new margin rules for offsets with 6, 8, 10, 12 and 14 legs that were called "complex option spreads."[3] After two revisions of this document,[4] the SEC approved these rules[5] and added them to NYSE Rule 431 in December 14, 2005. In August 2007, these rules were also recognized in Canada.[6]

The regulatory move of 2005 was a very important step in the development of the strategy-based approach because it demonstrated that four legs is not the final step. Moreover, as shown in [Matsypura et al., 2007], 14 legs is also not the final step, because there exists a method of designing offsets with any desired number of legs. In particular, it was shown that there exist offsets with a maximum of 134 legs if the number of different exercise prices of the options involved is at most four.

## 1.2 State of the Art: Strategy

Another reason for the high margin requirements obtained by the strategy-based approach is that the calculation of the minimum margin by using offsets with more than two legs, as shown in Section 3, is a (computationally) complex combinatorial problem that is not well understood. Despite the fact that margin regulations have a 75-year history dating from Regulation T in the Securities Act of 1934, the literature on margin calculations is surprisingly small.[7] We can point to only two books [Geelan and Rittereiser, 1998; Curley, 2008] and two papers [Fortune, 2000, 2003] devoted to margining practice, two papers [Moore, 1966; Luckett, 1982] studying the influence of margin requirements on investor's equity ratio, and two papers [Rudd and Schroeder, 1982; Fiterman and Timkovsky, 2001] devoted to margining algorithms. The vast majority of publications on margining consists primarily of regulatory circulares and manuals. Margin calculations have never attracted the attention of mathematicians or computer scientists because they have been traditionally considered a prerogative of brokerage accounting, something that is enforced and interpreted by security market lawyers.

Margin calculation systems, developed and used in the brokerage industry up to 2005, ignore highly effective and broadly applicable discrete optimization methods, such as integer and dynamic programming. In particular, the reduction of the margin-minimization-by-pairing problem to the minimum-cost network-flow problem [Rudd and Schroeder, 1982] was seemingly forgotten for more than 20 years.

---

[3] CBOE Regulatory Circular RG03-066, August 13, 2003.

[4] CBOE Regulatory Circulares RG04-90, August 16, 2004, and RG05-37, April 6, 2005.

[5] SEC Release 34-52738, December 14, 2005.

[6] IDA Bulletin 3654, August 13, 2007.

[7] This remark does not refer to the literature devoted to studying the relationship between margin requirements and market volatility. A survey of this literature can be found in [Kupec, 1998].

As a consequence, existing margin calculation technology, faced with the combinatorial complexity of the strategy-based approach, failed to take advantage of complex option spreads. The vast majority of margin calculation systems used in the brokerage industry, as our study shows, uses offsets with two legs only; and they are based on outdated heuristics proposed by brokers in the mid seventies [Cox and Rubinsein, 1985; Geelan and Rittereiser, 1998]. The most advanced margining systems recognize offsets with up to four legs by using heuristics that cannot guarantee the minimum margin. But the failure to use offsets with more that two legs, as we show in Sections 3.8 and 5, can result in a double margin charge or even increase the margin from zero to several thousands of dollars. We also show that the margin minimization problem with complex offsets can be efficiently solved by optimization packages like CPLEX.[8]

In our opinion, the strategy-based approach was unjustly discredited by the belief that the combinatorial problem stemming from the use of complex offsets could not be efficiently solved with the help of standard optimization packages. Naturally, the development of special optimization algorithms that can take into consideration particularities of the margining can be expected to bring much better results.

One of the goals of this paper is to rehabilitate the strategy-based approach and to show that it enables the calculation of much more appropriate margin requirements in comparison with those calculated by the *risk-based approach*.

### 1.3 The Pilot Program of 2005–2008: Risk

The risk-based approach[9] was proposed in 1989 by the OCC (Options Clearing Corporation) to calculate net capital requirements for brokers' proprietary portfolios of listed options.[10] It was implemented in 1996 in TIMS[11] and approved by the SEC[12] to be effective as of September 1, 1997. However, the approach was not used for margining customer accounts prior to 2005.[13]

---

[8]www.ilog.com

[9]The concept also appears in margin regulations and business-related literature under the names "portfolio margining approach" and "risk-based portfolio margining approach". (In what follows, we omit the adjective "margining" for compactness.) The argument standing behind the term "portfolio" is based on the *misrepresentation* of the strategy-based approach as a treatment of a margin account by considering individual positions only, while the "portfolio approach" treats a margin account as a whole. The strategy-based approach also treats a margin account as a whole; although it does so in a different way, it is still a portfolio approach. So we consider the term "risk-based approach" to be most appropriate. For the same reason, we do not think that the term "rule-based approach," frequently used on the Internet as synonymous with strategy-based approach, is suitable because the risk-based approach is also based on certain rules, such as Rule 15c3-1a or rules from Regulation T.

[10]SEC Release 34-27394, October 26, 1989.

[11]Recently the OCC replaced TIMS (Theoretical Intermarket Margining System) with STANS (The System for Theoretical Analysis and Numerical Simulations, which was approved by SEC Release 34-53322, February 15, 2006, on the basis "that the amount of margin it will collect under STANS will be significantly less than the amount of margin it currently collects under TIMS" and that "STANS identifies a more realistic correlative relationship among underlying assets than TIMS."

[12]SEC Releases 34-38248, February 6, 1997, see also [GAO/GGD-98-153, 1998]

[13]The SEC published the related NYSE proposal for public comments in SEC Releases 34-46576, October 1, 2002 and 34-50885, December 20, 2004, before approving the approach in July 2005.

Employing simple and liberal margin rules in contrast to the strategy-based approach, the risk-based approach produces substantially lower margin requirements. In the examples provided by the CBOE, the margin requirement for naked options and basic option spreads turns out to be at least two or three times lower.[14] (72 times lower for a long straddle!) After the NYSE proposals[15] on October 1, 2002, and December 20, 2004, the SEC approved the use of the risk-based approach to margining customer accounts under a temporary *pilot program* on July 14, 2005.[16]

The pilot program can be divided into three phases: Phase I permitted the use of the risk-based approach to margin accounts with only listed BBI (broad based index) and ETF (exchange traded funds) derivatives such as options, warrants, futures, and future options. Phase II included listed stock options and securities futures on July 11, 2006.[17] Phase III included equities, equity options, unlisted derivatives and NBI (narrow based index) futures on December 12, 2006,[18] to be effective from April 2, 2007. Phase III also permitted the use of a single portfolio margin account for all risk-based offsets. The pilot program was to expire on July 31, 2007. However, it was extended for one more year on July 19, 2007, and the risk-based approach was finally approved on July 30, 2008,[19] to be used permanently from August 1, 2008.

Monthly *margin debt* reports[20] published by the NYSE provide clear evidence of the influence of the pilot program on the equity market; see Figure 1. During the initial 20-month period of the pilot program from July 2005 through April 2007, the margin debt had increased by $82.22 billion compared to the $38.80 billion increase during the previous 20 months. Although the beginning of this period, between the starting points of Phases I and II, looks ordinary, the ending of this period, between the starting points of Phases II and III, is remarkable owing to the unusually high rate of increase in the margin debt and the trading volume volatility of the S&P 500 index.

During the subsequent four-month period, from the starting point of Phase III on April 2, 2007 through July 2007, the period when equities, equity options, unlisted derivatives and NBI futures joined the pilot program, the debt increase was $88.21 billion, i.e., at a rate at least five times higher. Thus, since April 2, 2007, the margin debt increased at a rate of more than $22.05 billion per month.[21]

Figure 1 clearly shows that the market credit in the period from April through July 2007 was *excessive* in the extreme. Even though the margin debt hit the level of $381.37 billion by the end of July 2007, the pilot program had, nevertheless, been extended for an additional year. In September 2007, it was clear that the growth in margin debt had lessened because in August 2007 it fell to $331.37 billion. This means

[14] www.cboe.com/margin, CBOE Rules 12.4, 9.15(c), 13.5 and 15.8A.

[15] SEC Releases 34-46576, October 1, 2002, and 34-50885, December 20, 2004.

[16] SEC Release 34-52031, July 14, 2005.

[17] SEC Release 34-54125, July 11, 2006.

[18] SEC Release 34-54918, December 12, 2006.

[19] Exchange Act Release No. 58251, July 30, 2008, 73 FR 45506, August 5, 2008.

[20] www.hyxdata.com/nysedata

[21] Our calculations show that the correlation between the margin debt and the S&P 500 index was 97.5% and 85.5% in the periods from November 2003 through May 2007 and October 2008, respectively.

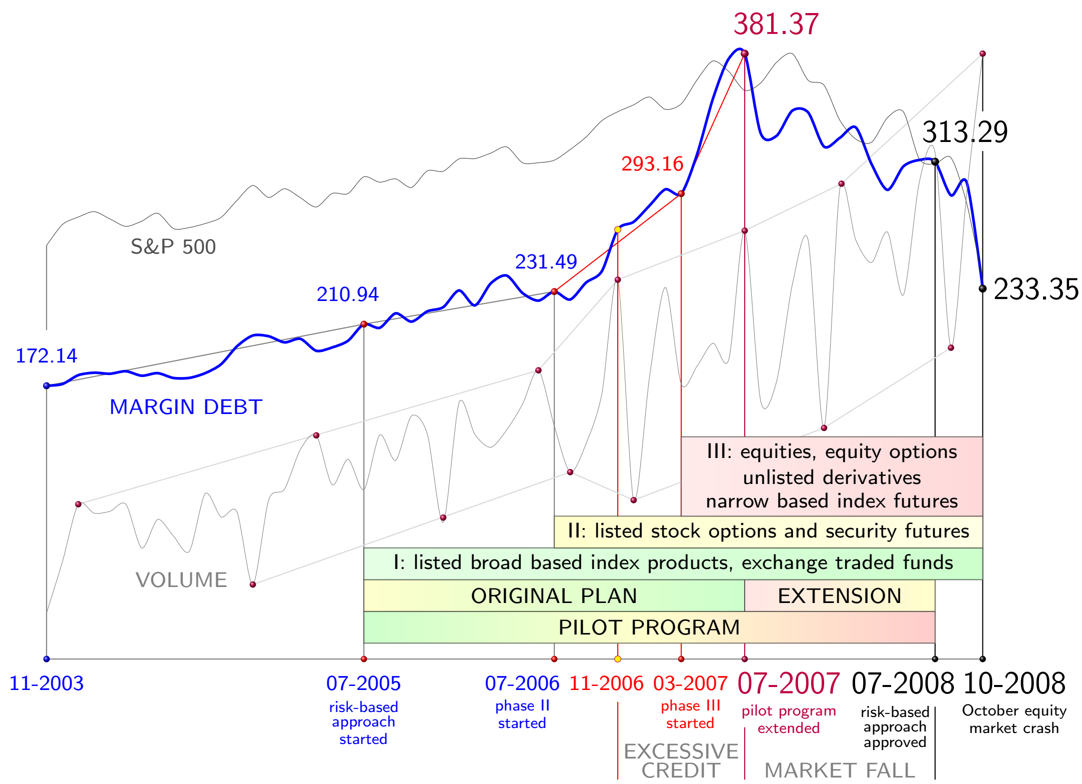


Figure 1: The margin debt in billions of dollars, and the level and trading volume of the S&P 500 index (scaled so as to align the maxima of the three curves) in the period from November 2003 through October 2008. The curves are drawn by smoothing the end-of-the-month data from `http://www.hyxdata.com/nysedata` and `http://finance.yahoo.com/`.

that in August 2007 investors received the $50.00 billion margin call and experienced astonishing losses together with *forced sales* from their under-margined accounts. By July 2008, the time of the final approval of the risk-based approach, the margin debt had plunged to $314.36 billion, and the equity market downfall was evident. In September 2008 the margin debt had another dramatic plunge from $299.96 billion to $233.35 billion, when investors received the $66.61 billion margin call.

