March 31, 2014

Elizabeth M. Murphy, Secretary
Securities and Exchange Commission
100 F Street, NE
Washington, DC 20549-0609

RE: File Number S7-03-13, Money Market Fund Reform

Dear Secretary Murphy:

We submit as commentary on the proposed Reform the attached paper: “Proposed Money Market Fund Regulation: A Game Theory Assessment.”

This paper presents a game theory analysis of the SEC’s June 2013 proposals for reform of Money Market Funds (the “Release”). Game theory is relevant to this policy debate as regulators, particularly FSOC, have depicted investor behavior using terminology of shareholder runs and first mover advantage – a framework classically employed in game models of bank runs.

The paper is responsive to various questions raised in the Release. We demonstrate that when implemented properly, the Fee/Gate alternative would effectively halt and even prevent runs from taking place. However, the alternative of moving to a fluctuating net asset value would neither halt nor prevent runs. The alternative of combining Fees/Gates with a fluctuating net asset value is found to be inferior to Fees/Gates alone because it would create an economically inferior product that would inevitably promote regulatory arbitrage without materially reducing run risk beyond the features of the Fee/Gate alternative.

The paper describes these issues in detail, both with regard to framing the final rule and in stating the requisite powers and responsibilities of directors. We believe that the current policy debate inside the Commission needs to reflect this perspective on the ability of Fees/Gates to provide a robust policy solution and adequately protect investors from first mover risks.

We thank you for your consideration.

Sincerely,

Anthony J. Carfang, Partner

Cathryn R. Gregg, Partner

Enclosures

cc: The Honorable Mary Jo White, Chair
The Honorable Luis A. Aguilar
The Honorable Daniel M. Gallagher
The Honorable Kara A. Stein
The Honorable Michael S. Piwowar
Proposed Money Market Mutual Fund Regulations: A Game Theory Assessment

ANTHONY J. CARFANG
CATHRYN R. GREGG
DAVID C. ROBERTSON
MICHAEL HUNSTAD
Treasury Strategies, Inc.
309 W. Washington Street, Suite 1300
Chicago, IL 60606

March 31, 2014
Abstract. This paper presents a game theory analysis of the SEC’s June 2013 proposals for reform of Money Market Funds (MMFs). Game theory is relevant to this policy debate as regulators have depicted investor behavior using terminology of shareholder runs and first mover advantage – a framework classically employed in game models of bank runs. The paper demonstrates that when implemented properly, the Fee and/or Gate alternative would effectively halt and even prevent runs from taking place. However, the alternative of moving to a fluctuating net asset value would neither halt nor prevent a run.
# Table of Contents

Introduction ........................................ 4  
I. Proposed Money Fund Regulations .............. 7  
II. Regulatory Objectives and the Game Methodology 8  
III. Expected Utility Theory ......................... 11  
IV. The Decision Framework ......................... 12  
V. The CNAV Model with Shadow Price and Market/Credit Risk 13  
VI. A Two-Period CNAV Model with Shadow Price, Market/Credit Risk and Switching Costs 14  
VII. The Two-Period CNAV Model with Switching Costs and Fire Sale Considerations 17  
VIII. The Regulatory Proposals and Game Outcome Summaries 18  
IX. Combining VNAV with Fees/Gates ............ 29  
X. Concluding Remarks ............................... 30  
References ............................................. 32  
Appendix A: The Prisoner’s Dilemma .......... 34  
Appendix B: Utility Functions .................... 37
Introduction

The Securities and Exchange Commission (SEC) recently developed a set of proposed regulations for money market funds (MMFs).¹ Throughout the debate surrounding the need for additional reforms, regulators have referenced elements of game theory with their discussion of “first mover advantage.” In particular, in its November 2012 Rule Proposal for MMFs, FSOC employed the first mover concept in justifying its proposed reforms, which differ from the SEC’s June 2013 proposals.² The Federal Reserve has similarly employed first mover concepts in arguing against the SEC’s Alternative 2 proposal (Fees and/or Gates) ³. For these reasons, it is logical to employ game theory to evaluate the efficacy of the SEC’s proposals (the “Release”).

Game theory is a well-established body of thought that is used to model competitive economic behavior. It describes how economic agents make rational decisions under uncertainty when the actions of other agents can also affect the outcome. Because of its sound methodological underpinnings and widespread applicability, game theory has been employed in numerous theoretical, business and military contexts. In the present context, it is useful in exploring the issue of run risk in MMFs. The SEC must balance this risk against other public policy objectives that are not captured by game theory, such as the effectiveness of MMFs in the capital markets.

The debate concerning MMF reform has now developed an extensive literature that addresses the costs and benefits of the proposed alternatives in terms of the impact on fund shareholders and issuers, as well as the broader macro effects on capital formation, efficiency, competition and systemic risk. For several viewpoints on potential risk and reform see Fisch and Roiter (2011),⁴ Bengtsson (2010),⁵ HSBC (2011),⁶ Jank and Wedow (2010),⁷ Kacperczyk and Schnabl (2012)⁸ and Investment Company Institute (2012).⁹ This paper contributes to the debate by providing a microeconomic analysis of investor behavior under the proposed alternatives. This is necessary in order to rigorously evaluate likely investor behavior under each regulatory regime and free the analysis from mere conjecture regarding the outcomes. This is particularly relevant in evaluating the issue of investor protection and board responsibilities under the various alternatives.

---

³ http://www.sec.gov/comments/s7-03-13/s70313-111.pdf
We analyze four primary questions:

- Will requiring a variable net asset value (VNAV) prevent or stop a run on money fund assets?
- Will maintaining a constant net asset value (CNAV) and providing MMF board discretion to impose liquidity or redemption fees (Fees) prevent or stop a run on money fund assets?
- Will maintaining a constant net asset value (CNAV) and providing MMF board discretion to impose a temporary redemption gate (Gates) prevent or stop a run on money fund assets?
- Will combining VNAV with Fees and/or Gates prevent or stop a run on money fund assets?

Conclusions

Our analysis considers the distinction between stopping a run in progress (“stopping”) versus preventing a run from starting (“preventing”). Public policy recognizes both objectives in order to avoid fire sales, prevent contagion and to protect investors from the “first mover” effects of other investors (which particularly relates to preventing runs). The rule-making challenge is to balance these goals in a way that preserves the integrity of MMFs as securities (and thus the allocational efficiency of markets, a primary SEC mandate) while minimizing risks to financial stability (a primary FSOC mandate). On the basis of our analysis, we reach several conclusions:

- A variable NAV structure is ineffective at either preventing or stopping a run. Compared with the SEC’s other proposals, it removes the apparent motivation to run that results from a potential shadow price deviation from $1. However, the neglect of real-world switching costs that investors face in changing investment programs causes this effect to be overstated. In a significant stress event, the effect is a minor determinant of behavior.
- Both the Fee and Gate alternatives are effective in stopping a run in progress. Gates do so by definition. Fees stop a run provided that the Fee is of sufficient magnitude. In either case, it is essential that fund boards have the latitude to implement Fees or Gates when they deem necessary. In particular, a requirement that boards wait until the end of day would render these alternatives ineffective.
- Fees and Gates are also effective in preventing runs, provided that boards are sufficiently preemptive in their actions. Investors must believe that they will be unable to redeem in a way that disadvantages other shareholders. Game theory provides an analytic prescription for how boards must set Fee/Gate policy that is based on setting investor expectations.
- The SEC has provided an extensive discussion of the factors relevant to implementing Fees or Gates. In particular, Section III.B of the Release identifies the key principles that underlie an effective policy and tradeoffs among them. We find that effective run prevention is attainable within the
approaches contemplated by the Release, while requiring that fund boards be
given discretion to take protective action.10

- Fees and Gates have been primarily analyzed in terms of their ability to stop or
  prevent runs – particularly those resulting from liquidity or credit stress events
  in which fund values are impaired, but markets remain continuous. However,
  recent developments make it apparent that markets may be discontinuous.
  Events such as cyber attacks, market shutdowns (or glitches) and extreme
  weather events, to list a few, disrupt market continuity with some frequency.
  This is especially important if the disruptive event does not impact all markets to
  the same extent or if some markets remain open. Regulators should therefore
  also consider allowing directors to impose very short-term Gates when there are
  business continuity events in which it may not be in the best interest of
  shareholders to make continuous redemptions. In these circumstances, fund
  directors and advisors may need a pause in order to make critical decisions on
  behalf of shareholders. Such power would complement the existing ability of
  funds to delay settlement for up to seven days.

- Fees and Gates fill what are currently gaps in Rule 2a-7. At present, a fund that
  breaks a dollar has no choice but to liquidate. Fees and Gates provide a
  framework for a fund to bridge such periods, and continue to operate afterwards.
  Fees enable investors to access their liquidity, but at a price. That price may (and
  probably will) exceed the discount of the shadow price to $1 during a crisis, but
  that is the cost of being able to assure that a stable NAV product will not cause
  contagion or fire sales during such periods. Gates do not impose an extra fee on
  shareholders but have the effect of restricting access to liquidity during critical
  periods. Together, Fees and Gates provide fund boards with powerful tools to
  prevent a run from materializing, to stop a run in progress, and to assure that a
  stress event does not cause contagion or fire sales.

- The proposal to combine VNAV with Fees/Gates would be effective at both
  stopping and preventing runs, due almost entirely to the effect of Fees/Gates. It
  would also eliminate any first mover effects of an initial shadow price beneath
  $1. However, it is likely that this proposal is completely ineffective as a
  practical matter due to the inevitable regulatory arbitrage. Revising Rule 2a-7 in
  this manner would largely eliminate the distinct meaning of a “Money Market
  Fund” as it is currently understood and would make these funds inferior to near
  substitutes that do not hold themselves out as 2a-7 funds.

