April 16, 2010

Elizabeth M. Murphy
Secretary
U.S. Securities and Exchange Commission
100 F Street, NE
Washington, DC 20549-1091

RE: File Number S7-02010, "Concept Release on Equity Market Structure"

Dear Ms. Murphy:

We are a group of scientists and business people who share a common interest in the application to financial systems of a scientific methodology called agent-based modeling (ABM). We believe this methodology represents a scientifically validated and powerful tool to facilitate the Commission's regulation of equity markets. We believe that its use would enable the Commission to make scientifically informed responses to questions posed in the Release and to anticipate both the intended and unintended consequences of proposed regulations. We also note that a number of other agencies of government now successfully employ this type of science.

The Commission's facilitation of the establishment of a national market system as directed by Congress in 1934 has been significantly affected by the emergence of new computing and communications technologies. These have affected the architectural structure of the system itself as well as how it functions. This is now reflected in a shift from manual trading to automated trading over increasingly faster time scales in increasingly more complex networks. The pace of this change has increased dramatically in only the last decade. As the Release notes, this now poses a number of important new regulatory issues for the Commission. ABM could help address these issues, as subsequently described in this letter.

1. Agent-Based Models are Test Beds

ABMs are computational scientific test beds. While they trace their scientific origins to the 1940's, in the 1990's they began to find growing and widespread application in a variety of fields. *Their growth has been enhanced by the same rapidly increasing computing and communication capabilities that have also dramatically affected the structure and functioning of our national equity markets resulting in the need for new regulations*.

ABMs are a class of computational model used to analyze the emergent behaviors of complex systems. They simulate the interactions of autonomous agents to assess their effects on both a system as a whole and on the individual agents and classes of agents which comprise it. These models are based on the *causal micro-structure* of systems that *macro-models* using differential equations and *aggregated* data do not reflect. Agents may be individuals or institutions. Models typically combine elements of game theory, the science of complex adaptive systems, the mathematics of emergence, computational sociology, and evolutionary programming. Most are architected to consist of numerous agents and agent classes with individual strategy spaces specified at various scales, decision-making heuristics, learning rules and adaptive responses, an interaction topology, and a superposed regulatory environment.

ABMs are a well-understood, widely accepted and powerful tool for analyzing the emergent behaviors of complex systems. They are currently being used successfully in a wide number of applications including: banking, supply chain optimization, logistics, consumer behavior, social network effects, distributed computing, workforce management, traffic congestion, military battlefield dynamics, the interdiction of terrorists and drug smugglers, biowarfare, epidemiology, and the regulation of electrical markets. As an indication of the extent of their use and the widening interest in them, a Google search on the topic of agent-based modeling yields about two million individual cites.

ABMs are also being increasingly employed by a growing list of government agencies including: the Federal Reserve Bank of New York, the Bank of Finland, the National Aeronautics and Space Administration, the National Institutes of Health, the National Science Foundation, and the U.S. military. An illustrative (but far from complete) list of such applications may be found at Appendix A.

1.1 An Early Equity Market ABM Test Bed

ABMs have been recommended in a number of financial domains in response to the recent financial crisis.[1] A number have also already been constructed dealing with various market issues.[2] But they have not yet found widespread use in the specific assessment of equity market regulatory initiatives. However, their use does have a successful, albeit it limited, history in this domain.

In 1998, an ABM was constructed to explore the combined effects of market microstructure and existing and proposed rules on the behavior of participants including market makers and traders in the Nasdaq Market. Although it did not obtain regulatory traction at the time, this particular model yielded six predictions in response to specific proposed market regulations. These predictions dealt with

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[1] See, for example, "The Economy Needs Agent-Based Modeling" by Farmer and Foley, *Nature,* Vol. 460, August 6, 2009, 685-686.

[2] See, for example, "Heterogeneous Agent Models in Economics and Finance," by Hommes, in *Handbook of Computational Economics, Vol 2: Agent-Based Computational Economics* by Tesfatsion and Judd, North-Holland/Elsevier, Amsterdam, 2006.

price discovery, parasitic trading, aggregate behavior, quote and spread clustering, wealth shifts between market participants, and abrupt dislocations (sometimes referred to in scientific jargon as *phase transitions*).