> "The task of the Board, as I see it,[22] is to formulate regulations with two principal objectives. One is to permit adequate access to credit facilities for security markets to perform their basic economic functions. The other is to prevent the use of stock market credit from becoming excessive. The latter helps to minimize the danger of pyramiding credit in a rising market and also reduces the danger of forced sales of securities from undermargined accounts in a falling market."

[22] From `http://fraser.stlouisfed.org/docs/ historical/martin/martin55_0314.pdf`, the speech of William McC. Martin, Jr., Chairman of the Board of Governors of the Federal Reserve System from April 2, 1951, through January 31, 1970, at the hearing on the study of the stock market before the U.S. Senate Committee on Banking and Currency on Monday, March 14, 1955.

In July 2007, it was clear that the market credit had been excessive during the preceding eight months. In July 2008, it was clear that the equity market had been falling for the previous twelve months. Yet the practice of using the risk-based approach, which evidently caused excessive market credit, was continuing.

### 1.4 State of the Art: Risk

As we will see in Section 4.2, the main mathematical model of the risk-based approach is asset pricing. The literature on this topic is overwhelming; see [Daffie, 2003]. The main option pricing models are the Black-Sholes model [Black and Scholes, 1973] for European-style options[23] and the Brennan-Schwartz model [Brennan and Schwartz, 1977] for American-style options.[24]

Although the SEC permits the use of any asset pricing algorithm and the software approved by the DEA (Designated Examining Authority),[25] there exists only one approved asset pricing software system today: the pricing module of STANS developed by the OCC. Thus, risk-based margining today is monopolized in the U.S. by the OCC.[26]

### 1.5 Contributions and Plan of the Remainder of the Paper

The contributions of this paper are 1) an argument showing that the risk-based approach is being misused, 2) an explanation of why the risk-based approach could have influenced the quick growth of the margin debt in the U.S. equity market in the period from July 2006 through July 2007, and 3) an identification of those portfolios for which the risk-based approach can work efficiently.

To develop these contributions, we introduce a mathematical model of portfolio margining in Section 2. Within this model the analysis of the strategy and risk-based approaches is given in Sections 3 and 4, respectively. In Section 4.5, we show that the risk-based approach being applied to security portfolios can in principle cause pyramiding credit. Computational experiments with both approaches are presented in Section 5. In conclusion, we balance the pros and cons of both approaches and discuss technological challenges in creating better margin calculation systems in Section 6.

## 2 A Model of Portfolio Margining

The existing practice of margin calculations uses the following four components that must be specified in any approach to margining portfolios:

- *Position Margining*, i.e., the method of calculating the margin requirement for an *individual* position, which is also called a *naked* position, in a margin account

[23] These can be exercised during a short period of time, usually one day, prior to the expiry date.
[24] These can be exercised at any time prior to the expiry date.
[25] Federal Register, Vol 73, no. 29, February 12, 2008.
[26] SEC Release No. 34-57270, February 5, 2008. Also, see footnotes in Section 1.3.

regardless of the presence of other positions in it. Position margining can also be considered as margining a portfolio that consists of only one position.

- *Offset Margining*, i.e., the method of calculating the margin requirement for a group of offsetting positions, which are simply called *offsets*. Since the positions offset each other, the margin requirement for an offset is lower than the total position margin in the offset. Therefore, offsets are the means of margin reductions. Offsets can be considered as portfolios of restricted sizes whose margin calculations are "manageable", i.e., margin requirements for offsets can be easily calculated by a simple formula or procedure.
- *Portfolio Partitioning*, i.e., the method of partitioning a portfolio into blocks of positions or offsets. Thus, if margin requirements for positions and offsets are defined, then the margin for the entire portfolio can be calculated as the total margin requirements of these blocks.
- *Portfolio Classification*, i.e., the method of defining portfolio types depending on what security types are in a portfolio's positions. Position margining and offset margining are usually dependent on security types and portfolio types.

Each margin rule in a rule book can be converted into an algebraic formula. The main problem here, as we illustrate in the next section, is text interpretation. Definitions related to portfolio partitions and portfolio classifications are then given in Sections 2.2 and 2.3.

## 2.1 Interpretation of Margin Rules

Over the entire history of margin regulations, even in the modern era when capital markets could not exist without computers, margin rules continue to be presented and maintained in text form. This phenomenon becomes the source of an enormous number of incorrect margin calculations and, as a consequence, costly margining system upgrades for the whole brokerage industry.

This difficulty, however, could be easily resolved if the texts of margin rules were appended by mathematical formulas providing formal definitions. This obvious step could have been taken 30 years ago, but it has still not been taken. Exchanges still provide explanations in their margin manuals by giving numerous examples for every particular case instead of giving only one general formula.

Those who have read margin regulations and tried to convert the texts of margin rules into mathematical formulas, hence to computer programs, will agree that this task is not always an easy one. To comprehend the often impenetrable text of margin rules, one needs a course in *hermeneutics* (the theory, art and practice of interpretation). As an example, which will be used in Section 3.8, let us consider the following strategy-based margin rule for a naked short position in options or warrants.[27] We will track only provisions for call options on stocks giving necessary quotations from tables (which we do not display here) in square brackets:

[27] NYSE Rule 431(f)(2)(D)(i).

"(D) The margin required on any put, call, currency warrant, currency index warrant, or stock index warrant issued, guaranteed or carried "short" in a customer's account shall be:

(i) In the case of puts and calls issued by a registered clearing agency, 100% of the current market value of the option plus the percentage of the current value of the underlying component specified in column II of this subsection (D)(i) below [20% for stocks]...

The minimum margin on any put, call, currency warrant, currency index warrant or stock index warrant issued, guaranteed or carried "short" in a customer's account may be reduced by any "out-of-the-money amount" (as defined in this subsection (D)(i) below), but shall not be less than 100% of the current market value of the option or warrant plus the percentage of the current value of the underlying component specified in column III of this subsection (D)(i) below [10% for stocks], ...

For the purposes of this subsection (D)(i), "out-of-the-money amounts" are determined as follows: (for call options) [Any excess of the aggregate exercise price of the option over the current market value of the equivalent number of shares of the underlying security.]"

After careful (and repeated) readings of these provisions, we can conclude that, after introducing the following notation,

$\mathsf{Cp}$ : call option market price,
$\mathsf{Ce}$ : call option exercise price,
$\mathsf{Up}$ : underlying stock market price,
$\mathsf{Co}$ : call option out-of-the-money amount, i.e.,

$$\mathsf{Co} = \max\{\mathsf{Ce} - \mathsf{Up}, 0\}$$

the margin requirement for a short position in a call option on stocks can be expressed by the formula

$$100 \cdot \mathsf{Cp} + 100 \cdot \mathsf{Cm}$$

where

$$\mathsf{Cm} = \max\{0.2 \cdot \mathsf{Up} - \mathsf{Co}, 0.1 \cdot \mathsf{Up}\} \tag{1}$$

In this formula, $100$ (shares) is the option contract size, $100 \cdot \mathsf{Cp}$ is the option premium, and $100 \cdot \mathsf{Cm}$ is the risk component.

By comparison with the associated text, the function in Formula (1) is much more compact, excludes ambiguous interpretations and calculates a unique margin requirement. On the other hand, Formula (1) (indeed, any mathematical formula) has, strictly speaking, no legal standing. We are left with an ambiguous legal foundation for formulas that calculate different margin requirements. Margin calculation systems developed up to now thus have their own interpretations of margin rules and therefore their own, possibly distinct, versions of margin requirements.

## 2.2 Portfolio Partitions

Let $S$ be the set of financial *securities* traded on the market. A *portfolio* on $S$ can be formally defined as a triplet $(P, i, q)$, where

- $P$ is a nonempty *position set*, i.e., the set of positions, also called *components*, in securities from $S$;
- $i$ is the *investment function* which defines an injection of the set $P$ into $S$, i.e., $i(p)$ is the security in position $p$, and $p \neq p'$ implies $i(p) \neq i(p')$; and
- $q$ is the *quantity function* on $P$, i.e., $q(p)$ is the number of units of $i(p)$.

The inequality constraint in the second bullet means that two or more positions in the same security are not allowed in portfolios for margining purposes; if such positions appear as a result of several buy transactions, they must be consolidated into one position with a netted quantity before margining.

Let $|P|$ be the *size* of portfolio $P$. The portfolio of size $|S|$ is the *market portfolio*. Portfolios of size one are *trivial*. In what follows, it will be convenient to associate portfolios with their position sets and trivial portfolios with positions. For the purposes of this paper, a *margin account* is just a portfolio.

Let $P_1$ and $P_2$ be two portfolios on $S$ with the investment functions $i_1$ and $i_2$ and the quantity functions $q_1$ and $q_2$. We say that the two portfolios are *similar* if $P_1 = P_2$ and $i_1 = i_2$. Thus, similar portfolios differ in position quantities only.

If $P_1$ and $P_2$ are similar portfolios and there exists an integer $n_1 > 1$ such that $n_1 q_1(p) = q_2(p)$ for all $p \in P_1$, then we say that $P_1$ is a *divisor* of $P_2$ and write $n_1 P_1 = P_2$. Portfolios without divisors are *prime*. A portfolio $P$ has exactly one similar prime portfolio; it is called a *base* of $P$ and denoted by $\dot{P}$. Thus, $P = n\dot{P}$ for some positive integer $n$, which we call the *multiplicity* of $\dot{P}$; the portfolio $P$ thus is a *multiple* of $\dot{P}$.

We say that $P_1$ is a *subportfolio* of $P_2$ or that $P_2$ *contains* $P_1$ if $P_1 \subseteq P_2$ and $i_1$ is a restriction of $i_2$ to $P_1$, i.e., $i_1(p) = i_2(p)$, and $q_1(p) \leq q_2(p)$ for all $p \in P_1$.

Let $P_1, \ldots, P_k$ be mutually disjoint subportfolios of some portfolio $P$ with quantity functions $q_1, \ldots, q_k$, respectively. Then we say that these subportfolios generate a *partition* of $P$ and write $P = P_1 + \ldots + P_k$ if

$$q(p) = \sum_{i=1}^{k} \sum_{p \in P_i} q_i(p)$$

for any position $p \in P$. If $n_1, \ldots, n_k$ are multiplicities of $\dot{P}_1, \ldots, \dot{P}_k$, respectively, then the sum

$$P = n_1 \dot{P}_1 + \ldots + n_k \dot{P}_k$$

is called a *prime partition* of $P$ on $\rho = \{\dot{P}_1, \ldots, \dot{P}_k\}$.

Let $\rho$ be a collection of prime portfolios on $S$ including all $|S|$ trivial prime portfolios, so that $|S| \leq k$. Then it is clear that a prime partition of any portfolio $P$ on $\rho$ always exists. If the maximum position quantity in the portfolio $P$ is $n$ then the number of all possible prime partitions of $P$ on $\rho$ is at most $(n+1)^k$.

### 2.3 Portfolio Classification

Let $I$ and $S$ denote the sets of *indices* and *securities* in the market, respectively. We distinguish these *financial instruments* because, unlike indices, securities (including exchange traded funds based on indices) are traded on the market, i.e., they have trading units. The market itself can be defined as the union $M = I + S$.

Let $u : S \to M$ be the *underlying function* on $S$, i.e., $u(s)$ is the *underlying instrument* for the security $s \in S$, and $s$ is a *derivative* from $u(s)$. Let $U$ denote the set of *underlying securities*, i.e.,

$$U = \{s \in S : \exists s' : u(s') = s\}$$

Then the sum $B = I + U$ is the set of *underlying instruments* of $M$, which we call the *base* of the market $M$. Let $u^{-1}(b)$ be the set of derivatives from the underlying instrument $b \in B$. Then the set

$$S_b = \begin{cases} \{b\} + u^{-1}(b) & \text{if } b \text{ is a security or} \\ u^{-1}(b) & \text{if } b \text{ is an index} \end{cases}$$

is a *security class* associated with the underlying instrument $b \in B$. We say that $P$ is a *class portfolio* based on the underlying instrument $b$ if all securities in $P$ belong to the class $S_b$. If not stated otherwise, the term "portfolio" refers to a class portfolio. Margin regulations distinguish the following three types of underlying instruments:

1: high-capitalization broad-based index and an ETF based on it,
2: low-capitalization broad-based index and an ETF based on it,
3: narrow-based index and margin eligible security.