- The combination of VNAV with Fees/Gates is an example of a policy
  prescription that weighs systemic risk concerns so heavily that investment utility

10 Note that rule 2a-7 already mandates board action in the relevant circumstances, a fact pointed out
in the comment file. See comments of Federated Investors, Inc., Section 4.3, pp. 9 – 11, (Sep. 16,
requires a Board to meet whenever it [“believes the extent of any deviation from the money market
fund's amortized cost price per share may result in material dilution or other unfair results to investors
or existing shareholders, [and] cause the fund to take such action as it deems appropriate to eliminate
or reduce to the extent reasonably practicable such dilution or unfair results.”]”
of the resulting product is undermined. Other rules could have a similar effect. For instance, our analysis shows that extreme preemptive action by boards in imposing Fees could be so effective that, while eliminating run risk, the resulting penalty to access liquidity could become unduly burdensome. In balancing these competing objectives, it is possible that directors should be allowed to set Fees that are an adequate deterrent to runs, but still with reasonable proportionality to transaction costs (and any current discount of the shadow price to $1), so they do not improperly penalize redemptions.

I. Proposed Money Fund Regulations

On June 5, 2013, the Securities and Exchange Commission (SEC) proposed regulations that would include additional restrictions for institutional prime money market mutual funds and tax-exempt money market funds (MMFs). One proposed reform alternative would require MMFs to sell and redeem shares based on a variable net asset value (VNAV) rather than the constant $1.00 per share net asset value determined by amortized cost (CNAV). A second reform alternative of the SEC proposal allows CNAVs, but would grant fund boards the discretion to impose liquidity fees (Fees) if the level of weekly liquid assets falls below a specified threshold. This alternative also would grant fund boards the discretion to suspend redemptions or "gate" the fund (Gates) if the level of weekly liquid assets falls below the proposed threshold.

These proposals follow significant and important Rule 2a-7 reforms enacted in 2010 that did much to address MMF risk. They substantially reduced the likelihood of a fund experiencing a run and were executed in a way that did not impair the money fund business. They mandated:

- Shorter maturity limits
- Periodic stress testing of circumstances that might result in the fund breaking a dollar
- Increased transparency of portfolio holdings and valuations
- The requirement that MMFs seek to determine liquidity needs of shareholders and plan accordingly (Know Your Customer requirements), and maintain a buffer of highly liquid assets from which to pay redemptions

With the success of these 2010 changes, the current proposals may be viewed as attempts to address remaining concerns. Regardless of which, if any, proposals are implemented, the ultimate success or failure of these proposed regulations will depend on how MMF investors respond to the new rules.

During a period of economic stress (a stress event), each individual investor's behavior is influenced by his perception of whether a certain action will make that investor better or worse off, and by the anticipated actions of other investors. This is true regardless of the stress event’s cause, which may be precipitated by a lack of liquidity, a negative credit event, or misinformation. As a result, the likelihood of a run on MMFs and contagion to other financial sectors depends on the decisions investors make in the face of a stress event.
II. Regulatory Objectives and the Game Methodology

There appear to be two primary drivers behind the SEC’s proposed regulations:

- Reduce what the SEC perceives to be money fund vulnerability to heavy redemptions in times of financial stress, sometimes referred to as a run. It believes these redemptions could trigger fear and contagion across other MMFs or asset classes, leading to widespread asset sales, volatility and losses – primarily on credit and credit-like instruments. As such, these proposed rules would apply to prime MMFs and municipal tax-exempt MMFs that carry explicit credit risk rather than federal agency or U.S. Treasury MMFs.

- Eliminate first mover advantage to prevent relatively unsophisticated investors from being exposed to losses or illiquidity that might be avoided by more proactive investors.

To assess the ability of the proposed reforms to meet these objectives, we adopt a game theory formulation similar to Diamond and Dybvig’s (1983) classic analysis of bank runs. For simplicity, assume there are only two investors and one fund in the market; however, results can be generalized to multiple investors and funds. For this illustration, we assume there is a stress event impacting the market that creates a perception of risk in a prime fund. Before the risk is resolved, investors are faced with the choice of remaining in the MMF or redeeming. The figure below illustrates the choices that each investor can make, and the four possibilities: both Stay, both Redeem, Investor 1 Stays but Investor 2 Redeems, and Investor 1 Redeems but Investor 2 Stays.

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Investor 2</th>
<th>Desired Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>Stay, Stay</td>
<td>First Mover Advantage</td>
</tr>
<tr>
<td></td>
<td>First Mover Advantage</td>
<td>Run</td>
</tr>
<tr>
<td>Redeem</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Regulatory objective
<table>
<thead>
<tr>
<th>The Diamond Dybvig Model</th>
</tr>
</thead>
</table>
| Diamond and Dybvig (1983)\(^{11}\) addressed the issues of financial contagion and first mover advantage in the context of bank runs. Many view their model as the classic model of run risk. In that paper, the authors present a two-period model of consumption which assumes two types of agents: some that only consume in period 1 and some that only consume in period 2. Each individual agent is given an initial endowment of money at time 0, but all are uncertain as to which period they will consume and will not learn so until period 1.

There also effectively exists at time 0 an option to purchase an interest-bearing asset (a bond with rate of return) with a maturity of two periods. If purchased, the initial endowment is unavailable until period 2, thus the asset is completely illiquid until maturity. If an agent purchases the asset and discovers he must consume in period 2, then his utility is increased by the amount of return generated. However, if the asset is purchased and he subsequently discovers he must consume in period 1, then his utility is reduced as he lacks the liquidity to consume. Conversely, if the asset is not purchased, then liquidity to consume exists at any time, but no return is earned and utility is not increased.

Diamond and Dybvig show that due to the differences in consumption preferences, agents actively seek to share risks. This risk sharing is facilitated through the creation of banks that provide liquidity and, thus, guarantee a reasonable return when the investor must withdraw before maturity (i.e., consume in period 1). The authors conclude that banks issuing demand deposits can provide better risk sharing among people who need to consume at different times.

However, Diamond and Dybvig also show that the demand deposit contract providing this improvement has an undesirable equilibrium during a stress event (i.e., bank run) in which all depositors panic and withdraw immediately, including those who would prefer to leave their deposits in if they were not concerned about the bank failing. These depositors are motivated by the desire to be first in line to withdraw so as to avoid absorbing losses associated with distressed asset sales. In short, the illiquidity of bond assets provides the rationale for both the existence of banks and for their vulnerability to runs.

One of the issues in bank runs is a two-way lack of transparency problem – depositors do not know what risks are in the bank’s portfolio and the bank does not know when depositors need to withdraw their deposits for liquidity needs. The 2010 SEC reforms for money market funds help mitigate the problems that make a bank’s portfolio vulnerable to runs: lack of transparency, illiquidity and credit risk.

Diamond and Dybvig see suspension of convertibility (halting withdrawals from bank accounts) as a reasonable solution when deposit insurance is not available. This is analogous to the idea of redemption Gates for MMFs during similar stress events.

---

Finally, Diamond and Dybvig cite Bernanke (1983), asserting that bank runs are significant and harmful because banks intermediate between savers and small business borrowers who otherwise have no alternative way to reach credit markets. The cost of bank runs has also been analyzed by Friedman and Schwartz (1963), Fischer (1911) and Bryant (1980). Money market funds, in contrast, invest in notes of large, rated and often public issuers who have alternative access to capital markets.

In the game shown in Figure 1, it is clear that the (Stay, Stay) strategy in the top left quadrant, where both investors choose Stay, achieves the most stable outcome. Both investors remain in the fund and a run is averted.

Consider (Redeem, Redeem) in the bottom right quadrant. Here, each investor chooses Redeem, probably as quickly as possible, so as not to incur future losses in case the stress event does cause a loss. As a result, the fund may need to liquidate portfolio securities at fire-sale prices, creating losses. This is the outcome regulators are trying to prevent or limit.

In the bottom left quadrant, Investor 1, seeking first mover advantage, chooses Redeem and Investor 2 chooses Stay (Redeem, Stay). To meet Investor 1’s redemption, the fund may have to liquidate some holdings, perhaps at a loss, even if the stress event results in no other losses. Investor 1 incurs no loss due to having redeemed first, at the $1 constant net asset value. However, Investor 2 may incur loss, if the actual fund value has dipped below $1. Vice versa for the top right quadrant (Stay, Redeem).

These two quadrants illustrate versions of the first mover advantage that regulators wish to avoid – where early redeemers leave remaining shareholders with a portfolio that may be impaired by losses or illiquidity. First mover is a concept of game theory that can define relative advantage in competitive games like the ones addressed in this paper.

Game theory analysis concerns how investors make their decisions to Redeem or Stay, and how the presence of other investors influences the outcome. One investor, in isolation, might choose Stay and ride out the storm. However, in the presence of another investor who chooses Redeem, the first investor may actually be better off by choosing Redeem. The run scenario is the plausible equilibrium outcome of the game, even though the loss associated with the stress event may not occur. This will be the case as long as the investor perceives the expected outcome of redeeming is better than that of any other choice.

16 This may also be the “best” outcome if the concern of a stress event turns out to be unfounded.
17 Note that in selecting a strategy, the investor does not know the exact behavior of other investors, but can assess the circumstances faced by all investors and deduce what might be optimal for other
One of the SEC’s regulatory objectives is to enact policies that incent investor choices that will prevent a run and possible follow-on contagion. In terms of the game above, that objective requires policies that would move the equilibrium to the top left quadrant (Stay, Stay). If this objective can be met, runs can be prevented, first mover advantage can be eliminated and overall utility can be maximized. In this paper, we will analyze whether any of the SEC’s proposed rules can actually achieve this objective. In Appendix A, we outline another classic model of game theory, the Prisoner’s Dilemma, that bears some formal similarities to the choices investors make in deciding to redeem or retain MMF holdings.

III. Expected Utility Theory

To model the games and test the effects of the proposed regulations, we must first understand how investors value the potential outcome of those games and make investment choices (Redeem or Stay) under uncertainty regarding future market events. Expected Utility Theory is a cornerstone of modern financial economics that was developed along with game theory as a framework for modeling behavior under uncertainty. As in most game theory applications, we employ that framework here.