Subsequent validation of this work based on independent data several years later directionally confirmed five of these predictions and was inconclusive, but not dismissive, as to the sixth. Several of these predictions were phase transition dislocational and of particular interest because the predictions predated the actual market dislocations presented by the so called dot.com Bomb and the subsequent validation analysis was completed after these dislocations had occurred and been measured in our equity markets.[3]

It is to some extent understandable that ABMs have not yet found their way into mainstream equity market regulatory thinking despite this one arguably successful albeit little known and somewhat dated undertaking. One succumbs to a tendency to categorize economic models as a single class of endeavor without distinguishing between the most well known of these which are macro-models based on aggregated data, and the newer but lesser known ABMs which are micro-models based on the causal microstructure of a particular system.

There is extensive literature on the predictive limitations of traditional macro-economic models which was only further ratified by the failure of the more notable of these to predict the latest financial crisis.[4] This understandably leads to skepticism about the predictive reliability of all economic modeling. However, there is a fundamental and significant distinction between micro- and macro-modeling, and with this distinction in mind, we believe that ABMs now deserve the Commission's consideration for application in the current environment.

1.2 ABM Test Beds are Well Suited for Equity Markets

ABMs are best suited for systems which are *contained* in the sense that their basic micro-architecture may be specified over a relevant predictive time scale. Such systems are sometimes also referred to as *closed* or *partially closed.* It is much more difficult to micro-model *open* systems than closed ones. For example, it would be a daunting task to attempt to micro-model the entire world economy. It is a massively open system involving an immense number of dynamically emerging micro-structures and new architectures at multiple levels in the system over a huge variety of time scales, together with emerging processes and flows within these dynamical structures which are themselves in a constant state of flux and adaption.

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[3] See "A Nasdaq Market Simulation: Insights on a Major Market from the Science of Complex Adaptive Systems" by Darley and Outkin, World Scientific, 2007**.**

[4] See, for example, "The Financial Crisis and the Systemic Failure of Academic Economics" ("Dahlem Report") by Colander et al., Kiel Working Paper 1489, February 2009.

Many therefore rightly argue that the only really predictive model for an entirely open system this extensive and complex must come quite close to duplicating the system itself which involves massive computing density rendering it impractical to undertake.

Our equity markets, however, are a smaller sub-system of the entire economy and this system is significantly closed *specifically by the wise fact of the Commission's regulatory oversight!* The structure of the system itself -- the markets which make it up and the types of securities involved -- is prescribed by regulation. So are many behaviors of participants within this structure -- the required and permitted flows of securities, money and information in the networks between participants and institutions in the system, as well as a number of the specific actions which may or may not be taken by individual participants. Our equity market system is thus at least significantly closed, and subject to certain constraints of due process, *the extent of this closure is within the purview of the Commission*. For this reason our equity markets are good candidates for ABM simulations as illustrated by the success of the Nasdaq ABM exercise.

Once a system to be modeled is sufficiently closed so that its architecture may be specified over relevant time frames, there still remains the issue of modeling the adaptive responses to the system by individual agents where the system remains open with respect to these behaviors. As the Release notes, this openness is in fact a specific objective of the Commission in providing fair access to and competition amongst the equity markets and their participants. As the Release further notes, in the equity markets these behaviors range from algorithmic trading responses in milliseconds on the fast end, to responses to informational flows on longer time scales where it takes participants time to prepare and digest information, to structural adaption by participants over longer time scales such as the emergence of ECN's and dark pools.

What is called the *granularity* of an ABM refers to the number and levels of classes for particular agent interactions into which these classes and levels self-organize over relevant time scales when the system is in operation. For an ABM to be predictive it is necessary to construct it representing these behaviors at a sufficiently granular level. As an example of granularity, the Release recognizes emergent categories of high frequency trading (HFT) strategies. These are Passive Market Making, Arbitrage, Structural and Directional. In scientific jargon these categories represent a higher level of granularity than simply HFT, although a somewhat more granular level than this would doubtless be required for predictiveness as next discussed.