The type of an underlying instrument $b$ defines the type of a portfolio $P$ based on $b$.

## 3 Strategy-Based Approach

The combinatorial essence of the strategy-based approach arises from the ability to partition a margin account in many different ways in accordance with a large variety of offsets given in the margin rule book. Each securities market follows its own margin rule book, for example, NYSE Rule 431 in the U.S.A. or Regulation 100 in Canada. The strategy-based offsets are of fixed size and imitate trading strategies.

In brokerage jargon, offsets of sizes 1, 2, 3 and 4 are *singles*, *pairs*, *tripos* and *quads*; the positions of the offsets are *legs*. Offsets of sizes at least two are *proper*. Proper offsets such as *calendar spreads*, *straddles* or *option-security combinations* are pairs, *butterfly spreads* or *box spreads* are quads (cf. [Cohen, 2005]). Complex offsets such as *calendar iron condor spreads*[28] have 14 legs.[29]

[28]SEC Release 34-52738, December 14, 2005.

[29]We should mention that it is possible to design *offsets-centipedes* with as many legs as desired. A method in [Matsypura et al., 2007] generates centipedes with up to 134 legs.

### 3.1 Position Margining: Strategy

The margin requirement for an individual position in the strategy-based approach is a certain percentage of the current security market value contained in this position. This percentage is called a *margin rate*. The higher the market price of the security, the more reliable it is considered; therefore, a smaller margin rate is assigned to this security in strategy-based margin regulations. The margin rate is also dependent on security type, rating, expiry date, the market where it is traded, whether the position is long or short, and other factors. Margin rules for positions in derivatives are not simple at times, and the related margin requirements can have nontrivial margin formulas.[30]

Fixed-income securities like bonds, debentures and notes have much lower margin requirements than equity securities because they are much less volatile. Stocks are the most volatile equity securities and have the most stringent margin requirements. For example, U.S. initial and maintenance margin rates for stocks in long positions are 50% and 25%, respectively.[31] Note that the Regulation T initial rate of 50% has remained unchanged since 1974, and it has never been lower than 40% since 1940. In Section 4.1, we will see that the risk-based approach assigns both initial and maintenance margin rates for stocks as low as 15%.

### 3.2 Hedging Mechanism of Strategy-Based Offsets

*Trading strategies*, called *strategies* for short, are special portfolios with known "exits". The positions of a strategy appear in a margin account as a result of simultaneous buy and sell trades. Strategies are designed not only to maximize the profit in favorable scenarios but also to provide the possibility to *liquidate* all or some of their positions in unfavorable scenarios.

Offsets in the strategy-based approach *imitate* strategies. A single exit is also associated with an algorithm for liquidating (closing) all or some of the positions of the offset with a *minimum loss*. In accordance with the strategy-based approach, margin regulators choose the *worst exit* and calculate the *maximum minimum loss*, i.e., the minimum loss in the *worst case scenario*. This loss constitutes the *minimum regulatory margin requirement* for the offset.[32]

We can formally define an offset as a portfolio with the associated collection of liquidation algorithms that we call a *hedging mechanism* for the offset. Each liquidation algorithm is usually associated with a subset of positions in *exercisable securities* in the offset (such as options, futures, warrants, convertible bonds) and represents a sequence of *trades* (*buy* and *sell* transactions) which must be triggered once all exercisable securities in the subset are exercised.[33] Thus, offsets in the strategy-based approach are

[30]See NYSE Rule 431 for U.S., Regulation 100 for Canada, web sites of particular securities markets.

[31]Examples of margin rates in different countries can be found in [Roll, 1989].

[32]The worst case scenarios for the offset holder and the offset holder's broker can be different. That is one of the reasons why margin rules for brokers and their customers are usually not the same.

[33]Note that the exercises are not to be performed during margin calculations, they are virtual only, as in a simulation of reality.

supported by hedging mechanisms for exiting from dangerous situations. Since every liquidation algorithm is a sequence of trades, the hedging mechanism defines necessary and sufficient *cash flows* and *security flows* that must be run to satisfy all *parties* involved in the offset liquidation.

It is important to observe that the hedging mechanism for a prime offset $\dot{O}$ generates the hedging mechanisms for all its multiples because the hedging mechanism for $m\dot{O}$ is just the hedging mechanism for $\dot{O}$ applied $m$ times to the offset $O$. Margin regulations therefore describe only prime offsets. Any *margin rule* represents actually a prime offset and the margin requirement for it, and any margin *rule book* is a collection of margin rules. In what follows, we will associate a rule book with a collection of prime offsets $\rho$ that includes all $|S|$ trivial prime offsets and keep in mind that the margin requirements for the prime offsets are given.

The more offsets there are in the rule book, the more opportunities exist for margin reductions. It is important to emphasize that offsets make margin reductions possible not at the expense of increasing the risk but, in contrast, through hedging against the worst case scenario, using the *hedging mechanisms* provided for that purpose.

### 3.3 Hedging Mechanism of a Call Spread

Let us consider, for example, the hedging mechanism of a *calendar call spread* involving a short position in a call option $C$ and a long position in another call option $D$ on the same underlying security $U$, at the same expiry date, exercise price $e$ and contract size $s$. The mechanism consists of only one liquidation algorithm which must be triggered once the holder of $C$ exercises the option:

- the *buy* of $s$ units of $U$ by the holder of $D$ from the writer of $D$ at the price $e$;
- the *sell* of $s$ units of $U$ by the holder of $D$ to the holder of $C$ at the price $e$.

Thus, the holder of the spread is protected by this hedging mechanism against the underlying security price fluctuations. The maximum loss here is the cost of the spread paid at the time when the position in this spread was entered.

Each of these trades implies the *cash flow* of quantity $se$ from a buyer to a seller and the underlying *security flow* of quantity $s$ from a seller to a buyer. It is also important to notice that this pair of trades involves (apart from the brokers) the following *three* parties:

- the writer of $D$;
- the holder of the call spread, i.e., the holder of $D$ who is also the writer of $C$; and
- the holder of $C$.

Thus, the calendar call spread holder can trigger the hedging mechanism in order to avoid a loss larger than the spread cost in the case when the holder of $C$ decides to exercise the option. The calendar call spread holder in this case plays the role of an *intermediary* (third party) between the holder of $C$ who buys $s$ units of the underlying security at the price $e$ and the writer of $D$ who sells them at the same price. The hedging mechanism, therefore, determines who receives the money in the amount of $se$, who receives $s$ units of the underlying security and from where, when the calendar call spread must be closed.

We have considered the hedging mechanism of the simplest offset. In general, the hedging mechanism of an offset with many positions involves a series of trades (cf. [Cohen, 2005]), but the main point is that it makes clear the *money distribution* (money side) and the underlying *security distribution* (security side) among the parties involved in the related set of trades, when the offset must be closed. That is why the offsets in the strategy-based approach are called *two-sided offsets*.

In the above example we considered *security options*, i.e., options on a security, such as an equity, ETF, bond, currency, or commodity. Exercising security options triggers a cash flow and a security flow. Exercising *index options*, i.e., options on an index, triggers a cash flow only. For example, exercising an index call option brings to the option holder the difference between the index price and the option exercise price. Thus, the hedging mechanism of an index call spread is much simpler because it entails only the movement of money.

### 3.4 Cross Margining: Strategy

If an offset is a class portfolio (see the definition in Section 2.3), i.e., all its positions have the same underlying instrument, then we call it a *class offset*. The vast majority of offsets created for margin calculations are actually class offsets. There also exist *cross offsets* whose positions are based on different (crossing) underlying instruments, which do not necessarily belong to the same securities market. This explains the term *cross margining* when cross offsets are permitted. Cross offsets are usually permitted if they are underlain by highly correlated market indices or ETFs based on these indices. Therefore, cross offsets are cross-index products.

Cross offsets can be easily designed from class offsets by allowing some positions to have different underlying instruments. Margin requirements for cross offsets are usually more stringent and take into consideration the correlation between the crossing underlying instruments. Although class offsets are often more advantageous than cross offsets, there is no priority to class offsets in the strategy-based approach. If cross offsets turn out to be more efficient than class offsets in margin reductions then they are allowed to be used first. As we will see in Section 4.2, the situation is different in the risk-based approach because it gives priority to class offsets.

### 3.5 Account Offsetting and Margining: Strategy

The essence of the strategy-based approach is to find a hedging mechanism for a margin account by the consolidation of the hedging mechanisms of the offsets contained in the account. Let $A$ be a margin account, let $\rho = \{\dot{O}_1, \ldots, \dot{O}_k\}$ be a rule book, and let

$$A = x_1\dot{O}_1 + \ldots + x_k\dot{O}_k$$

be a prime partition of $A$ on $\rho$; see Figure 2 for an example. Since the hedging mecha-

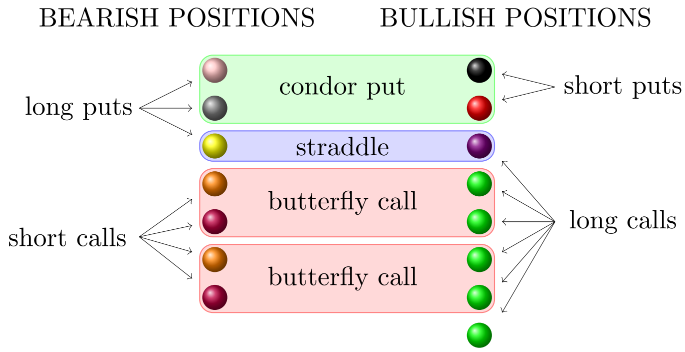


Figure 2: An account $A$ that consists of only one class portfolio with nine positions in options which are marked by nine different colors. Each option contract is depicted by a ball. The account $A$ thus has total of fifteen option contracts. The figure shows the prime partition $A = \dot{O}_1 + \dot{O}_2 + 2\dot{O}_3 + \dot{O}_4$ on NYSE Rule 431 that involves the prime offsets $\dot{O}_1, \dot{O}_2, \dot{O}_3$, and $\dot{O}_4$ which are a condor put spread, a long straddle, a butterfly call spread, and a naked long position in one call option contract, respectively.

nisms of the prime offsets $\dot{O}_j$ generate the hedging mechanisms of the multiples $x_j\dot{O}_j$, the hedging mechanism of $A$ is just a consolidation of the hedging mechanisms for the multiples. Thus, the hedging mechanisms of the prime offsets $\dot{O}_j$ must be applied $x_j$ times to the positions of $A$ that are covered by these offsets and, since the multiples $x_j\dot{O}_j$ cover all position quantities in the account $A$, the entire account becomes hedged.

If $m_j$ is the margin requirement for the prime offset $\dot{O}_j$ then,in accordance with the strategy-based approach, the margin requirement for the margin account $A$ is

$$m_1x_1 + \ldots + m_kx_k$$

Thus, a margin requirement for $A$ is associated with a prime partition of $A$, and every prime partition of $A$ gives a margin requirement for $A$. Hence, finding a minimum regulatory margin requirement for an account is finding its prime partition, i.e., multiplicities $x_1, \ldots, x_k$, such that the latter sum achieves the minimum. We will refer to this problem as the *account margin minimization* (AMM) problem. If the maximum offset size in the rule book $\rho$ is $d$ then we will say that the AMM problem is of dimension $d$. Accordingly, the AMM problem of dimension two is the AMM problem by pairing.

Thus, finding the margin requirements for margin accounts in the strategy-based approach means solving the AMM problem. Achieving the minimum is important for both the brokers and the account owners because the knowledge of the regulatory minimum against the real margin requirement gives the exact estimate of the overcharge. On the other hand, the minimum represents an adequate level of risk for the account.