We assume each investor possesses a utility function based on wealth that reflects the preference over return (wealth) outcomes. The wealth utility function measures how much better off an investor is, as his wealth rises. Utility rises with wealth. However, it rises at a diminishing rate as shown in Figure 2, a property called concavity. As a consequence, a loss of a given magnitude reduces utility by more than the same gain increases utility. For this reason, investors would not make an investment that gives them an equal chance at the same dollar gain or loss. A positive expected return is required in order to induce the investor to take a risk. This is how risk aversion is reflected in financial decisions and asset prices.

Figure 2. Illustrative Utility of Wealth Function

investors to do. In the Redeem, Redeem scenario, each investor may try to be the “first mover,” even though each may anticipate that others are doing the same.
Most financial models of investor behavior begin by specifying how different investment choices lead to different return outcomes or distributions. In concept, the investor: (i) identifies the differing wealth outcomes associated with each possible investment decision; (ii) assigns a probability to each outcome; and (iii) calculates the utility of each outcome. The investor selects the investment that maximizes the expected, or probability-weighted utility. We employ this framework here.

Our game analysis begins with an unspecified stress event. That is, something in the marketplace happens that may possibly impact an MMF’s value. In the analysis, each investor has two choices: to keep his investment in the fund or to redeem his investment. The outcome of either choice is uncertain, because the investor cannot be 100% sure of the value of the fund due to the stress event. However, the investor estimates possible outcomes, the associated probabilities, and the expected utility of each choice. He then selects the action he believes will maximize expected utility. Additional characteristics and implications of the investor utility function are discussed in Appendix B.

IV. The Decision Framework

In this section, we discuss the situation faced by MMF investors and the decisions they could make in the context of a game theory model. We then summarize the outcome of that game, in the context of a stress event within a single fund.

The illustration, shown in Figure 3 below, develops our game methodology using a highly simplified exposition. We first model the current MMF CNAV structure – that is, without new regulations – and illustrate the issue regulators believe they are addressing with these proposals. We then introduce a more detailed framework that captures the complexity of the current debate.

We assume a market stress event that creates the risk of loss. Investors must decide whether to stay invested in the MMF or redeem, and they select the course of action that maximizes expected utility. Faced with a stress event, each investor can either redeem now at $1 per share, or stay invested in the fund but face a possible decline in value below the $1 constant NAV (i.e., the stress event may turn out to be real and negatively impact the fund’s portfolio value). If both investors stay in the fund, they each face a possible outcome of their investment value dropping below the $1 NAV. They would share any loss equally. If one investor stays in the fund and the other redeems, the one that stays faces the possible outcome of less than $1 NAV, while the other redeems at $1 NAV and has no loss. If there were a loss, the investor that stays would bear the entire loss himself. If both investors redeem, they would both exit at $1 NAV, because the redemption occurs before the loss, if any, is realized.18

Each investor assesses probabilities for each return outcome. In the scenario we have depicted, the outcome that maximizes utility for each is (Redeem, Redeem). The reason is simple: by redeeming, neither investor can be made worse off and might be better off. This is labeled in the figure as a Nash Equilibrium, an important concept in game theory studied by the noted mathematician John Nash. When each investor knows the choices available to

18 This illustration assumes a simplistic framework by shareholders and ignores the potential that if both investors redeem, a fire sale of assets could cause losses. This possibility is discussed in later sections as we introduce complexity into the model.
the others, and has made a decision (Stay or Redeem), and neither investor can be made better off by a unilateral change, then the current choices and the resulting outcomes are a Nash Equilibrium. Economists view such outcomes as strongly predictive because the theoretically optimal behavior is so unambiguous.

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>Possibly &lt;$1, Possibly &lt;$1</td>
<td>Possibly &lt;$1, $1</td>
</tr>
<tr>
<td>Redeem</td>
<td>$1, Possibly &lt;$1</td>
<td>$1, $1</td>
</tr>
</tbody>
</table>

Nash Equilibrium

Figure 3. Investor payoff matrix

Regulators have argued that, as a practical matter, first movers create a contagion effect that draws others into the run, and can even spread across funds. In the case discussed here, the Nash Equilibrium is for both investors to Redeem. If regulations can be designed that eliminate the optimality of running, then regulatory objectives are met, even if some residual (or idiosyncratic) first movers still remain. Clearly, the SEC’s intent with the proposals is to achieve this. The question is whether any of alternatives will be effective.

The introduction of a variable NAV or Fees and Gates can alter the investor’s expectation of return outcomes and shift optimal behavior. For instance, in a Fee scenario, the probable return associated with a decision to Redeem must be adjusted by the Fee to be paid. In a Gate scenario, the probable return of a decision to Stay or Redeem must be adjusted by the consequence of temporarily not having access to one’s liquidity. In the subsequent sections, we further develop our game framework and then summarize each of the SEC’s rule proposals to determine their effect on run risk and investor behavior.

V. The CNAV Model with Shadow Price and Market/Credit Risk

The model shown in Figure 3 makes highly simplified assumptions regarding the choices investors face. We now introduce additional features that provide a more realistic depiction of the current CNAV model. In this section, we consider the shadow NAV and interest rate/credit risk.19

In exchange for accepting a small (but non-zero) risk, MMFs provide investors with liquidity and a competitive rate of return that approximates the average yield on very short-term securities. Treasury funds are thought to contain interest rate risk only, while prime funds contain both interest rate and credit risk. MMFs now generally report the shadow

---

19 Shadow NAV is the good faith estimate of the fair value of the fund share price. It is generally computed as the unrounded mark-to-market or mark-to-model net asset value fund share.
NAV (that we denote “S”). S is the good faith estimate of the fair value of the fund share price, but is subject to inaccuracies due to the data limitations in the models employed.20

Shadow NAVs continuously fluctuate, even if infinitesimally. This is because of ongoing market-wide fluctuations in interest rates or general spreads (up or down). We will express this risk by writing the shadow price as $S = 1 + \varepsilon$, where $\varepsilon$ (called “epsilon”) represents a price deviation that can be positive or negative. In rare events, such as during September 2008, credit events can be an additional source of risk in shadow NAVs. Here, any unforeseen credit losses that are concentrated in a particular sector or security can cause further losses in a prime MMF. We express this price component as $\eta$ (called “eta”). The overall prime fund shadow price is therefore expressed as $S = 1 + \varepsilon + \eta$, where $\varepsilon$ and $\eta$ represent price deviations due to changes in the overall level of rates (or spreads) and specific credit events, respectively.

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Investor 2</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>S, S</td>
<td>S, $1</td>
<td></td>
</tr>
<tr>
<td>Redeem</td>
<td>$1, S</td>
<td>$1, $1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.

The model including these extensions is expressed in Figure 4. When faced with a possible stress event (market-wide shock to rates/spreads or a specific credit event), investors must decide to Redeem at $1 or stay in the fund and realize S (if the event occurs). In either case, the optimal decision is for both investors to Redeem. Shareholders are not worse off, but might be better off by choosing to Redeem.

VI. A Two-Period CNAV Model with Shadow Price, Market/Credit Risk and Switching Costs

In applying the model depicted in Figure 4, regulators also consider the distinction between: (i) the current or starting value of the shadow price, versus (ii) the value that investors might expect it to be at the end of the next investing period. This is because it is common for investors to remain in a fund after there has already been a deviation of the shadow price (from $1) caused by prior period market or credit events. Regulators have pointed out that, if the starting value of the shadow price is below $1, and further losses are feared, this gives investors an even stronger reason to redeem, thus serving as an accelerant to a run. In fact, regulators sometimes suggest that this is the origin of “first mover advantage”. We will use

20 In all but the most extreme circumstances, deviations of the shadow price from $1 are de minimis. This is why cost/benefit analysis favors the use of amortized cost (or $1) for shareholder transactions, rather than the shadow price.
S° to represent the current (starting) value of the shadow price and S¹ to denote the value at the end of the next investment period. 21

From the foregoing sections, one may think it is optimal for the investor to Redeem whenever the current shadow price is less than $1. Using our definitions, this is expressed as S° < $1. In fact, the broad historical record demonstrates that shadow prices have fallen below $1 due to increases in the general level of interest rates or credit spreads (i.e., ε, η < 0), but with little or no correlation to redemption activity. Moreover, because changes in prices reflect changes in yields, one cannot claim that any small correlation is driven by a desire to avoid loss rather than a desire to obtain higher yield. There appears to be a materiality threshold that must be crossed before investors redeem, provided there are no immediately expected events that would drive S still lower (when investors have remained in the fund with the shadow price at S°).

The explanation for this effect is that shareholders experience costs in switching investment programs. When shareholders redeem for an unplanned reason, there is time and expense incurred in finding an acceptable alternative. Furthermore, large institutional investors have client relationships with the MMF advisor that could be jeopardized by frequent trading in the affected MMF. This is because advisors are under no obligation to take any given investor’s future subscriptions, and they have a fiduciary obligation to refuse future investments from investors who are deemed disruptive to the fund.

We express the switching costs that investors incur from unplanned redemptions as a reduction in the proceeds received from redemption. Only this net amount is available for reinvestment into an acceptable alternative. Using π (called “pi”) as the expression of switching costs, the net proceeds from unplanned redemption are $1 − π. Using this definition, investors would remain in the fund after a prior period market event as long as S° > $1 − π.

As an illustration, say a shareholder invests at the CNAV of $1 and overall market rates then rise by 25 basis points. Assuming the maximum permitted 60-day average maturity for the fund, the shadow NAV would be expected to decline to approximately $0.9996. Would it make sense for the shareholder to redeem at $1 and reinvest the proceeds at a 25 bp higher rate? If π = 10 bp, the shadow NAV would have to fall to below $.9990, or market rates would have to rise by more than 60 basis points, before it is worthwhile for the investor to redeem and invest in an alternative asset. So in the case of a 25 bp rate increase, the investor would not redeem.