There are two ways in which regulated equity markets are particularly ideal candidates for ABM applications seeking to model at sufficient granularity levels for predictiveness. One is the nature of the distribution in which trading strategies so far typically appear to present themselves, and another is the emergence of algorithmic trading itself.

When the Nasdaq ABM was constructed in a simpler era, despite a large number of individual trading strategies, statistical investigation discovered that a significant majority of market volumes were in fact spawned from less than a dozen emergent categories of basic strategies by human traders. While such emergent categorizations must be statistically and rigorously investigated in the actual construction of a predictive ABM, the empirical fact that emergent categories are often distributed in what are called *power law distributions* is helpful to the micro-structure modeler. These distributions, which are expressed mathematically as exponential functions – hence the adjective power – are sometimes also colloquially referred to as the "eighty-twenty" rule – where 80% of the activities may be represented by 20% of the categories. The frequency with which power law distributions appear to so far emerge in trading environments as they did in construction of the Nasdaq ABM is a helpful fact in making equity markets well suited for ABM simulations at predictive levels of granularity.

The emergence of algorithmic trading also renders ABMs particularly well suited to our equity markets in their current state. These algorithms themselves are to a growing extent the actual "agents" at the bottom of the system and human traders employing them are "meta-agents" at a higher level of the system. There has understandably been a good deal of discussion in the scientific literature about the mysteries of human behavior in ABM simulations, particularly in systems in which emergent power law distributions do not present themselves to facilitate granularity (although they often do in trading markets as just noted). This is due to the challenges of pre-stating the micro-structure of human behavior under various circumstances in advance.[5] However, in the algorithmic trading setting, *the modeler enjoys the additional advantage that a trading algorithm has already been pre-programmed in advance!* Even if the algorithm is proprietary, it may still be simulated for overall market effect while at the same time protecting the confidentiality of its proprietary code to preserve innovation and competition. Moreover, in addition to being pre-statable in advance, trading algorithms also appear to present themselves in emergent power law distributions as their human trading forebears a decade ago did, placing predictive levels of granularity further within reach of the diligent modeler.

Once an ABM has been constructed at a micro level to represent both the architectural structure and agent adaptive responses of the system, it is generally then *tuned* by running this description of the system using past data to see if the macro-data that result conform to macro-data which actually occurred. Tuning is generally done by inserting a limited number what are called *parameters* into the model which is the mathematical equivalent of a carpenter leveling up the windows in a new house with a few wedges before putting the trim on. In systems which are relatively more closed and modeled at higher levels of granularity, fewer

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[5] See, for example, "Natural Rationality in the Economy as an Evolving Complex System," by Darley and Kauffman, Addison-Wesley, 1997.

parameters are generally required in this tuning process. When the tuned system simulates the known past in a satisfactory way, proposed changes to its micro-structure are then made and the model is run to determine how these adjustments affect the system as a whole and how they affect the individual agents which comprise it over relevant future time scales.

3. An ABM Test Bed Would be Reponsive to Specific Concept Release Issues

We do not argue here that ABMs are a panacean answer for all financial and economic questions. With the explosive growth in the use of ABMs, one may also observe significant variation in both the quality of the modeling and the scientific rigor applied from project to project. As noted in **1.2** above, there are also domains in which open systems present intractable problems for micro-structure modelers, although this has not precluded laudable and well-intentioned attempts in such domains.

However, ABMs are an increasingly accepted scientific tool in an increasing number of other domains which, like our equity markets, present problems for which they are particularly well suited and in which relatively simple rule and other micro-architectural changes may spawn both extremely complex and in many instances unintended as well as intended consequences. If constructed with scientific rigor, *we believe an AMB test bed holds promise for assisting the Commission in reaching informed conclusions on a number of questions specifically posed in the Release*:

(i) An ABM test bed would facilitate analysis of the impact of proposed regulations on market quality metrics, including quoted spreads, effective spreads, realized spreads, execution speeds, and time scale separations, as well as provide the ability to test proposed new market quality metrics;

(ii) An ABM test bed would facilitate analysis of short-term volatility and its relative impact on various classes of market participants including shifts of wealth between them;