## 3.6 Mixed Integer Program for Strategy-Based Margining

We show next that the AMM problem reduces to a mixed integer program (MIP). This reduction will allow us to use standard optimization software for the calculation of strategy-based margins in our computational experiments presented in Section 5.

Let a margin account have $m$ positions $i = 1, 2, \ldots, m$ with quantities $q_i$ and $n$ prime offsets $j = 1, 2, \ldots, n$ from $\rho$. Prime offset $j$ can be represented by the column vector $\mathbf{o}_j$ whose $i$th component $o_{ij}$ is the quantity of the $i$th components of prime offset $j$.[34] Note that $o_{ij} = 0$ if and only if position $i$ is not involved in prime offset $j$. We assume that $m \leq n$ and that prime offsets $j = 1, 2 \ldots, m$ are trivial, i.e., they represent the positions. Thus, the column vectors have $o_{jj} = 1$ as the only nonzero elements.

Let $m_j$ be the margin requirement for prime offset $j$. Thus, $m_j$ is the margin requirement for a single unit of the security in position $j$ if $1 \leq j \leq m$, or the margin for prime offset $j$ of size more than one if $m+1 \leq j \leq n$. Let $x_j$ denote the multiplicity of prime offset $j$. If we introduce the vectors[35]

$$\mathbf{m} = \|m_1, \ldots, m_n\|^\top \qquad \mathbf{q} = \|q_1, \ldots, q_m\|^\top \qquad \mathbf{x} = \|x_1, \ldots, x_n\|^\top$$

which we call a *margin*, *quantity* and *variable vectors*, respectively, and the *offset matrix*

$$\mathbf{O} = \|\mathbf{o}_1, \mathbf{o}_2, \ldots, \mathbf{o}_n\|$$

then the AMM problem can be formulated as finding an $\mathbf{x}$ which solves

$$\min \left\{ \mathbf{m}^\top \mathbf{x} : \; \mathbf{O}\mathbf{x} = \mathbf{q}, \; \mathbf{x} \geq \mathbf{0} \right\} \tag{2}$$

where $\mathbf{0}$ is a zero column vector. Note that the size of $\mathbf{O}$ is $m \times n$. In this MIP, components of the variable vector $\mathbf{x}$ from $m+1$ to $n$ must be integers because they define offset multiplicities of offsets with sizes more than one, while components from 1 to $m$ can be real. They are integers if they are quantities of positions in such securities like stocks (shares), options, warrants, but they can be real if they are, for example, bond quantities. The quantity vector $\mathbf{q}$ is also integer but the margin vector $\mathbf{m}$ is real.

Without loss of generality, we assume that $\mathbf{O} = \|\mathbf{I}, \mathbf{P}\|$, where $\mathbf{I}$ and $\mathbf{P}$ are the *identity matrix*, i.e., the matrix of trivial prime offsets, and the matrix of *proper prime offsets*, i.e., the matrix of prime offsets of size at least two, respectively.

Note that MIP (2) is an extension of the transportation model introduced in [Rudd and Schroeder, 1982] for the calculation of the minimum account margin by pairing, i.e., in the case where the matrix of proper prime offsets is a matrix of prime pairs.

## 3.7 Margining Practice: Strategy

Surprisingly, current margin calculation practice still uses heuristics designed in the mid seventies based on brokers' intuition and taste, even for margining by pairing. In the general case, the most advanced heuristics take advantage of the result from [Rudd and Schroeder, 1982] by using minimum-cost network-flow algorithms. There exist, however, counterexamples (cf. Section 3.8) showing that these heuristics can double

[34] Component quantities of prime offsets are integers, unlike the quantities of convertible securities with non-integer conversion ratios.

[35] In what follows, $A^\top$ denotes the transpose of matrix $A$. Thus, the transpose of a row vector is a column vector and vice versa.

regulatory minimum margin requirements or even raise them from zero to catastrophic margin calls. Millions of margin accounts maintained by prime brokers with the use of the strategy-based approach therefore are substantially over-margined, just because there has been no serious attempt to find efficient optimization algorithms.

Despite the fact that the AMM problem was posed (originally as a problem of margining accounts with options only) more than thirty five years ago, it has not been well studied and remains one of the most intractable problems in the investment industry. Neither useful theoretical results nor solution techniques with reasonable computing times are known. The only exception is the paper [Rudd and Schroeder, 1982] devoted to the AMM problem by pairing. It was shown that this case reduces to the well-studied bipartite minimum-cost network-flow problem.

An analysis of the literature suggests that the AMM problem has never been considered in the MIP form, one of the more natural and popular ways of solving discrete optimization problems. However, in Section 5, we show that mixed integer programming algorithms yield very good results for the AMM problem.

### 3.8 Advantage of Offsets with Larger Numbers of Legs

In this section, we show that the advantage of using even four-leg offsets over two-leg offsets in margin calculations can be significant.

Let us consider a margin account which consists of a long position in one call option $\mathsf{A}$, a long position in one call option $\mathsf{B}$ and a short position in two call options $\mathsf{C}$. The options' exercise prices and market prices are, respectively,

$$\begin{aligned} \mathsf{Ae} &= \$70.00, & \mathsf{Ap} &= \$55.90,\\ \mathsf{Be} &= \$90.00, & \mathsf{Bp} &= \$40.90,\\ \mathsf{Ce} &= \$80.00, & \mathsf{Cp} &= \$50.60. \end{aligned}$$

Each of the options expires by the end of day, January 15, 2010, and has the contract size of $\mathsf{100}$ shares. The market price of the underlying stock is $\mathsf{Up} = \$123.62$.[36] Since

$$\mathsf{Ae} < \mathsf{Ce} < \mathsf{Be} \ \text{ and } \ \mathsf{Ce} - \mathsf{Ae} = \mathsf{Be} - \mathsf{Ce}$$

the account represents a long butterfly spread

$$\mathsf{A} + \mathsf{B} - 2\mathsf{C}$$

whose components are calendar spreads[37]

$$\mathsf{A} - \mathsf{C} \ \text{ and } \ \mathsf{B} - \mathsf{C}$$

Next, we show that the margin requirement for this account is zero if considered as the long butterfly spread, or $\mathsf{\$1000}$ if considered as a consolidation of the two calendar

[36] The data are taken from [`http://finance.yahoo.com`] as of the end of day, May 21, 2008, at NYSE for the symbol IBM.

[37] We follow NYSE Rule 431.

spreads. Indeed, the margin requirement for a long butterfly spread is the net market value of the options if it is positive, and zero otherwise. In our case, it is

$$\begin{aligned} & \mathsf{100} \cdot \max\{\mathsf{Ap} + \mathsf{Bp} - \mathsf{2Cp}, \mathsf{0}\} \\ = \; & \mathsf{100} \cdot \max\{\$55.90 + \$40.90 - 2 \cdot \$50.60, 0\} \\ = \; & \$0 \end{aligned}$$

The margin requirement for a calendar spread is the lesser of the short option margin requirement and the spread out-of-the-money amount. The spread $\mathsf{A} - \mathsf{C}$ is in-the-money, therefore its out-of-the-money amount, $100 \cdot \max\{\mathsf{Ae} - \mathsf{Ce}, 0\}$, and hence its margin requirement is zero. The spread $\mathsf{B} - \mathsf{C}$ is out-of-the-money, and its out-of-the-money amount, $100 \cdot \max\{\mathsf{Be} - \mathsf{Ce}, 0\}$, is \$1000.

Using Formula 1 form Section 2.1 we can conclude that the margin requirement for the short position in the option $\mathsf{C}$ is

$$100 \cdot \mathsf{Cp} + 100 \cdot \mathsf{Cm} = \$5060.00 + \$2472.40$$

where

$$\begin{aligned} \mathsf{Cm} &= \max\{0.2 \cdot \mathsf{Up} - \mathsf{Co}, 0.1 \cdot \mathsf{Up}\} \\ &= \max\{0.2 \cdot \$123.62 - \$0.00, 0.1 \cdot \$123.62\} \\ &= \$24.724 \end{aligned}$$

Therefore, the margin requirement for the spread $\mathsf{B} - \mathsf{C}$ is \$1000. Thus, if the account is considered as a consolidation of the two calendar spreads, the margin requirement for the account is \$1000.

As can be seen, if a margin calculation system is not capable of identifying four-leg offsets, it can generate unjustifiable margin calls which can exceed the minimum regulatory requirement by thousands of dollars even for small margin accounts with four positions. The rationale behind complex offsets with six or more legs is similar to that behind four-leg offsets.

## 4 Risk-Based Approach

The risk-based approach has a much lighter optimization component (for cross margining only), but is dependent on the use of probabilistic models of asset pricing theory (for pricing options, warrants, futures, convertible securities) and related Monte Carlo simulations of the price movements of underlying instruments. To decide what portfolios qualify for cross margining, the method looks through historical data of the correlation of the instruments underlying the portfolios.

As we will see, the combinatorial complexity of the strategy-based approach is replaced in the risk-based approach by the complexity of simulations and the difficulties in obtaining a high-level of statistical significance. In addition to rejecting the entire rule book of *fixed-size offsets* based on trading strategies, it uses much looser *variable-size offsets* based on grouping positions by underlying securities. Details of the risk-based approach are as follows.

## 4.1 Position Margining: Risk

Similarly to the strategy-based approach, the risk-based approach uses the current market price of a security for margin calculations, and in addition, it considers price variations within certain ranges in an attempt to catch the worst-case price movement for the entire portfolio. This technique is called *portfolio choking.*

According to the risk-based approach, in order to calculate the margin requirement for each unit of a security $s$ in a position whose underlying security $u$ has the market price $c$, it is necessary to consider in addition to $c$, ten more *valuation points*

$$c(1 - ai) \text{ and } c(1 + bi)$$

where $i = 1, 2, 3, 4, 5$, and the constants $a$ and $b$ depend on the type of the underlying instrument as follows:

- $a = 0.016$, $b = 0.012$ for type 1,
- $a = 0.020$, $b = 0.020$ for type 2,
- $a = 0.030$, $b = 0.030$ for type 3.

Thus, the lowest (highest) valuation points appear to be 8%(6%), 10%(10%), 15%(15%) lower (higher) than the current market price $c$ for types 1, 2, and 3, respectively.[38]

It is important to observe that, in accordance with this rule, the risk-based margin rate for stocks and margin eligible equities in customer margin accounts is only 15%, the lowest margin rate since 1929. Before the equity market crash in October 1929 it was 10%. The current strategy-based margin rates are 50% (initial) and 25% (maintenance).

In our opinion, this significant margin reduction in margining individual stock positions together with the faulty hedging mechanism of risk-based offsets for stocks were the main reasons for the equity market crash on October 2007, as noted in Section 1.3. We consider the hedging mechanism of risk-based offsets in Sections 4.4 and 4.5.

Let $c_v$, $1 \leq v \leq 11$, be one of the eleven valuation points including $c$. If $s = u$, i.e., $s$ is the underlying security, then the difference $o_v = c_v - c$ or $c - c_v$ shows the outcome (*gain* if positive, and *loss* if negative) associated with point $c_v$ for long or short positions in $s$, respectively, for each security unit.[39]

If $s$ is a derivative, then the outcome associated with the valuation point $c_v$ should be calculated in accordance with the mechanism of the derivative. In most cases, $o_v$ is a function of $c_v$, $e$ (the exercise price of the derivative) and $p_v$ (the market price of $s$ estimated at the valuation point $c_v$).[40] The estimated price $p_v$ must be calculated using a qualified *theoretical pricing model.*[41]

[38]These percentages follow Rule 15c3-1a, section (b)(1)(i)(B).

[39]These gains and losses are called "theoretical gains and losses" in SEC Release 34-53577.