These relationships may now be expressed in Figure 5 below. When S° > $1 − π, investors who stay in the fund until the end of the second period realize the value S¹ = (S° + ε¹ + η¹). Those who redeem receive $1 − π. Under normal circumstances, investors anticipate that η = 0 and |ε| < π so that S¹ > $1 − π and it is not optimal for an investor to redeem. However, when there has been an adverse outcome (or the risk of a sufficiently adverse future outcome) for ε or η, then switching costs do not dissuade an investor from redeeming.

21 Using our definitions, S° is equal to $1 + ε⁰ + η⁰ where ε⁰ and η⁰ are the market-wide or specific credit events that may have caused the current shadow price to deviate from $1. We can write S¹ = S° + ε¹ + η¹, where ε¹ and η¹ are the market-wide or specific credit events that occur during the next investment period that may cause the shadow price to further deviate from its starting value of S°.
because of the risk that $S^1 < S^1 - \pi$ in the future. It would then be optimal for one or both investors to redeem, depending upon their risk assessments for future outcomes. 

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$S^1$, $S^1$</td>
<td>$S^1$, $S^1 - \pi$</td>
</tr>
<tr>
<td>Redeem</td>
<td>$S^1 - \pi$, $S^1$</td>
<td>$S^1 - \pi$, $S^1 - \pi$</td>
</tr>
</tbody>
</table>

Figure 5.

In Figure 5, we have adopted a simplified depiction of the game that reflects investor perception of the payoffs. A more complex alternative payoff matrix that could be more economically exact is shown in Figure 5a. In this matrix, when one investor redeems and the other stays, the remaining investor receives $S^1 - (S^1 - S^0)$ because he is adversely impacted by the dilutive effect of the exiting shareholder redeeming at $S^1$ rather than $S^0$. (Here we assume also that the redemption is not met in a manner that adversely concentrates risk for the remaining shareholder, e.g. by funding the redemption from liquidity while leaving the remaining shareholder with a more concentrated portfolio that may be impaired.) In addition, when both investors redeem, they receive $S^0$ rather than $S^1$ (less switching costs, if applicable).

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$S^1$, $S^1$</td>
<td>$S^1 - (S^1 - S^0)$, $S^1 - \pi$</td>
</tr>
<tr>
<td>Redeem</td>
<td>$S^1 - \pi$, $(S^1 - S^0)$</td>
<td>$S^0 - \pi$, $S^0 - \pi$</td>
</tr>
</tbody>
</table>

Figure 5a.

While this matrix is more mathematically accurate, it is notationally more complex and does not necessarily better reflect investor beliefs, particularly when we consider this game as an approximation to a more realistic N-person format. For instance, the effect on remaining investors of a first mover redemption in an N-person format (with equal investments) becomes $S^1 - (S^1 - S^0)/(N-1)$, which is negligible for large $N$ and small initial departures of $S^0$ from $S^1$. More importantly, Figure 5a does not lead to conclusions that are different from the depiction of the game in Figure 5. In particular, in subsequent sections, our conclusion that it is optimal for investors to be first movers (under particular circumstances) is only increased in Figure 5a due to the fact that redeeming shareholders

---

For example, in the face of a material event such as experienced in September 2008, an existing small deviation of the shadow price from $S^1$ is a minor portion of the loss that investors are hoping to avoid by redeeming.
further dilute remaining shareholders. The equilibrium outcome does not change. For these reasons, we will continue to employ the simpler payoff assumptions as expressed in Figure 5.²³

VII. The Two-Period CNAV Model with Switching Costs and Fire Sale Risk

The model shown in Figure 5 incorporates the relevant features for analyzing the SEC’s rule proposals – in particular, will the Commission’s rule proposals eliminate the risk of runs? However, the model can also be used to depict FSOC’s systemic risk concerns. First, FSOC’s arguments imply that switching costs play little or no role in investor decisions (i.e., π is zero or at most a few basis points). As a result, if the initial shadow price is less than $1 (i.e., $S^0 < 1$), investors will be quick to redeem at an assumed $1 if there is any investment concern for the period ahead. Second, there is the risk of fire sales when both investors redeem.²⁴ We express this cost as “ω” (called “omega”), which appears as a reduction in redemption proceeds when both shareholders redeem.²⁵ These relationships are captured in Figure 6, where we have assumed that $\pi = 0$.

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$S^1, S^1$</td>
<td>$S^\circ, 1$</td>
</tr>
<tr>
<td>Redeem</td>
<td>$1, S^1$</td>
<td>$1-\omega, 1-\omega$</td>
</tr>
</tbody>
</table>

Figure 6.

The analogy between the game depicted in Figure 6 and the Prisoner’s Dilemma described in Appendix A is clear. Even when faced with a possible loss, it is possible that investors would be better off by cooperating (i.e., remaining in the fund) and receiving $S^1$ rather than both realizing $1 - \omega$.

²³ Note also that (in Figure 5a) depicting the payoff in the (redeem/redeem) cell as ($S^0, S^0$) suggests that investors may be less likely to redeem (when they believe the other shareholder is also redeeming) because of the fact that they have both already incurred a loss of $1- S^0$. We do not believe this characterizes investor expectations or behavior.
²⁴ In an N-person game, this would occur when larger numbers of shareholders redeem.
²⁵ Furthermore, fire sales can create a social cost or externality not anticipated or borne solely by these investors. It is imposed on the rest of the economy through contagion that adversely impacts other markets or investors.
VIII. The Regulatory Proposals and Game Outcome Summaries

In this section, we developed a series of three game sets to understand better the effectiveness of the three main SEC proposals, which we summarize as:

- VNAV
- Fees
- Gates

In each game set, we discuss the situation faced by investors and the decisions they could make, and then summarize the outcome of that game, based on a stress event within a single fund.

**Game 1: Will VNAVs Prevent A Run On Money Fund Assets?**

This game considers a regulatory environment where VNAVs are required and MMFs have changed their valuation method from amortized cost to “mark to market.” When there is a stress event, each investor must make the decision to either stay in the fund or redeem, possibly sparking a run. As before, we assume there are just two investors in the fund. This assumption can easily be generalized to more than two players so it is not restrictive. The table below illustrates the choices and payoffs associated with this game.

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Investor 2 Stay</th>
<th>Investor 2 Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>VNAV(^1), VNAV(^1)</td>
<td>VNAV(^1), VNAV(^\circ)-(\pi)</td>
</tr>
<tr>
<td>Redeem</td>
<td>VNAV(^\circ)-(\pi), VNAV(^1)</td>
<td>VNAV(^\circ)-(\pi), VNAV(^\circ)-(\pi)</td>
</tr>
</tbody>
</table>

Figure 7.

Notice that this game is similar to the depiction of the CNAV game in Figure 5. The return relationships, and the associated probabilities, are largely unchanged simply by changing the pricing convention. Therefore, the Nash Equilibrium is for investors to run when faced with similar risks as were discussed under the CNAV game scenario.

This result may be counterintuitive, so we will use a numerical example. We start by re-examining the CNAV game in Figure 5. Assume that market events have already caused the shadow price to decline by 5 bp, so that \(S^0 = 0.9995\). We now assume that investors fear a potential loss of 50 – 100 bp that could result from impairment in a large MMF holding. Based on these assumptions, one or both investors may redeem the fund depending on the probability assigned to the loss.

Now consider the same scenario, but under a variable NAV as in Figure 7. The only difference is that investors have already realized a 5 bp loss due to market events (i.e.,
VNAV\(^0\) = $.9995), but still have not redeemed.\(^{26}\) This realized loss eliminates any motivation to redeem in order to avoid the 5 bp loss, as might be the case under CNAV (holding aside the materiality threshold created by switching costs). However, the potential future loss due to a possible credit event remains at 50 to 100 bp. If investors are assumed to have a similar loss aversion as under the CNAV case, the likely outcome will be similar – one or both investors will redeem.

The only reason to expect that the outcome of the two games would be different would be if investor aversion to losses were different. However, while a differing regulatory regime and pricing convention may change risk awareness, there is no reason to expect a change in risk tolerance among investors that pursue the safety and liquidity of MMFs.

We can conclude that in the face of a stress event, VNAV would not prevent a run on MMFs. In particular:

- The rationale behind the proposal incorrectly presumes that the conversion of CNAV to VNAV would cause investors become more risk tolerant.
- The proposal incorrectly presumes that it is primarily the ability to redeem a CNAV fund at $1 (when the shadow NAV may be less) that creates the motivation to run, rather than the risk of subsequent greater loss.

Game 2: Will CNAV with Board Discretion to Impose Redemption Fees Stop or Prevent a Run?

This game considers a regulatory environment where the MMF board has the right to impose a redemption Fee. Although the decision on Fee imposition rests with fund boards, the ultimate impact of a decision will depend on how investors react. If Fees are imposed, they accrue to the benefit of investors who stay in the fund until the redemption Fee is lifted. We denote the redemption Fee by “\(\delta\)” (called “delta”). The redemption Fee need not be a fixed number that the shareholder knows in advance, if it is imposed.

There are two methods for imposing a redemption Fee. In the first method, when a fund board decides to impose the Fee, it becomes effective after that day’s close of business. We call this the \textit{end-of-day method}. In the second method, when a board decides to impose the Fee, it is effective at the next intraday order settlement. Thus, it impacts all outstanding redemption orders that were not yet settled at the time of the Fee decision. We call this the \textit{intraday method}.

In either of these methods, we must distinguish between: (i) the ability of the Fee to stop a run in progress, versus (ii) the ability of the Fee to prevent a run from occurring.