[40]For example, if the security $s$ is an option then, after calculating its in-the-money amount, $i_v = \max\{c_v - e, 0\}$ if $s$ is a call option or $\max\{e - c_v, 0\}$ if $s$ is a put option, its outcome in long positions and short positions can be calculated as $i_v$ and $p_v - i_v$, respectively, multiplied by the option contract size for each option.

[41]The model must be approved by the DEA (Designated Examining Authority). Currently, only the OCC model implemented in STANS is approved, see Federal Register, Vol 73. No. 29, February 12, 2008.

## 4.2 Account Offsetting and Margining: Risk

Unlike the strategy-based approach, the risk-based approach uses much more aggressive offsets. Their margin requirements are simply net losses of the involved positions. To calculate the margin requirement for an account $A$ we need to perform the following six steps, where Steps 2 through 5 should be repeated for each valuation point.

*Step* 1. *Account Partition:* Find the partition of $A$ into the portfolios $P_1, \ldots, P_k$ associated with the underlying instruments involved in $A$. Note that this operation is free of any combinatorial element because it is equivalent to the enumeration of all positions in the account.

*Step* 2. *Finding Positions' Theoretical Gains/Losses:* Calculate the outcome $o_{pv}$ for each position $p \in A$.[42]

*Step* 3. *Netting of Positions:* Calculate the outcome $n_{uv}$ for each portfolio $P_u$, $u = 1, \ldots, k$, using the formula

$$n_{uv} = \sum_{p \in P_u} o_{pv}$$

Thus, $n_{uv}$ is the net outcome of all positions in $P_u$ for the evaluation point $c_v$.

*Step* 4. *Finding Gain and Loss Portfolios:* At this point, each portfolio $P_u$ is either a *loss portfolio* with $n_{uv} < 0$ or a *gain portfolio* with $n_{uv} > 0$ or a *null portfolio* with $n_{uv} = 0$. In what follows, the notation

$$L_{1v}, \ldots, L_{mv} \text{ and } G_{1v}, \ldots, G_{nv}$$

will stand for the loss and gain portfolios, respectively. Their lower case counterparts, i.e.,

$$l_{1v}, \ldots, l_{mv} \text{ and } g_{1v}, \ldots, g_{nv}$$

will stand for their losses and gains.

*Step* 5. *Cross Margining:* Find the *guarantee-guarantor deficit* $d_v$ for the valuation point $c_v$ (see Section 4.3) and assign it as the margin requirement for $c_v$. Note that, if cross margining was not allowed, the guarantee-guarantor deficit would be the total loss through all loss portfolios, i.e.,

$$d_v = l_{1v} + \ldots + l_{mv} \text{ for each } v = 1, \ldots, 11$$

*Step* 6. *Account Margining:* Assign the margin requirement for the account $A$ to be $\max\{d_0, d_1, \ldots, d_{11}\}$ with

$$d_0 = \sum_{i=1}^{l} \max\{0.375, p_i\} \cdot q_i r_i + 0.375 \cdot \sum_{i=l+1}^{k} q_i r_i$$

[42] Note that since we add the position index $p$ to $o_v$ to distinguish the outcomes from different positions, the same index must be added to the parameters $c_v$, $e$ and $p_v$ in Section 4.1.

where $1, \ldots, l$ and $l+1, \ldots, k$ are, respectively, long and short positions in $A$ in derivatives, and where $p_i$, $q_i$ and $r_i$ are the position $i$ unit price (per underlying unit), quantity and derivative contract size, respectively.

### 4.3 Cross Margining: Risk

As is evident from Step 3 of the above algorithm, the risk-based approach allows us to fully offset position losses by gains of another position inside each portfolio. In addition, it allows us to offset, to a certain extent, losses of index portfolios of type $i$ by gains of index portfolios of type $j$ if the pair $(i, j)$ is either $(1, 1)$, $(2, 2)$ or $(3, 3)$ (see the definition of types in Section 2.3). Since these offsets connect different security classes based on different indices, this technique is called *cross margining*.

It is important to note that cross margining offsets between portfolios are permitted only if the OCC determines that the *crossing* indices associated with types 1, 2 and 3 are sufficiently correlated.[43] Furthermore, the offset of an index by an ETF based on it is permitted only if the ETF holds securities in the same proportion as the index.[44]

- 90% of the gain of a type 1 can offset the loss of type 1,
- 75% of the gain of a type 2 can offset the loss of type 2,
- 90% of the gain of a type 3 can offset the loss of type 3.

These percentages follow the *haircut* rule[45] that must be applied before netting. Thus, prior to solving the guarantee-guarantor deficit problem, one needs to reduce the gains $g_j$ by 10% for all gain portfolios $G_j$ of type 1 or 3, and by 25% for all gain portfolios $G_j$ of type 2. We consider at this point that the haircut has been applied.[46]

The problem of the distribution of gains to cover losses is not trivial. We now show that a solution can be found by formulating it as a linear program. Let $R$ denote the guarantee-guarantor relation between the loss portfolios and the gain portfolios of the same type defined by the OCC. Specifically, let $1 \leq i \leq m$ and $1 \leq j \leq n$, then

$$\begin{aligned} (i, j) \in R \text{ if and only if the loss portfolio } L_i \text{ (guarantee)} \\ \text{can be offset by the gain portfolio } G_j \text{ (guarantor)} \end{aligned}$$

Let $x_{ij}$ denote the portion of the loss $l_i$ in the loss portfolio $L_i$ that is assigned to be offset by the gain portfolio $G_j$. Then the guarantee-guarantor deficit is

$$d = \sum_{i=1}^{m} \left( l_i - \sum_{j=1}^{n} x_{ij} \right) = \sum_{i=1}^{m} l_i - \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}$$

[43]The recent OCC's methodology implemented in STANS even disregards portfolio types associated with product groups; see SEC Release 34-53322, February 15, 2006.

[44]SEC Release 34-53577, March 30, 2006.

[45]The haircuts are used as cushions against market falls; see SEC Rule 15c3-1a (b)(1)(v)(B).

[46]In this section we remove the subscript $v$ from the notation related to gains and losses bearing in mind that the guarantee-guarantor deficit must be found for each of the eleven valuation points $c_v$.

Since the gain portfolio $G_j$ can offset at most the $g_j$ loss of the loss portfolios,

$$\sum_{i=1}^{m} x_{ij} \leq g_j \tag{3}$$

for all $j$ such that $(i, j) \in R$; and, since the loss portfolio $L_i$ needs at most $l_i$ to be offset by the gain portfolios,

$$\sum_{j=1}^{n} x_{ij} \leq l_i \tag{4}$$

for all $i$ such that $(i, j) \in R$. If we are interested in reducing the guarantee-guarantor deficit as much as possible, we need to solve the linear program that maximizes

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}$$

with nonnegative variables $x_{ij}$ under Conditions 3 and 4.

With the exception of the aggressive offsetting[47] at Step 3 of the algorithm described in Section 4.2, nothing contradicts the strategy-based approach. Portfolio choking, however, has never been used in the strategy-based approach – although it could be. Without portfolio choking, the risk-based approach is just a simplified case of the strategy-based approach with substantially lower margin requirements for singles and offsets. Further, for each class portfolio, the risk-based approach squeezes the entire rule book of the strategy-based offsets into only one offset, which is the class portfolio itself; see Figure 3 for an example. If Step 3 were replaced by the procedure for creating strategy-based offsets, the risk-based approach would turn into a hybrid approach. It would have been natural to use this approach in the pilot program for a "cushioned" transition.

## 4.4 Hedging Mechanism of Risk-Based Offsets

As can be seen from the algorithm in Section 4.2, a risk-based offset is the entire portfolio. Its hedging mechanism is just netting position losses and gains. It is important to observe that, if the underlying instrument of the portfolio is a security, the netting does not show security movements in the case when the portfolio must be liquidated.

The risk-based approach, however, works perfectly for portfolios with index options because their exercises do not trigger any security movements.[48] To confirm this fact we can observe that the margin debt growth was quite moderate in the period between Phases I and II when only portfolios with index products were involved in the pilot

[47] Note that netting is the most radical way of offsetting because it brings the maximum advantage to covering losses. On the other hand, netting "cares" about money movement only. As we show in Sections 4.4 and 4.5, netting is very risky for security portfolios.

[48] Upon the exercise of an index call or put option, its holder receives, respectively, $\max\{u - e, 0\}$ or $\max\{e - u, 0\}$, where $u$ is the underlying index price and $e$ is the option exercise price.

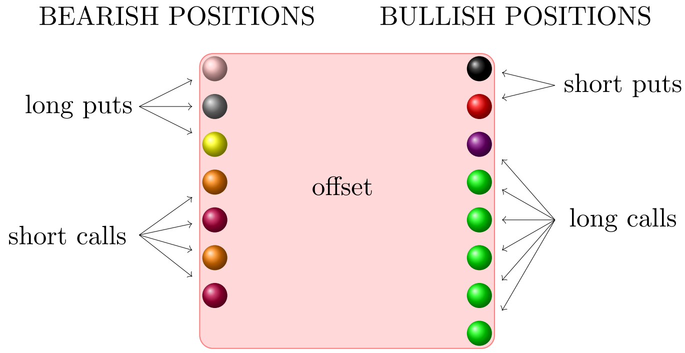


Figure 3: The account from Figure 2. It has only one risk-based offset which is the union of all four strategy-based offsets shown in Figure 2.

program, see Figure 1. However, once portfolios with security options and futures joined the program, the debt started growing very fast.

Next, we consider a situation with simple security portfolios showing that the ignorance of security movements enforces the borrowing of money at an exponential rate, which in turn leads to the so-called *pyramiding credit* effect mentioned in Section 1.3.

Let us consider a portfolio $P$ with the following three positions: a long position in a call option $C$, a short position in a call option $C_0$ and a short position in a call option $C_1$. In what follows, it will be convenient to denote a long (short) position in an option $C$ as $+C$ $(-C)$, respectively, and a portfolio as a formal sum of positions. Also, it will be natural to omit the $+$ symbol, as accepted in formal sums. Thus,

$$P = C - C_0 - C_1$$

All three options are on the same underlying stock and have the same expiry date and contract size of 100 shares. We assume that the expiry date is distant enough, so that the options' current prices are much smaller that the current price $u$ of the underlying stock, and therefore can be disregarded in the calculation of gains and losses. We assume that the options $C$, $C_0$, and $C_1$ are in the money and have exercise prices

$$u - \frac{1}{2}u, \quad u - \frac{1}{4}u \quad \text{and} \quad u - \frac{1}{4}u,$$

respectively. Thus, the portfolio holder experiences gain in the amount of

$$50u = 100(u - u + \tfrac{1}{2}u)$$

on $C$ and loss in the amount of

$$50u = 100(u - u + \tfrac{1}{4}u) + 100(u - u + \tfrac{1}{4}u)$$

on $C_0$ and $C_1$. Following the risk-based approach we should net the losses and the gain and conclude that the margin requirement for this portfolio is zero. However, let us try to liquidate the portfolio $P$ exercising all three options.

Upon exercising the options, the portfolio holder has zero cash balance, because $50u - 25u - 25u = 0$, 100 shares of the stock from exercising the option $C$ and the obligation to deliver $100 + 100$ shares to the holders of options $C_0$ and $C_1$. Thus, the portfolio holder is 100 shares short, and these shares must be purchased to liquidate the portfolio. Since the risk-based approach allows us to do this on 85% credit (the margin requirement is just 15%), the portfolio liquidation requires borrowing $85u$.

Thus, risk-based offsets do not provide a real hedge and encourage borrowing money for portfolio liquidation. Note that the strategy-based approach provides a complete hedge for this portfolio, as it catches the spread $C - C_0$ or $C - C_1$ and requires a proper margin charge for the naked short position $-C_1$ or $-C_0$, respectively.

## 4.5 Pyramid of Credit

Now we assume that the holder of the call option $C_0$ is short in two call options $C_{00}$ and $C_{01}$, and the holder of the call option $C_1$ is short in two call options $C_{10}$ and $C_{11}$. Each of these four options has the exercise price $u - \frac{1}{8}u$. Thus, these holders are actually the holders of the portfolios

$$P_0 = C_0 - C_{00} - C_{01} \quad \text{and} \quad P_1 = C_1 - C_{10} - C_{11}$$

Each of the portfolio's holders experiences a gain in the amount of

$$25u = 100(u - u + \tfrac{1}{4}u)$$

on the long position and a loss in the amount of

$$25u = 100(u - u + \tfrac{1}{8}u) + 100(u - u + \tfrac{1}{8}u)$$

on the short positions. Again, each of the portfolios holders is in the same situation as the portfolio $P$ holder and must purchase 100 shares of the stock to liquidate the portfolio. Together they should borrow $2 \cdot 85u$.

The short options $C_{00}$, $C_{01}$, $C_{10}$, $C_{11}$ in their turn can be long options in the next layer of portfolios $P_{00}$, $P_{01}$, $P_{10}$, $P_{11}$, in which options in short positions have exercise price $u - \frac{1}{16}u$; see Figure 4.

Continuing by induction we can conclude that layer $k$ contains $2^k$ portfolios, which we call *credit blocks*, whose options in short positions have exercise price

$$u - \frac{1}{2^{k+1}}u$$

and require the amount $2^{k-1} \cdot 85u$ for closing these positions. The credit blocks thus build a pyramid with the total credit

$$\left(2^0 + 2^1 + \ldots + 2^{k-1}\right) \cdot 85u = \left(2^k - 1\right) \cdot 85u$$

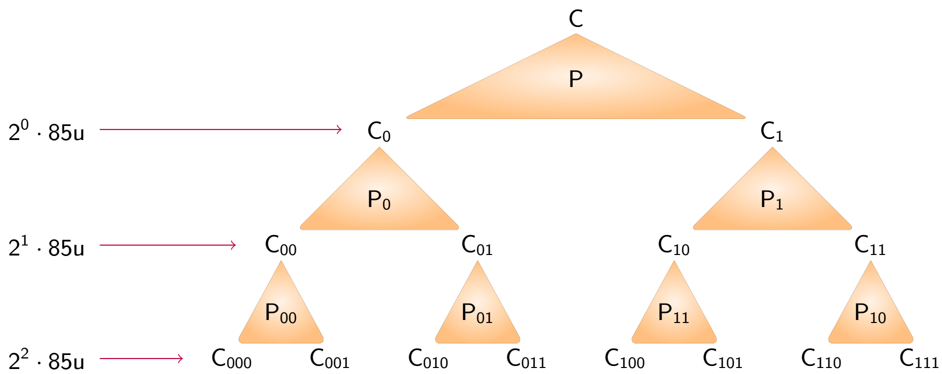


Figure 4: Pyramid of credit with three layers and total credit requirement $(2^3 - 1) \cdot 85u = 595u$. The arrows point to layers $k = 1, 2, 3$ with credit requirements $2^{k-1} \cdot 85u$.

If layer $k$ is the base of the pyramid then exercising all $2^k$ options in layer $k$ requires borrowing not only the amount $2^{k-1} \cdot 85u$ to close short positions in these options, but an additional amount

$$\left(2^k - 1\right) \cdot 85u - 2^{k-1} \cdot 85u = \left(2^{k-1} - 1\right) \cdot 85u$$

to close all of the other short positions of this pyramid. Thus, the borrowed amount is almost doubled. We should emphasize that our example involves only the simplest credit blocks, which are actually tripos, requiring credit of $85u$.

In practice, credit blocks could be more credit demanding. For example, the quad with a long position in a call option with exercise price $u - \frac{1}{2}u$ and three short positions in call options with the same exercise price $u - \frac{1}{6}u$ has zero risk-based margin but requires the credit $170u$ to be liquidated. The base of the exponential growth associated with the pyramid of credit built from these blocks then becomes three.

# 5 Computational Experiments

The strategy-based approach cannot compete with the risk-based approach in the speed of computations because the latter is free of any combinatorial quantities. Nor does it give an advantage in *initial* margin requirements, which are substantially higher than those obtained by the risk-based approach; see Sections 1.3 and 4.1. But an answer to the question of whether the strategy-based approach always yields higher *maintenance* margin requirements in comparison with the risk-based approach is not obvious.

The results of our experiments show indeed the opposite: the strategy-based approach yields substantially lower maintenance margin requirements for large portfolios. To demonstrate this, we compare the behavior of the portfolio *maintenance* margin requirement as a function of the portfolio size that is calculated by both the strategy-based approach and the risk-based approach. As before, we follow NYSE Rule 431.

Another goal of our experiments is to contrast different strategy-based algorithms and clarify to what extent offsets with numbers of legs more than two can reduce maintenance margin requirements and increase computing time in comparison with two-leg offsets. Note that the strategy-based approach accommodates many margining algorithms, while the risk-based approach has only one hard-coded margining algorithm.

## 5.1 Design of the Experiments

As we discussed in Section 3, the strategy-based approach uses offsets of fixed sizes. We say that a strategy-based margining algorithm is of *dimension* $d$ if it solves the AMM problem of dimension $d$, i.e., it takes into consideration all regulatory offsets of sizes $1, 2, \ldots, d$. Note that an algorithm of dimension one just calculates the total positions margin. We consider dimensions one, two and four only.[49] Dimension four is maximum in this experiment because we demonstrate the capability of the strategy-based approach up to December 2005, when offsets of sizes more than four were not permitted. Dimensions higher than four will be the subject of a future work.

The main idea of portfolio variation is to first build a representative *maximal portfolio* and then randomly remove its positions to provide a monotonic reduction of its size. The experiment performed the following steps:

*Step* 1. A group of 16 call options and a group of 16 put options on a stock were selected such that exactly 8 options inside each group were in the money; see Table 3 in Appendix A.

*Step* 2. The maximal portfolio with 32 positions was built by creating 8 long positions in randomly[50] chosen 8 call options and 8 short positions in the remaining 8 call options; the other 16 positions in put options were created in the same way.

*Step* 3. The number of option contracts in each position was randomly generated in the range from 1 to 10.

*Step* 4. Step 3 was repeated 100 times to get 100 different portfolios with the same position enumeration.

*Step* 5. A single position was randomly selected to remove it from all 100 portfolios to get another set of 100 portfolios.

*Step* 6. Step 5 was repeated 29 times to get a total of 30 sets of 100 randomly generated portfolios with sizes monotonically decreasing from 32 to 3. The last generated set of 100 portfolios was used as a basis for generating the next set of 100 portfolios, each of which has one position less.

---

[49]Note that dimension three does not deserve special consideration because it excludes very efficient offsets of size four. Also note that the existing margin regulations allow one to design strategy-based algorithms of maximum dimension 14; see Section 1.1.

[50]For random selections we used the Excel function Randbetween(a,b), which yields numbers uniformly distributed between a and b.

*Step* 7. The margin requirement was computed for each portfolio in each set by using the three strategy-based algorithms of dimensions one, two and four, plus the portfolio-based algorithm.

*Step* 8. The margin requirements in each set were averaged. Hence, we calculated 30 averaged margin requirements for each algorithm.

Thus, we computed margin requirements for 3,000 unique and randomly generated portfolios overall. The experiment described in Steps 1 through 8 has a *symmetric scenario* in that the positions quantities chosen at Step 3 are distributed uniformly between 1 and 10. Because of this step, the symmetric scenario generates more or less *balanced portfolios* where the numbers of call and put options, in-the-money and out-of-the-money options, long and short positions are approximately the same. Generating 100 balanced portfolios at Step 4 with subsequent averaging at Step 8 assures that the *balance* is statistically significant.

We also performed this experiment in six *asymmetric scenarios* to model *unbalanced portfolios* with different kinds of asymmetry. We performed the same steps in the above algorithm except for Steps 3 and 4, which were replaced by a single step assigning fixed numbers of options in the positions in the following three scenarios:

8lo +2sh : eight/two option contracts for each long/short position,
8ca +2pu : eight/two option contracts for each position in call/put options,
8bu+2be : eight/two option contracts for each bullish/bearish position.

The other three asymmetric scenarios were obtained by transposing eights and twos.[51] Strategy-based algorithms of dimensions two and four were CPLEX algorithms[52] solving MIP (2) from Section 3.6, where only two-leg offsets and four-leg offsets along with two-leg offsets were used, respectively.

## 5.2 Comments on Experimental Results

The results of the experiments are presented in Appendix B, where all figures present maintenance margin requirements in thousands of dollars. The notation S1, S2, S4 and R stands for margin requirements obtained by the strategy-based algorithms of dimensions one, two, four, and the risk-based algorithm, respectively. The summary is given in Tables 1 and 2.

The main conclusion we can draw from the experiment is that the strategy-based approach gave better results than the risk-based approach in four scenarios out of seven; see arrows in the tables. In addition, the strategy-based approach yields substantially lower margin requirements for balanced portfolios of sizes 20 and more; see Figure 6.

[51] Recall that long positions in call options and short positions in put options are bullish, long positions in put options and short positions in call options are bearish.

[52] We used ILOG OPL Development Studio 6.1.1 with CPLEX 11.2.1 on an Apple iMac with 2.4 GHz Intel Core 2 Duo CPU and 2 GB RAM running Mac OS X 10.5.3. To run ILOG CPLEX, we used Parallels Desktop for Mac 3.0 (build 5600), Virtual Machine with 1 GB RAM, and Windows XP.

Table 1: Average maintenance margin requirements (in thousands of dollars) over portfolio sizes from 3 through 32 for strategy-based algorithms of dimension one (S1), two (S2), four (S4) and the risk-based algorithm (R) for seven scenarios. Arrows point to minimum requirements in columns.

| algorithm | symmetric | 8lo+2sh | 2lo+8sh | 8ca+2pu | 2ca+8pu | 8bu+2be | 2bu+8be |
|---|---|---|---|---|---|---|---|
| S1 | 135.5 | 48.9 | 195.7 | 137.4 | 107.2 | 107.2 | 137.4 |
| S2 | 52.5 | 4.4 | 127.7 | 55.9 | 31.7 | 65.3 | 101.6 |
| S4 | → 33.1 | → 2.1 | 112.7 | 27.9 | → 20.8 | → 54.8 | 91.9 |
| R | 43.1 | 6.0 | → 76.9 | → 12.5 | 66.1 | 74.9 | → 5.9 |

Table 2: Average percentage margin reductions over portfolio sizes from 3 through 32 for strategy-based algorithms of dimension two (S2), four (S4) and the risk-based algorithm (R) related to the total position margin for seven scenarios. Arrows point to maximum reductions in columns.

| algorithm | symmetric | 8lo+2sh | 2lo+8sh | 8ca+2pu | 2ca+8pu | 8bu+2be | 2bu+8be |
|---|---|---|---|---|---|---|---|
| S2 | 53.1 | 75.9 | 32.0 | 50.9 | 58.3 | 34.8 | 22.9 |
| S4 | → 65.2 | → 84.5 | 38.1 | 67.8 | → 66.9 | → 44.1 | 28.6 |
| R | 63.7 | 79.3 | → 57.4 | → 89.3 | 25.3 | 19.1 | → 93.1 |

In general, the approach works better for the bullish portfolios. In contrast, the risk-based approach is preferable for the bearish portfolios. The scenario 2bu+8be gives an exceptional example in which the strategy-based requirement is $125,975.80 against zero risk-based requirement for a portfolio with 32 positions; see Figure 12. This scenario shows that the risk-based approach is very advantageous for speculators playing bear.

The risk-based approach can give, however, very bad results for bullish portfolios. For example, the risk-based margin requirement for the portfolios with less than 10 positions in the scenarios 2ca+8pu and 8bu+2be is higher than even the total position strategy-based margin; see Figures 10 and 11. The scenario 2ca+8pu turns out to be pathological for the risk-based approach because it requires $93,663.28 against zero strategy-based requirement for a portfolio with 32 positions; see Figure 10.