Game 2a: Stopping a Run in Progress

If the Fee has already been imposed, the analysis is simple: a run will be stopped if the Fee exceeds the potential loss the investor is seeking to avoid. In this case, the Nash Equilibrium will be Stay/Stay (provided that both investors hold a similar view of the potential loss in relation to the Fee). If it is less, the investor will have a motivation to run. The

\(^{26}\) They have not yet redeemed due to the materiality threshold discussed in Section VI (i.e., switching costs) or because they may be content to own the fund at this yield.
corresponding investor payoff matrix is shown in Figure 8, where for simplicity we initially assume that switching costs are zero (i.e. \( \pi = 0 \)). If both investors stay they receive \( S^1 \). If one stays and one redeems, the redeeming investor receives \( $1 - \delta \), and the remaining investor receives \( S^1 + \delta \). If both redeem, they receive \( $1 - \delta \). Investors will stay if \( S^1 > $1 - \delta \) (or \( \delta > S1-S^1 \)). If switching costs are positive, then the condition becomes \( S^1 > $1 - \delta - \pi \) (or \( \delta + \pi > $1 - S^1 \)). The suggested Fee in the SEC’s proposal is 2%, which seems designed to exceed losses that might be realistically expected, so that Stay/Stay is a Nash Equilibrium.  

<table>
<thead>
<tr>
<th>Investor 2</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>( S^1, S^1 )</td>
<td>( S^1+\delta, $1-\delta )</td>
</tr>
<tr>
<td>Redeem</td>
<td>( $1-\delta, S^1+\delta )</td>
<td>( $1-\delta, $1-\delta )</td>
</tr>
</tbody>
</table>

Figure 8.

In the foregoing discussion, the end-of-period value \( S^1 \), and hence the potential loss that investors face (\( $1 - S^1 \)), is of course uncertain. Therefore, the condition \( \delta > $1 - S^1 \) is probabilistic in the investor’s mind. That is, it is true, or not, with a certain probability. Game theory (and its expected utility underpinnings) provides guidance on setting the Fee under these conditions. Specifically the Fee need not exceed the worst-case loss that investors fear. Indeed, it is doubtful that setting the Fee in this way would be in the best interest of investors, or public policy, since it could be unduly burdensome for those who are not running to avoid loss, but simply need access to liquidity. Instead, the expected utility of wealth after imposition of the Fee must less than the expected utility from remaining in the fund. Using this reasoning, risk aversion implies that the Fee must exceed the mathematical expectation of the loss (that is, the probability weighted loss), but may be less than the worst-case fund loss itself, since it is not certain.

As a practical matter, boards may take several approaches in determining the Fee. One approach would simply be to pick a fixed number (such as 2%) that will be seen as large in relation to any expected loss. A more general method would be to base the Fee either on a Value-at-Risk calculation (e.g. a 99% confidence level); or on stress tests using market-implied default probabilities reflected in the yield spreads of the securities in the fund portfolio. A very direct method (that would be closer to a worst-case loss) would be to assume a default of one or more securities (with reasonable recovery rates) and to calculate

27 Note that if the Fee imposed is too small in relation to the feared loss, Redeem/Redeem may be a Nash Equilibrium. However, whether or not it is will be influenced by the fact that one investor may be inclined to stay if he knew the other was redeeming (due to Fees accruing to the remaining shareholder.)

28 Using conceptual notation, we can express this as \( U($1 - \delta) < E[U(S^1)] \), where \( U(\cdot) \) denotes the investor’s utility function and \( E[\cdot] \) denotes expectation. We would then require \( \delta > $1 - U^{-1}(E[U(S^1)]) \) for every investor, where \( U^{-1}(\cdot) \) denotes the inverse of the utility function. This expression states that the Fee must be set such that the utility of the assured wealth upon redeeming (and after the Fee) must be less than the expected utility of remaining in the fund.
the portfolio loss that would result. In calibrating Fees to severe adverse outcomes, boards would need to be cognizant of the tradeoff between eliminating run risk and unduly penalizing investors for accessing liquidity. The current or proposed stress tests required under Rule 2a-7 could easily be adapted to compose an acceptable Fee estimate that is effective but economically fair. We will denote the Fee that satisfies the requirements for Stay/Stay to be a Nash Equilibrium as $\delta^N$.

**Game 2b: Preventing a Run from Occurring**

The analysis is more complex when the Fee has not yet been imposed and the investor must decide a course of action, knowing that a Fee may be imposed in the near future. This is the question we now examine. We first address the issue of whether the Fee can be effective if it is imposed either at the end-of-day or intraday.

**End-of-day Method** – As before, we assume there are two investors in the fund. There is a stress event with possible implications for the MMF. Regulations permit the fund’s Board to impose a redemption Fee, which goes into effect after all today’s trades are settled.

In the face of that stress event, an investor has some expectation of how big a loss the fund may incur and whether other investors are seeking to redeem. He may also possibly benefit from Fees paid by other investors who redeem after the Fee is imposed, if he stays. He knows his own liquidity needs; if he redeems today, he knows that he will pay no Fee, but after today, and until the Fee is lifted, he will incur it if he decides to redeem and if the fund directors have imposed the Fee.

He must balance all these factors together to determine his maximum utility. Figure 9 illustrates the payoffs that each investor faces based on the choice Stay or Redeem during the current decision period (i.e., during the day before a Fee is potentially imposed at end-of-day).

![Figure 9](image.png)

Figure 9.

Notice that this payoff matrix is identical to that shown in Figure 5, that is, the game without a redemption Fee. In this scenario, and under the assumption that a stress event is at hand, the Nash Equilibrium is (Redeem, Redeem) before the end of the day, whenever the expected loss exceeds the switching costs. It is clearly therefore suboptimal for boards to be required to wait until the end-of-day to impose a Fee. The Fee would then have no ability to prevent a run, although it may be able to stop a run in progress from continuing on the second day, depending on the magnitude of the Fee.

**Intraday Method** – Again, there are two fund investors, and a stress event with possible implications for the MMF. Assume under this method that regulations permit the
fund’s Board to impose a redemption Fee, which takes effect during the day and accrues to the benefit of investors who remain in the fund.

In considering a possible redemption, an investor does not know whether he will incur a Fee or not. The Fee itself is therefore uncertain. This makes it similar to $S^1$ in that it is an unknown prior to imposition by the board. When subject to this uncertainty, we will denote the Fee by $\delta^*$. Faced with this uncertainty, the investor must consider the full range of outcomes in deciding whether or not to redeem. He will also weigh the possibility of having to redeeming after today, but before the Fee is lifted. Finally, he will consider the possibility of benefiting from Fees paid by the other redeeming investor.

Figure 10 shows the payoffs to each investor. The game is similar to that shown in Figure 8 except that the uncertain Fee ($\delta^*$) now replaces the certain Fee ($\delta$). (And we have also explicitly shown switching costs, $\pi$.) As before, this game has two possible equilibria. The relative size of the anticipated Fee versus the potential loss determines whether either is a Nash Equilibrium.

- If one investor redeems but the other stays, the one who redeems may have to pay a Fee. The one that stays will experience any potential future loss, but will benefit from Fees paid by the first mover, if any. The probability of the Fee will be important in the investor’s decision process.

- If both investors stay, then each will suffer the potential future investment loss, and each may benefit (or lose) from the effect of any future Fees that are assessed on either or both investors, should one or both redeem in the future.

- If both investors redeem, both face a possible Fee. Here, investors may anticipate the Fee would have a greater likelihood of being imposed than if just one investor redeems.

- As in our prior analysis, investors will select the strategy that maximizes expected utility.

<table>
<thead>
<tr>
<th>Investor 1</th>
<th>Investor 2</th>
<th>Stay</th>
<th>Redeem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stay</td>
<td>$S^1, S^1$</td>
<td>$S^1 + \delta^<em>, S^1 - \pi - \delta^</em>$</td>
<td></td>
</tr>
<tr>
<td>Redeem</td>
<td>$S^1 - \pi - \delta^<em>, S^1 + \delta^</em>$</td>
<td>$S^1 - \pi - \delta^<em>, S^1 - \pi - \delta^</em>$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 10.

To summarize the outcome using utility concepts, if for every investor the expected utility of the payoff $1 - \pi - \delta^*$ is less than the expected utility of the payoff of $S^1$, then staying is a Nash Equilibrium.\(^{29}\) If the opposite is true, redeeming will be the Nash Equilibrium. To illustrate these conclusions, consider a situation in which investors believe that, $\delta^* > 1 - S^1$. For simplicity we assume that $\pi = 0$ (and $S^1 < 1$).

\(^{29}\) That is, if $E[U(1 - \pi - \delta^*)] < E[U(S^1)]$. 

22
Investor 1’s optimal strategy would be:

- If Investor 2 stays, then $S^1 > 1 - \delta^*$, so Investor 1’s optimal strategy is to stay.
- If Investor 2 redeems, then $S^1 + \delta^* > 1 - \delta^*$, so Investor 1’s optimal strategy is to stay.

Similarly, Investor 2’s optimal strategy would be:

- If Investor 1 stays, then $S^1 > 1 - \delta^*$, so Investor 2’s optimal strategy is to stay.
- If Investor 1 redeems, then $S^1 + \delta^* > 1 - \delta^*$, so Investor 2’s optimal strategy is to stay.

Since either investor will choose to stay regardless of the actions of the other investor, (Stay, Stay) is a Nash Equilibrium.

Now consider a situation where investors believe that the potential loss exceeds the potential Fee such that $\delta^* < 1 - S^1$ and for simplicity $\pi = 0$ (and $S^1 < 1$).

Investor 1’s optimal strategy would be:

- If Investor 2 stays, then $S^1 < 1 - \delta^*$, so Investor 1’s optimal strategy is to redeem.
- If Investor 2 redeems, then $S^1 + \delta^* < 1 - \delta^*$, so Investor 1’s optimal strategy is to redeem.

Similarly, Investor 2’s optimal strategy would be:

- If Investor 1 stays, then $S^1 < 1 - \delta^*$, so Investor 2’s optimal strategy is to redeem.
- If Investor 1 redeems, then $S^1 + \delta^* < 1 - \delta^*$, so Investor 2’s optimal strategy is to redeem.

Under these circumstances, either investor will choose to redeem, again regardless of the action of the other. Hence, this scenario (Redeem, Redeem) is a Nash Equilibrium.