With regard to the strategy-based approach, we can conclude that using pairs along with tripos and quads (i.e., strategy-based algorithms of dimension four) yields a substantial margin reduction even in comparison with using only pairs; it is especially noticeable for balanced portfolios. Besides, in all scenarios except 2lo+8sh and 2bu+8be, the trend of the strategy-based requirement is to be *smaller* for *larger* portfolios. Note that these two scenarios reveal the same but weaker trend for the risk-based approach.

Obtaining the enormous margin reduction owing to offsets with three and four legs in the strategy-based approach is computationally time consuming. CPU time for portfolios of sizes between 3 and 20 (our study shows that the average size of customers' margin portfolios in the U.S. is smaller than 12) is approximately 0.5 sec; see Figure 5. If we multiply this time by a million (this is a good estimate of customers' margin accounts under the custody of a large prime brokerage firm), then we get a bit more than 138 hours of CPU time. Note that this does not include data retrieval time.

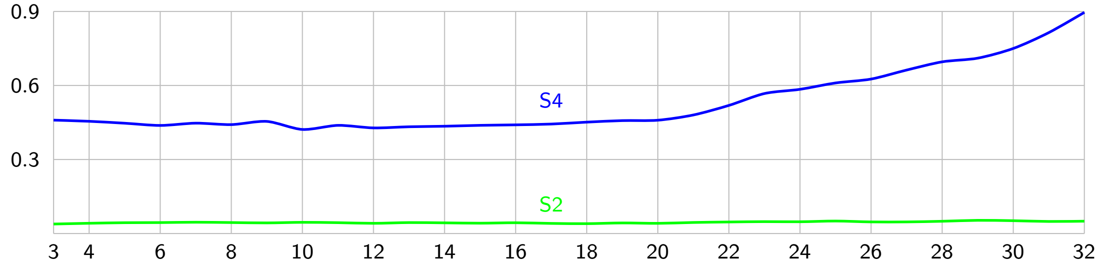


Figure 5: CPU times (in seconds) of the strategy-based algorithms of dimensions two and four in the symmetric scenario. CPU times of the same algorithms in the asymmetric scenarios are graphically indistinguishable. CPU times of the strategy-based algorithm of dimension one and the risk-based algorithm are negligible in all scenarios, and not shown.

This means that, in order to use the CPLEX optimization package, a large brokerage firm, say in New York, will require a very significant amount of computational resources and massive parallelization because there are only 17.5 hours available to produce margin account status reports by the morning (9:30am) of the next business day, from the end (4pm) of the previous business day. At this point, it is not clear whether a simple increase in computing resources will solve the problem. In addition, the question of massive parallelization is not trivial and requires special study. Hence, modern strategy-based margining technology is in serious need of efficient specialized optimization packages.

# 6 Concluding Remarks

Our general observation concerns margin rules for naked positions in underlying securities. In both the strategy and risk-based approaches, the rules are based on assigning a margin rate, i.e., a fixed percentage of the current security market value. This percentage usually depends on the security type (in all rules known today), market price bands (Canadian rule), and expiry date (bonds, debentures, and notes).

For example, in accordance with NYSE Rule 431, the margin rate for a naked long stock position is 50%/25% (initial/maintenance) in the strategy-based approach and 15% in the risk-based approach; see Sections 4.1 and 3.1.

The main question regarding margin rates is how they are assigned. This question is valid for any approach to portfolio margining because it is always possible to have a trivial portfolio with only one security position. Why choose rates of 50%, 20%, 15% or, say, 100% as was done in 1946? This fundamental issue must be addressed in further research on margining. As far as we know, a model, method, or justifiable technique for assigning securities margin rates does not exist today.[53] So far, this issue has been a prerogative of margin regulators, their intuition and experience. Margin rates are

[53] A theory of the investor's optimal margin account based on a model of optimal borrowing for the entire margin account can be found in [Luckett, 1982].

being assigned now by "gut feeling", the same way that option prices were assigned before the appearance of the Black-Scholes formula.

The strategy-based approach is, at this point, the most appropriate one for margining security portfolios in customer margin accounts. Minor reductions of margin rates for equity securities in naked positions, using existing and developing new offsets of higher sizes, such as complex option spreads, and related strategy-based algorithms of higher dimensions, can eliminate over-margining by the strategy-based approach.

The risk-based approach can work efficiently for margining only index portfolios in customer margin accounts and inventory portfolios of brokers, because the liquidations of such portfolios do not involve security movements. It is possible in the former case because the underlying instrument is an index, which does not have trading units, and in the latter case because the security movements are needed only inside the inventory of a single broker. The application of the risk-based approach to security portfolios in customer margin accounts is in fact very risky and can have the effect of pyramiding credit. We suggest that Phases II and III of the Pilot Program of using the risk-based approach for margining security portfolios in customer margin accounts were major contributors to the pyramiding credit in the period from December 2006 through July 2007 and the subsequent equity market crash in October 2008.

The main task for strategy-based margining technology is to take advantage of complex option spreads. For this purpose, new special optimization algorithms must be developed which can meet the computing-time demands made by margining large batches of accounts. Note that margining batches of accounts can be performed in a much more efficient way than solving a sequence of AMM problems for separate accounts [Oron et al., 2009].

The practice of margin calculations has a long history. However, the science and art of margin calculations has only just begun to evolve. We hope that this paper will attract the attention of margin regulators and academic researchers who are involved in studying efficient exits from the current economic crisis.

## 7 Acknowledgments

The authors are grateful to Patrick Nobel for his help on pricing options and Eddie Anderson whose critical comments helped to improve this paper.

## References

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *J. Pol. Econ.*, 81:637–654.

Brennan, M. J. and Schwartz, E. S. (1977). The valuation of American put options. *J. Financ.*, 32(2):449–462.

Cohen, G. (2005). *The bible of options strategies: the definitive guide for practical trading strategies*. Financial Times Prentice Hall. Pearson, New Jersey.

Cox, J. and Rubinsein, M. (1985). *Options Markets*. Prentice-Hall.

Curley, M. T. (2008). *Margin trading from A to Z: A complete guide to borrowing, investing and regulation*. John Wiley & Sons, New Jersey.

Daffie, D. (2003). Intertemporal asset pricing theory. In Constantinides, G. M., Harris, M., and Stulz, R. M., editors, *Handbook of the Economics of Finance*, volume 1B of *Handbooks in Economics*, chapter 11, pages 632–742. Elsevier, 1 edition.

Fiterman, A. E. and Timkovsky, V. G. (2001). Basket problems in margin calculations: modelling and algorithms. *European J. Oper. Res.*, 129:209–223.

Fleckenstein, W. A. and Sheehan, F. (2008). *Greenspan's Bubbles: The Age of Ignorance at Federal Reserve*. McGraw Hill Professional, New York.

Fortune, P. (2000). Margin requirements, margin loans, and margin rates: practice and principles. *New England Econ. Rev.*, pages 19–44.

Fortune, P. (2003). Margin requirements across equity-related instruments: how level is the playing field? *New England Econ. Rev.*, pages 29–50.

GAO/GGD-98-153 (July 1998). *Risk-based capital: regulatory and industry approaches to capital and risk*. U.S. General Accounting Office, Washington D.C. 20548.

Geelan, J. P. and Rittereiser, R. P. (1998). *Margin regulations and practices*. NYIF Corporation, Division of Simon & Schuster, Inc., New York, 4 edition.

Kupec, P. H. (1998). Margin requirements, volatility and market integrity: what have we learned since the crash? *J. Financ. Serv. Res.*, 13(3):231–255.

Luckett, D. G. (1982). On the effectiveness of the federal reserve's margin requirements. *J. Financ.*, 37(3):783–795.

Matsypura, D., Oron, D., and Timkovsky, V. G. (2007). Option spreads: centipedes that cannot have more than 134 legs. Technical report, The University of Sydney.

Moore, T. G. (1966). Stock market margin requirements. *J. Polit. Economy*, 74(2):158–167.

Oron, D., Steiner, G., and Timkovsky, V. G. (2009). The bipartite margin shop and maximum red matchings free of blue-red alternating cycles. *Discrete Optim.*, 6(3):299–309.

Roll, R. (1989). Price volatility, international market links, and their implications for regulatory policies. *J. Financ. Serv. Res.*, 3:211–246.

Rudd, A. and Schroeder, M. (1982). The calculation of minimum margin. *Manage. Sci.*, 28(12):1368–1379.

# A Input Data

Table 3: Maximal Portfolio with 32 options on IBM stock at $84.92 selected as of the end of day of January 16, 2009, from `http://finance.yahoo.com/`. All options expire on Friday, April 17, 2009. For pricing options we used interest rate 0.3% and historical volatility 15% (since 2000).

| # | exercise price | call option | last price | put option | last price |
|---|---|---|---|---|---|
| 1 | 45 | IBMDX.X | 39.70 | IBMPX.X | 0.45 |
| 2 | 50 | IBMDU.X | 35.50 | IBMPU.X | 0.67 |
| 3 | 55 | IBMDV.X | 31.90 | IBMPV.X | 1.00 |
| 4 | 60 | IBMDL.X | 25.30 | IBMPL.X | 1.45 |
| 5 | 65 | IBMDM.X | 21.50 | IBMPM.X | 1.90 |
| 6 | 70 | IBMDN.X | 17.30 | IBMPN.X | 2.70 |
| 7 | 75 | IBMDO.X | 13.50 | IBMPO.X | 3.90 |
| 8 | 80 | IBMDP.X | 10.10 | IBMPP.X | 5.34 |
| 9 | 85 | IBMDQ.X | 7.10 | IBMPQ.X | 7.38 |
| 10 | 90 | IBMDR.X | 4.63 | IBMPR.X | 10.00 |
| 11 | 95 | IBMDS.X | 2.85 | IBMPS.X | 14.83 |
| 12 | 100 | IBMDT.X | 1.75 | IBMPT.X | 17.02 |
| 13 | 105 | IBMDA.X | 0.95 | IBMPA.X | 21.50 |
| 14 | 110 | IBMDB.X | 0.50 | IBMPB.X | 26.03 |
| 15 | 115 | IBMDC.X | 0.20 | IBMPC.X | 28.40 |
| 16 | 120 | IBMDD.X | 0.15 | IBMPD.X | 32.90 |

# B Graphs of Margin Requirements

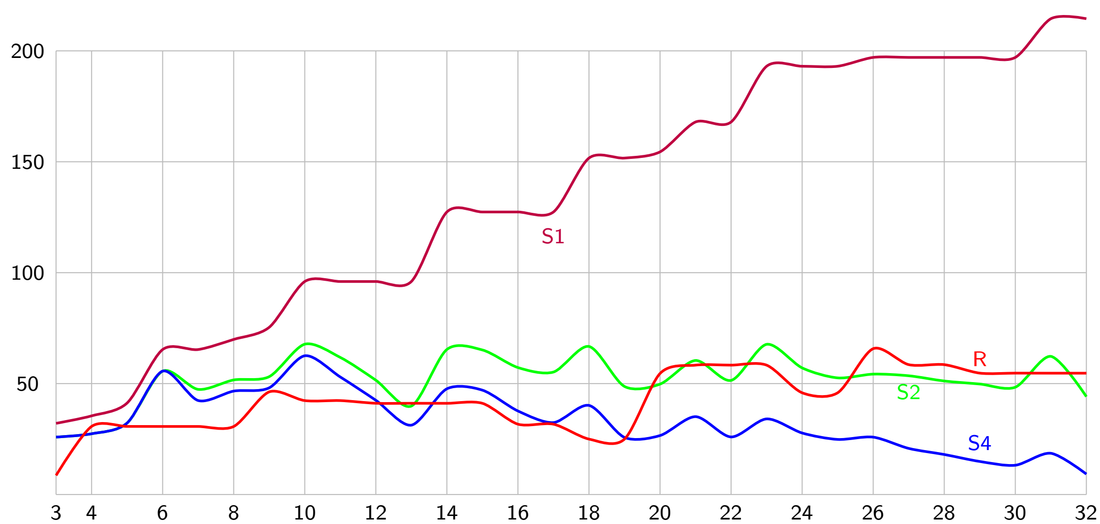


Figure 6: Symmetric scenario.