Game 2c: Setting Fee Policy Using the Intraday Method
The success of Fees has been shown to critically depend on the magnitude of the Fee in relation to the potential loss. When a Fee has already been imposed, the stated Fee must exceed a probability (and utility) weighted measure of the anticipated loss. (In Section VIII.2.a we provided practical guidance on how this could be done.) When the Fee has not yet been imposed, we have shown that the corresponding probability (and utility) weighted anticipated Fee (or the Fee that the investor anticipates will be realized on a future redemption) must exceed the probability (and utility) weighted measure of anticipated loss. We now discuss how to frame Fee determination in this case. Specifically, utility theory provides practical guidance regarding how fund boards must set Fees in this scenario.
Recall that the Fee ($\delta^*$) is not necessarily predetermined in timing or magnitude and that the board can implement the Fee as and when it deems necessary. In Section VIII.2.a, we defined $\delta^N$ as the Fee that, when already imposed, is sufficient to imply that Stay/Stay is a Nash Equilibrium.  

Game theory (and its utility underpinnings) demonstrate that a Fee ($\delta^*$) will be effective at preventing a run if shareholders believe that:

$$E(\delta^*) \geq \delta^N$$

This formula states that the expected Fee must equal or exceed the fixed Fee ($\delta^N$) that is necessary (when imposed) to deter redemptions. The board has some latitude in determining its Fee policy provided that the above expression is satisfied.

In practice, directors would not state a probability of redemption Fee to shareholders. They disclose their policy (as appropriate) and operate in a manner that (along with disclosures from the advisor) enables investors to estimate what Fees will be and when they will be imposed. If the directors wished to reduce some uncertainty and adopted a particular redemption Fee, such as 2% (and disclosed this), then the directors must adopt other policies or practices that cause the probability of implementation to satisfy the above formulas.

The question of how to set Fee policy to stop or prevent runs is a key issue in evaluating the efficacy of the SEC’s proposals. Thus far, there appears to be an understanding that, once imposed, Fees can stop runs. However, the ability of Fees to prevent runs may not be fully appreciated in the policy debate. This is highly relevant for determining that Fees or Gates can provide adequate protection for investors while eliminating contagion and fire sales.

As a practical matter, boards can meet the requirements for Fees to be effective if they are adequately preemptive in reacting to market developments. For this to be true, the investor must believe that the probability of Fee imposition is not determined by the directors simply as a function of how many investors have already redeemed. This could encourage some investors to be first. Investors must believe that even a first redeemer is subject to the Fee risk. The probability of imposition must be primarily determined by characteristics of the portfolio in a way that cannot be gamed by first movers. Directors should have a reasonable conception of the materiality threshold that their customer base employs and be able to exercise fiduciary duty in determining whether allowing redemptions is in the best interest of shareholders. In particular, forms of activity that are harmful should be prohibited by policy or rule and disclosed to shareholders, as are many other requirements of Rule 2a-7.

---

30 That is, $\delta^N$ is larger than the mathematical expectation of the loss but not necessarily as large as the worst case outcome. It is sufficient to meet the utility conditions described in footnote 28.

31 This is a result of the concavity of utility functions and Jensen’s inequality.

32 A particular application of this formula illustrates the Fee necessary to deter redemptions simply on account of the current shadow price being below $1$, (i.e., if $\pi = 0$). In this case the redemption Fee and associated probability of imposition must satisfy $E(\delta^*) \geq 1 - S^\ell$. If the Fee were imposed under any circumstance, the probability of imposition is 1, and the requirement becomes $\delta^* \geq 1 - S^\ell$. This amounts to implementing a VNAV structure, since the Fee must always be at least the current shadow price discount to $1$. 

24
As an example of how the SEC could proceed within the parameters of the Release, funds adopting a Fee could evaluate two Fee measures: (i) a fixed default fee, such as 2%; and (ii) an alternate fee (usually smaller) that is based on market-determined measures reflecting liquidity, transaction costs, default risk, etc. This alternate measure could be developed and maintained using the stress test methodology that is required under any new rule. Trigger events would require that boards consider the implementation of either of the above fees, or a different fee (including zero) at the discretion of the board. Mandatory trigger events could include a liquidity threshold test, as in the Release. However, the SEC could also provide guidance with respect to additional policies and procedures that boards should adopt to protect investors. The policies would provide specific actions that a board must take if it determined that making redemptions at $1 could be harmful to remaining shareholders. Note that rule 2a-7 already mandates board action in the relevant circumstances, a fact pointed out in the comment file. “Rule 2a 7(c)(8)(ii)(A) already requires a Board to meet whenever it [“believes the extent of any deviation from the money market fund's amortized cost price per share may result in material dilution or other unfair results to investors or existing shareholders, [and] cause the fund to take such action as it deems appropriate to eliminate or reduce to the extent reasonably practicable such dilution or unfair results.”]” 33

Under such a regime, investors would know that redemptions are not on autopilot and that any potentially harmful redemption is under scrutiny. These measures would give the board specific requirements, but also rely on policy guidance with respect to fulfilling the board’s fiduciary responsibility. (Over time, and as necessary, additional specific rules could be adopted based on the experience gained through the process.) The effect of these changes would be to ensure that investors, even first movers, do not believe they will be able to force losses onto other shareholders. The effect of such board policies could be to place new safeguards on fund liquidity, portfolio management or other administrative procedures. However, this is the means by which the cost of investor protection and financial stability is internalized by the funds, while still retaining stable NAV.

33 See comments of Federated Investors, Inc., Section 4.3, pp. 9 – 11, (Sep. 16, 2013), http://www.sec.gov/comments/s7-03-13/s70313-130.pdf. An example: a board could adopt an “anti-fire sale” policy, in which a Fee is implemented preemptively in lieu of the advisor selling securities at a loss to fund redemptions. Specific procedures could be designed to implement the policy while not unduly hampering investment management. Similarly, policies could also require that any redemption (even funded from daily liquidity) that has the effect of reducing the shadow NAV must be determined (by the board) to be in the best interest of shareholders. Other procedures could require that, in the event of a trigger, the board would have information that is necessary to determine whether to apply the fixed Fee or the alternate Fee (where the alternative Fee may better correspond to current market events and represent a more economically fair price of liquidity).
Game 3: Will CNAV With Board Discretion To Impose A Redemption Gate Stop or Prevent A Run?

This game considers a regulatory environment where the MMF board has the right to impose a redemption Gate. No redemptions are permitted until the Gate is lifted. There are two methods for imposing the Gate. In the first method, the board decides to impose the Gate, and it becomes effective after today’s close of business. Thus, the Gate impacts all redemptions beginning tomorrow and thereafter, until lifted. We call this the end-of-day method. In the second method, when the board decides to impose the Gate, it is effective at the next intraday order settlement. It impacts any outstanding redemptions that were not yet settled at the time of the Gate decision and thereafter, until lifted. We call this the intraday method. In either of these methods, if the Gate has already been imposed, the run has clearly been stopped. The question is how investors react before a Gate has been imposed.

**End-of-day Method** – As in our earlier formulations, there are two investors in the fund and a stress event. Regulations permit the fund’s Board to impose a redemption Gate, which becomes effective after all today’s trades are settled.

In the face of that stress event, an investor has some expectation of how big a loss the fund may incur. He can exit the fund today. But if he waits until tomorrow, he cannot exit until the Gate is lifted. He loses access to his liquidity in the fund, and suffers a proportional share of the potential losses. In this scenario, the Nash Equilibrium is (Redeem, Redeem) for similar reasons that the end-of-day method fails under Fees:

- If one investor redeems but the other stays, the one that redeems pays nothing. The one that stays will suffer the entire loss if there is one, and loses access to his liquidity until the Gate is lifted.
- If both investors stay, then each will suffer part of the potential loss and lose access to their liquidity temporarily.
- If both investors redeem, neither suffers any impairment.

Thus the end-of-day Gate does not prevent a run from taking place. Both investors would choose to redeem.

**Intraday Method** – Again, there are two fund investors, and a stress event with possible fund losses. Regulations permit the fund’s Board to impose a redemption Gate, which becomes effective during the day (today) at a time unknown to the investors. Gates impose a cost on investors similar to a Fee, but it is not explicit. Gates impair investor liquidity for an unknown period of time. This could prevent them from meeting other obligations and certainly prevent them from avoiding future losses. We will denote the investor’s perceived cost of a Gate as $\theta^*$ (called “theta”). As with the redemption Fee ($\delta^*$), the probability is uncertain and may vary depending upon various factors.

We consider two formulations of gating rules in order to illustrate the importance of investor expectations regarding the trigger event. In the first formulation, the Gate is

---

34 The specificity of the weekly liquidity threshold in the proposal is not a necessary precondition for its success. As in the case of Fees, it could become a counterproductive accelerant for a run if it motivates first movers to redeem as weekly liquidity declines, even if the decline is unrelated to stress in the fund.
imposed based on the volume of redemptions that are experienced. This will be shown to have an unstable outcome. In the second formulation, the Gate is triggered based on economic criteria that are unrelated to the volume of redemptions. This will be shown to have a favorable outcome if the economic criteria are appropriately determined.

**Game 3a: Gate Imposed Based On Volume Of Redemptions**

Figure 11 shows the payoff matrix associated with the first formulation. We consider the case that there is a potential loss greater than $\pi$. If investors stay, they both avoid the Gate and realize $S_1$. We assume that if they both try to redeem, the Gate will be imposed and both realize $S_1 - \theta^*$. But if one stays and the other tries to redeem, we assume that the Gate is not imposed and the redeeming shareholder loses only the switching cost. Under these assumptions, while both have an incentive to cooperate, Stay/Stay is not a Nash Equilibrium because either has an incentive to redeem if the other stays.

![Figure 11. First Formulation of Gates](image)

It is clear from Figure 11 that this game is qualitatively different from the games with Fees. In the case of Fees (shown in Figure 10), the Fee is assumed to be set on a criteria of potential loss in the portfolio and the Fee applies only to redeemers. There is no motivation for investors to cooperate. By contrast, in Figure 11, the Gate is imposed based on a criteria of the volume of redemptions and applies to all investors. Here, there is a motivation to cooperate. Clearly, the investors are better off if both stay rather than if they both redeem.35

---

35 In assessing the likelihood of the different outcomes in Figure 11, an investor does not know if he will encounter the Gate or not. If investors believe they can escape the Gate, the outcome is the same as in the end-of-day method, and the equilibrium is (Redeem, Redeem). The possibility of gating may induce investors to remain in the fund, just as the imposition of a redemption Fee may lead investors to stay. This will be the case if both investors realize (and understand through fund disclosures) that if both redeem then a Gate is very likely. If investors stay, the event risk may quickly dissipate and the potential for being rendered illiquid is eliminated. Therefore, there are benefits to cooperating.

However, it is difficult to prove that the gating rule in Figure 11 will as effectively dissuade investors from running as a high Fee will, since $\theta^*$ is potentially less costly to investors (although there can be significant cost to illiquidity, particularly during a stress event). For example, suppose a fund were gated for 30 days. The investor would lose liquidity and risk further loss. Now compare this to a 2% redemption Fee imposed with the same probability. For these two outcomes to be equal in the investor’s eyes implies a very high convenience cost for liquidity: 2% which is an annualized rate of 24% for those thirty days.

In the case of Fees, a redeeming investor would still be able to avoid further losses, which would not be possible under gating. This could have a material effect. As with redemption Fees, Gates
However, it may be more difficult for directors to impose gating under this type of rule in a manner that causes (Stay, Stay) to be a Nash Equilibrium (i.e., to prevent a run). Although it may be in the best interest of investors to cooperate and remain in the fund, there is an incentive for one investor to redeem if he believes that he might be first to redeem or the other investor might stay. This is illustrated in the off-diagonal cells in the matrix. Clearly, however, once the Gate is imposed, the only possible outcome is (Stay, Stay).

**Game 3b: Gate Imposed Based On Alternate Criteria**

Figure 11 illustrates the difficulty in achieving a stable outcome when the probability of a Gate is based on the number of investors redeeming. Therefore, boards are likely to employ alternative criteria that are based, not on the numbers redeeming, but on portfolio characteristics. An alternative rule would be to impose a Gate, in lieu of a Fee, whenever a Fee would have been imposed in Figure 10. Figure 12 illustrates the payoffs in this game scenario.

![Table](image)

**Figure 12. Second Formulation of Gates**

In this second formulation, the payoff matrix is more complex. The outcome will depend on whether a Gate is imposed (with the entry on the left representing the payoff when a Gate is not imposed, and on the right the payoff when it is imposed). In this formulation of gating, when the Gate is imposed, \( \theta^* \) takes on a value and the entries on the right sides of each cell become applicable.

Several observations are in order. First, if investors believe that the Gate is triggered based on similar criteria as a Fee policy (i.e., one that assures Stay/Stay is a Nash Equilibrium), then shareholders will believe that they will not be able to redeem in a way that disadvantages other shareholders. As soon as they have a return motivation to run, they will be unable to. Here, there is also no incentive to cooperate. In particular, the Gate can will be more effective if investors do not believe the imposition of a Gate is partly determined by whether some investors have already redeemed. If this is the case, an investor will be incented to run before the Gate is lowered. To avoid this, every redeemer (including a potential first mover) must face the same risk.

Using our utility methodology, a gate would certainly be imposed when \( U(1-\pi) \geq E[U(S')] \), or when the utility of redeeming exceeds the expected utility of staying. This implies that a gate would already have been imposed if \( 1-\pi \geq S^0 \). A more exacting criteria must reflect the fact that investors will anticipate that a gate may be imposed. In this case the criteria becomes \( U(1-\pi) \geq E[U(S' - \theta^*)] \). That is, the ability of gates to prevent runs must reflect investor aversion to the gate itself. In this case, investors that perceive a gate as very costly (i.e. \( \theta^* \) is large even if the probability of imposition is
be applied even when both investors stay. As with Fees, Gates are effective at preventing a run when boards are sufficiently preemptive and the trigger is designed so that it cannot be gamed by investors. However, our analysis (see footnote 36) also demonstrates that investors will include the possibility of a Gate being imposed, along with possible future losses in the fund, as part of a decision to stay or redeem. Some investors may prefer a gated fund (i.e., with an ex ante de minimis risk of a Gate being imposed, but no Fees) to a fund without Gates (but with a possible Fee). These investors would perceive $\theta^*$ as small (so $\theta^*$ would not materially affect the gating analysis). But others may be intolerant of any risk of illiquidity (seeing $\theta^*$ as larger). For this reason, regulators may wish to permit both options, or allow funds to designate a primary method, so that investors may self-select into their preferred form of fund.\textsuperscript{37}

IX. Combining VNAV with Fees/Gates

One of the SEC’s proposals calls for combining VNAV with Fees/Gates. A VNAV has the ability to protect investors from the risks of first movers who are motivated by an initial departure of the shadow NAV from $1. However, it would not be effective at preventing a larger run. Conversely, Fees/Gates can stop a run in progress. However, if Fees/Gates were improperly implemented, they may not adequately prevent harm from first movers. The apparent logic in combining these features would be to marry the strengths of each. While intuitive, this is simplistic reasoning since the combination alternative will be challenged by the more fundamental economic force of arbitrage.

Investors will see the transition from CNAV to VNAV as a loss of utility that already fully reflects the market price of liquidity. Additional restrictions, like Fees or Gates in combination with VNAV, would be viewed as a noneconomic punitive feature similar to capital controls. While there may be a purported public policy reason for wanting to restrict MMF asset sales in a crisis, no other investment products, and certainly no mutual funds are encumbered in this way.\textsuperscript{38} The combination would create an inferior economic product with many near substitutes that would draw investor assets. In particular, if VNAV is combined with Fees/Gates, then the current meaning of a 2a-7 fund would be lost and there would be no advantage for a fund to qualify as a “Money Market Fund”. Thus, it is possible to make the rules so onerous that no rational investor would ever invest in MMFs. Other substitute vehicles would quickly absorb the assets that would otherwise have remained in MMFs.

These conclusions are most obviously arrived at by concepts of economic or regulatory arbitrage. However, the utility theory framework employed in our game theory models implies the same outcomes when the analysis is expanded to include alternative products. In particular, our methodology is predicated on the theory of expected utility in which investors array the investment choices and select the investment with the highest probability weighted utility of small), may redeem very early. This methodology demonstrates the more general observation that investors may self-select out of funds that have gates in preference of alternative vehicles. In this case, investors preemptively redeem by not investing in the fund to begin with.\textsuperscript{37} In particular, investors that perceive $\theta^*$ to be large in relation to $\pi$ will be expected to select a Fee fund for investment, or redeem very quickly from a gated fund.\textsuperscript{38} Some mutual funds, such as certain high yield funds, have redemption Fees. However, these are implemented to restrict frequent trading in assets that have high transaction costs.
wealth. When we configure the choices to be not “Stay” vs. “Redeem”, but instead “MMF with VNAV and Fees/Gates” vs. “Near MMF with VNAV and no Fees/Gates”, investors will unambiguously choose the latter. (Similarly, investors’ revealed preference of current 2a-7 MMFs over alternative VNAV products, along with their public commentary supporting the Fee/Gate alternative over the VNAV alternative, suggest that CNAV with Fees/Gates would be preferred over VNAV alternatives by utility optimizing investors.)

X. Concluding Remarks

Our analysis has employed utility and game theory to derive important conclusions regarding the efficacy of the SEC’s proposals for the reform of money market funds. We specifically addressed the ability of the proposals to stop or prevent runs, as well as the strength of these conclusions. We found that VNAV could not stop a run, although it could mitigate first mover advantage associated with the motivation to run that results from small shadow price departures from $1. We found that Fees and Gates can stop and prevent runs, provided that they are implemented effectively through policy and preemptive action by fund boards. In particular, Fees and Gates can be implemented so that remaining in the fund is a Nash Equilibrium, a strongly predictive economic result. We believe that the SEC has provided an extensive discussion of the factors relevant to implementing Fees or Gates. In particular, Section III.B of the Release identifies the key principles that underlie an effective policy and tradeoffs among them. We find that highly effective run prevention is attainable within the approaches contemplated by the Release, while requiring that fund boards be given discretion to take protective action. This is the mechanism by which Fees/Gates cause MMFs to internalize the cost of investor protection, while preserving the utility of current CNAV vehicles. We found that combining VNAV with Fees and Gates would ultimately be ineffective because arbitrage and investor preference would drive investors to other products. Cumulatively, these findings lend support to Fees and Gates as the preferred means of protecting investors and eliminating the related systemic risks of contagion and fire sales.

Fees and Gates have been primarily analyzed in terms of their ability to stop or prevent runs – particularly those resulting from liquidity or credit stress events in which fund values are impaired, but markets remain continuous. Recent developments, including cyber attacks, market shutdowns and extreme weather events, make it apparent that markets may be discontinuous. Regulators should therefore also consider allowing directors to impose very short-term Gates when there are business continuity events in which it may not be in the best interest of shareholders to make continuous redemptions. In these circumstances, fund directors and advisors may need a pause in order to make critical decisions on behalf of shareholders. Such power would complement the existing ability of funds to delay settlement for up to seven days.

Fees and Gates fill what are currently gaps in Rule 2a-7. At present, a fund that breaks a dollar has no choice but to liquidate. Fees and Gates provide a framework for a fund

39 While this paper does not take up economic cost/benefit analysis of the alternatives, our conclusions with respect to combining VNAV with Fees/Gates supports the marginal cost/benefit argument that adding VNAV to a regime of Fees/Gates provides a net negative economic result.
to bridge such periods, and continue to operate afterwards. Fees enable investors to access their liquidity, but at a price. That price may (and probably will) exceed the discount of the shadow price to $1 during a crisis, but that is the cost of being able to assure that a stable NAV product will not cause contagion or fire sales during such periods. Gates do not impose an extra Fee on shareholders, which is appealing to many shareholders, but have the undesirable effect of restricting access to liquidity during critical periods. Together, Fees and Gates provide fund boards with powerful tools to prevent a run from materializing, to stop a run in progress, and to assure that a stress event does not cause contagion or fire sales.