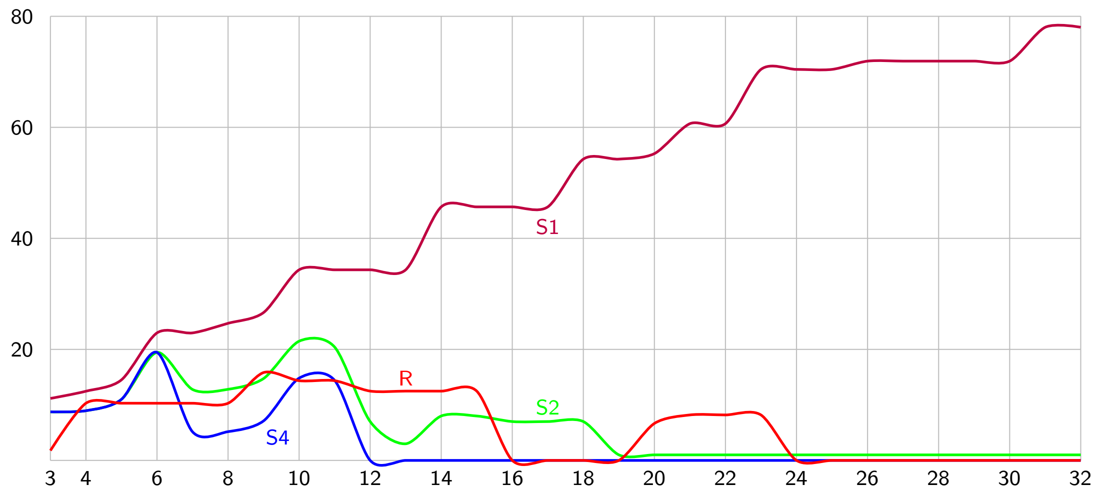


Figure 7: Asymmetric scenario 8lo+2sh.

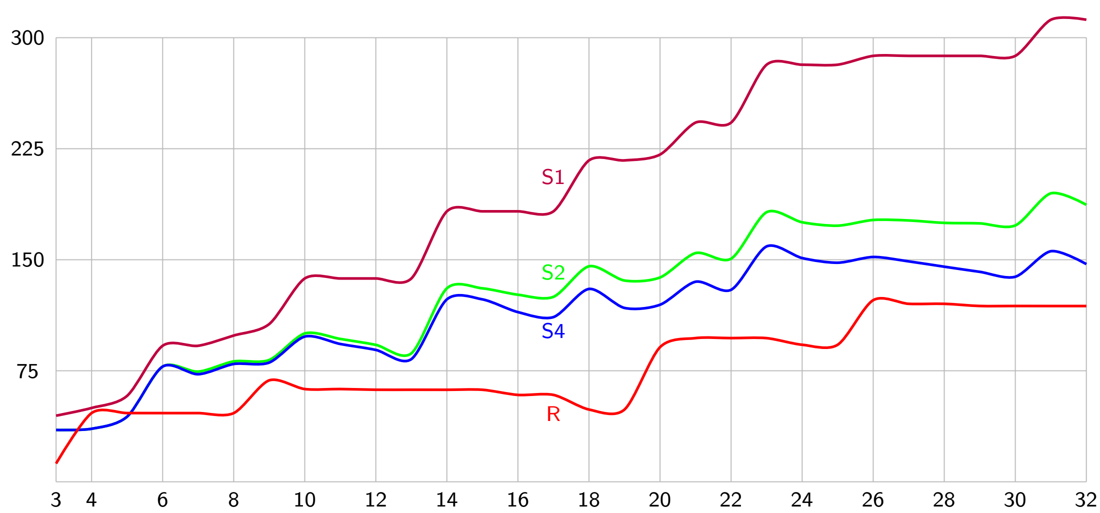


Figure 8: Asymmetric scenario 2lo+8sh.

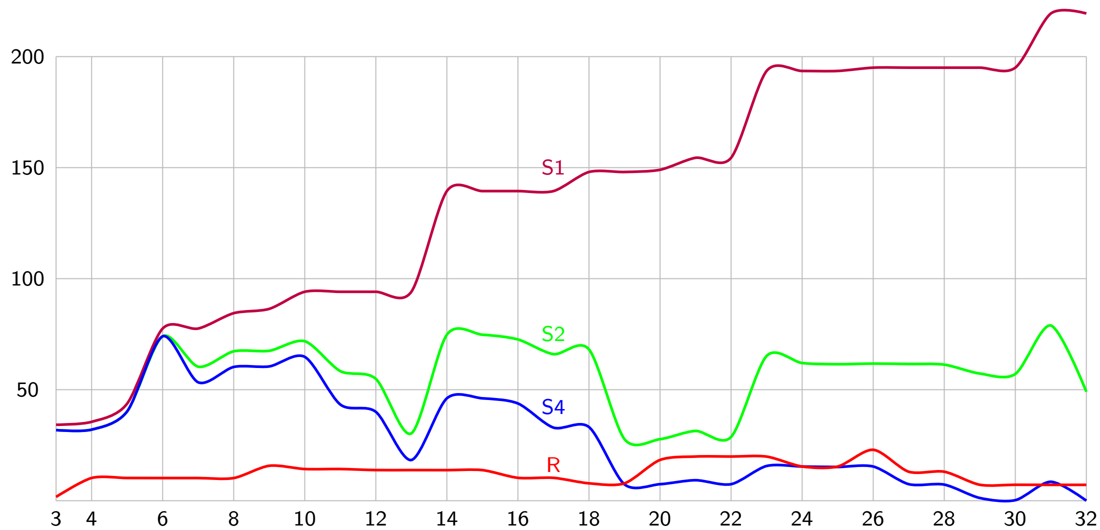


Figure 9: Asymmetric scenario 8ca+2pu.

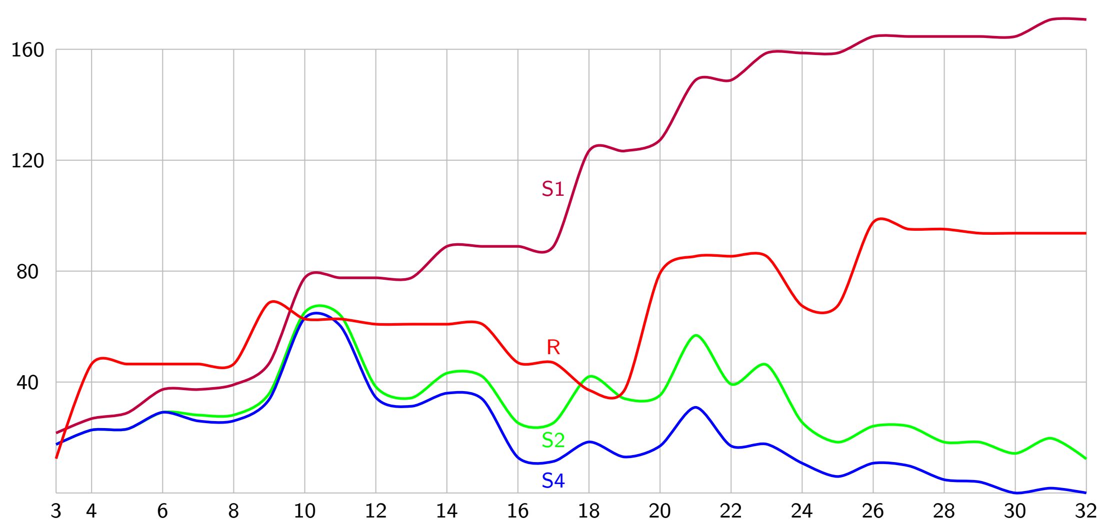


Figure 10: Asymmetric scenario 2ca+8pu.

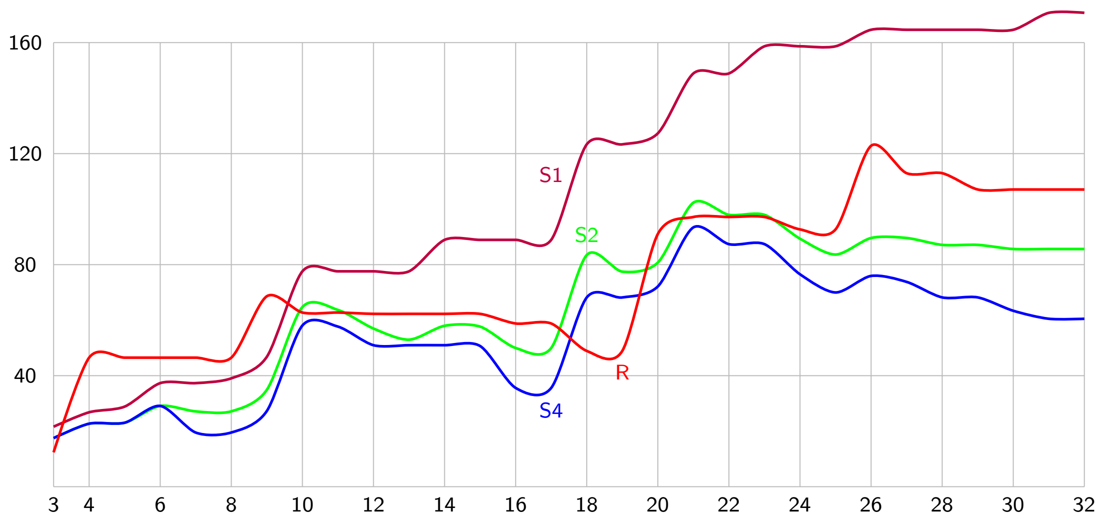


Figure 11: Asymmetric scenario 8bu+2be.

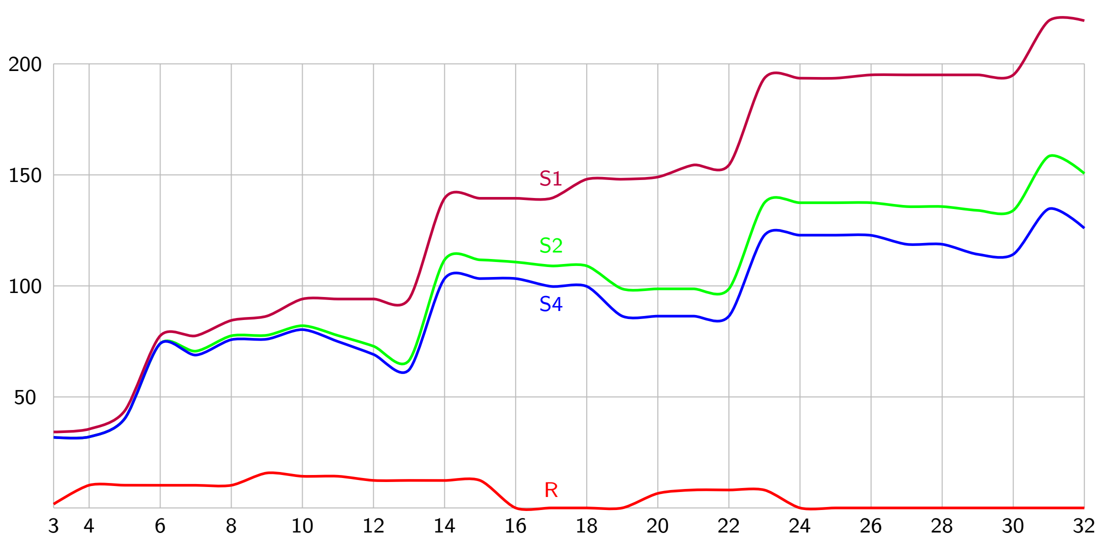


Figure 12: Asymmetric scenario 2bu+8be.