The combination of VNAV with Fees/Gates is an example of a policy prescription that weighs systemic risk concerns so heavily that investment utility of the resulting product is undermined. Other rules could have a similar effect. For instance, our analysis shows that extreme preemptive action by boards in imposing Fees could be so effective that, while eliminating run risk, the resulting penalty to access liquidity could become unduly burdensome. Policy makers must consider how best to balance these competing objectives. It is possible that directors should be allowed to set Fees that are an adequate deterrent to runs, but still with reasonable proportionality to transaction costs (and any current discount of the shadow price to $1), so they do not improperly penalize redemptions.

---

40 In particular, boards may be more likely to take preemptive action when the corrective action (after Fee/Gate imposition) is a loss recognition that does not necessarily entail liquidation or abrupt change in operation. For instance, a fund that experienced a 25 basis point credit loss (but did not “break the buck”) and had instituted a fee or gate, could be allowed to reverse split back to a $1 NAV and continue operation.
References


(9) HSBC (2011). Run risk in money market funds.


Appendix A

Prisoner’s Dilemma

A simple two-player game is typically described in tabular form as a set of outcomes for strategies for the Row player $r$ and the Column player $c$.

In the classic Prisoner's Dilemma, two suspects Row and Column have been captured by police and accused of jointly participating in a crime. The suspects must decide if they will confess to the crime or accuse the other player of the wrongdoing.

If both Row and Column confess, i.e., (confess, confess) then they will each receive just one year in prison. However, if one player confesses and the other player accuses, i.e., (confess, accuse) or (accuse, confess), the accuser will go free while the confessor will receive four years in prison. On the other hand, if both accuse, (accuse, accuse) each other then they each receive three-year prison sentences.

Their strategies are, therefore, either confess or accuse. The payoffs to these actions are detailed in Table A1:

<table>
<thead>
<tr>
<th>Prisoner's Dilemma</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Column</td>
</tr>
<tr>
<td>Confess</td>
</tr>
<tr>
<td>Confess</td>
</tr>
<tr>
<td>Accuse</td>
</tr>
</tbody>
</table>

Table A1. Prisoner’s Dilemma

How do we solve the Prisoner's Dilemma? As stated previously, this involves finding the Nash Equilibrium to the game. A formal definition follows:

Nash Equilibrium. A Nash Equilibrium consists of probability beliefs $(\pi_r, \pi_c)$ over strategies, and probability of choosing strategies $(p_r, p_c)$, such that:

1. the beliefs are correct: $p_r = \pi_r$ and $p_c = \pi_c$ for all $r$ and $c$; and,

2. each player is choosing $(p_r)$ and $(p_c)$ so as to maximize his expected utility given his beliefs.
Where expected utility is given as:

\[
E(u_r) = \sum_r \sum_c p_r\pi_r u(r,c)
\]

\[
E(u_c) = \sum_c \sum_r p_c\pi_c u(r,c)
\]

As such, it is apparent in this definition that a Nash Equilibrium is equilibrium in both actions and beliefs. In equilibrium each player correctly foresees how likely the other player is to make various choices, and the beliefs of the two players are mutually consistent.

Using this definition it is clear that there is only one Nash Equilibrium to the Prisoner's Dilemma. This outcome corresponds to the strategy (accuse, accuse) and is determined as follows:

- If Column confesses, it is in Row's best interest to accuse since Row will go free.
- If Column accuses, it is in Row's best interest to accuse since doing so will lead to just three years in prison instead of four if Row confesses.
- If Row confesses, it is in Column's best interest to accuse since Column will go free.
- If Row accuses, it is in Column's best interest to accuse since doing so will lead to just three years in prison instead of four if Column confesses.

The only strategy that is common to both players is (accuse, accuse), thus it is a Nash Equilibrium of the game. While this particular game had just one Nash Equilibrium, it is certainly possible to have more than one. A widely held "folk theorem" (i.e., unproven conjecture) of game theory is that all games have an odd number (e.g., 1, 3, 5, etc.) of Nash Equilibria as demonstrated by Lemke and Howson (1964)\(^1\) and Wilson (1971)\(^2\) with sufficient conditions provided by Li and Wang (2004)\(^3\). For simplicity we will concern ourselves only with "Pure Strategy" Nash Equilibria and avoid the more cumbersome and, in this case, unimportant "Mixed Strategies."

---

**Pure Strategy Nash Equilibrium**: A Nash Equilibrium in pure strategies is a pair \((r^*, c^*)\) such that:

- \(u_r(r^*, c^*) > u_r(r, c^*)\) for all Row strategies \(r\), and
- \(u_c(r^*, c^*) > u_c(r^*, c)\) for all Column strategies \(c\).

The existence of multiple Nash equilibria creates something of a problem. If there is more than one outcome to the game, which one is the "best"? To answer this question, economists have developed the concept of Pareto optimality:

- A feasible outcome \((r, c)\) is **weakly Pareto Optimal** if there is no other feasible outcome \((r^*, c^*)\) such that all players strictly prefer \((r^*, c^*)\) to \((r, c)\).

- A feasible outcome is **strongly Pareto Optimal** if there is no feasible allocation \((r^*, c^*)\) such that all players strictly prefer \((r^*, c^*)\) to \((r, c)\), and some player strictly prefers \((r^*, c^*)\) to \((r, c)\).

Note that in the Prisoner's Dilemma, while \((\text{accuse, accuse})\) is the Nash Equilibrium and, hence, logical outcome of the game, it is *neither strongly nor weakly* Pareto Optimal. Both players could be made strictly better off by switching to \((\text{confess, confess})\), because their jail time would each be reduced by two years. However, \((\text{confess, confess})\) is not a Nash Equilibrium and, as a result, is not a viable outcome.

Economists have long held that government regulation is only justified if it moves games of strategic interaction away from Nash Equilibria that are not Pareto optimal to those that have either strong or weak optimality. In this sense, the market couldn't gravitate naturally to an optimal outcome, so regulation is required to nudge it in that direction. Regulation that produces outcomes that are not Pareto Optimal are not justifiable in an economic sense.

Finally, it is worth noting that the classic lawyer’s solution to the prisoner’s dilemma is to have the two prisoners use the same lawyer. This would force cooperative behavior and a more positive outcome. MMF Board authority to impose Gates on behalf of the interests of all shareholders, without a specific numerical trigger, does the same thing.
Appendix B
Utility Functions

Investment utility functions are quantitative expressions of the preferences of investors. As such, we implicitly require that preferences be:

- **Complete.** If a decision maker has two choices, he or she has the knowledge to compare these choices and choose between the choices. The decision maker is never clueless.

- **Transitive.** Preferences are transitive when a decision maker is internally consistent (e.g., decision maker prefers a faster car. Given three cars, x, y and z, if x is faster than y, and y is faster than z, then x is faster than z.

- **Reflexive.** Preferences are reflexive when a particular good/option is at least as well liked as any other same good/option.

- **Continuous.** Preferences are continuous when not only a utility representation exists, but also a continuous representation exists. Preferences are continuous when they have no “jumps” or “reversals” when the options/goods being compared change slightly.

- **Strongly Monotonic.** More of a good thing is good.

If these requirements are met, then there must exist a continuous utility function \( u: R^k \rightarrow R \) which represents those preferences. For a formal proof see Varian (1989)\(^{44}\).

The money fund investor’s utility function is characterized as: \( u_i(w_i) \). Where \( w_i \) is the total wealth of investor \( i \). We would expect that the derivative of the utility function is positive such that: \( \frac{du_i}{dw_i} > 0 \) The greater the wealth, the greater the overall utility.

We would also expect that the second derivative of the utility function is negative such that as wealth increases, utility increases at a diminishing rate.

Note that this utility function is not defined specifically for money funds; it can be generalized for all types of cash and short-term investments.

---

Utility Function Implications
Careful analysis of this utility function shows it has far-reaching implications. In fact, it brings into question the viability of the objectives motivating the SEC's proposals.

The expressed goal is to prevent runs on MMFs and potential contagion. Yet this utility function suggests that tighter regulation of MMFs begs the larger issue. These regulations will not control large sums of sophisticated, extremely liquid and extremely interest rate-sensitive assets from moving within the financial system during periods of credit market stress. Assets would just move to other instruments that meet similar utility functions (i.e., bank deposits, T-bills, direct commercial paper, etc.), especially if regulations are changed and the equilibrium shifts.

Second, it highlights that none of the proposed regulations operate directly on the investor utility function. Instead, they simply limit the viability of one particular investment vehicle (MMFs). All investors are utility maximizers. VNAVs, liquidity Fees and exit Gates reduce the utility of money funds since they represent a reduction in liquidity, safety of principal and perhaps even rate. Therefore, we would realistically expect large outflows from MMFs into other vehicles.

This exodus could be extremely problematic to the financial system as a whole. MMFs are, in general, designed to be very liquid and, as such, take very little interest rate risk. The typical duration of a money fund is usually less than 60 days such that redemptions would rarely, if ever, force the fund to realize losses due to rate movements. In this way they are perhaps the ideal vehicles for short-term liquidity.

Banks, on the other hand, are structured such that they take considerably more rate risk with deposits. If deposit proceeds are invested for a significantly longer duration (years) than the deposit itself, the bank's balance sheet could be at risk and the bank could be forced to realize significant losses to support deposit withdrawals. Bank products are less well-suited than MMFs for highly liquid deposits.

In a recent Treasury Strategies study (2012), we demonstrated that MMFs were more stable than other short-term investment vehicles during the financial collapse of 2007-2008. MMFs were the last domino standing among many in the financial collapse. From a game theoretic perspective, they optimized investor utility functions better than any other non-government insured short-term instrument\(^4\)\textsuperscript{5}.

\(^{45}\) Treasury Strategies. (2012). Dissecting the financial collapse.