(iii) An ABM test bed would permit metrics for small orders and block orders to be studied and investigated and the testing of alternative metrics;

(iv) An ABM test bed would enhance measurement of institutional investor transaction costs and the consequences of proposed regulations affecting these;

(v) An ABM test bed would facilitate analysis of trends in quality metrics as well as provide evidence as to whether such trends behave linearly over relevant time scales, or emerge in phase transition distributions representing more extreme dislocations over the same time scales. (As noted in **1.1** above, phase transition dislocations in certain market metrics were predicted in the Nasdaq ABM exercise and later validated subsequent to the dot.com Bomb); and,

(vi) An ABM test bed would provide insight into the dynamics of order flow, and the system's impact on order fragmentation, best execution, and order interaction.

4. Research and Research Tools

We understand that there is a great deal of what is generally categorized as "research" conducted about equity markets. This is done by the Commission itself, under other Government auspices, and privately by a number of scientists, economists and other academics. Some of this is undertaken to inform the Commission's regulatory initiatives. Other research is undertaken by other agencies of Government to inform their various responsibilities. And still other research is undertaken to inform participants in the system about the most adaptive strategies which they might adopt. These are valuable undertakings. We commend them and in no wish to diminish their significance.

However, while the construction of an ABM equity market test bed may sound at first like research in this same broad sense, there is a distinction. Construction of an ABM test bed is the construction of a *research tool* which may be used multiple times for specific research undertakings as well as fine-tuned and adjusted along the way to reflect lessons from its use. Such a process represents the leveraged dynamical application of the scientific method. It is easily understood that breakthrough biological research is leveraged by the use of laboratories and scientific equipment. Similarly, ABM test beds, when constructed and refined with scientific rigor, are leverage points for a continuously improving research agenda for equity markets to permit increasingly well informed regulatory response.

5. Implementation of an ABM Test Bed Would Enhance the Commission's Regulation of Equity Markets

We recommend that the Commission give serious consideration to constructing an ABM Equity Market Test Bed for the further enhancement of market regulation and that it do so either within the Commission, or independently but subject to its supervision.

Eventually more academic, public interest, and private ABM test beds for equity markets may emerge in various forms as they have in other domains. As this occurs these may well be funded by special interests or emerge in open source environments which may have varying interests differing from those of the general public as represented by the Commission.

By constructing such a test bed, we believe the public and the Commission would earn a significant return on a small investment. It would be a small investment because this work could be managed by a small group of senior scientists with input from a scientific advisory board. For example, the Nasdaq simulation test bed was constructed over a year by an average of only five scientists and a handful of advisors. It would earn a significant return because at a time when many Americans are questioning the ability of various agencies in our government to predict both the intended and unintended consequences of their policies in various economic settings, the credibility of the Commission can only be enhanced by affording itself the realistic possibility of making regulatory decisions informed by the application of the scientific method in the form of an ABM test bed to a system for which is well suited at the demonstrated current level of scientific capability.

We appreciate the opportunity to provide our scientific thoughts and would be happy to further discuss them, including with the appropriate Commission staff.[6]

Sincerely,

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[6] Professor Zoe-Vonna Palmrose can help coordinate such discussions. Her contact information is zpalmrose@marshall.usc.edu or 213-740-5019.

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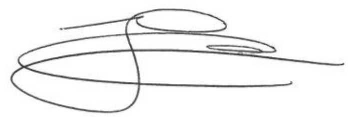

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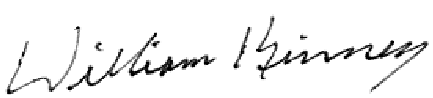

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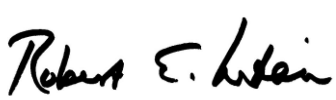

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Cc: The Honorable Mary L. Schapiro, Chairman, U.S. Securities and Exchange Commission

The Honorable Luis A. Aguilar, Commissioner, U.S. Securities and Exchange Commission

The Honorable Kathleen L. Casey, Commissioner, U.S. Securities and Exchange Commission

The Honorable Troy A. Paredes, Commissioner, U.S. Securities and Exchange Commission

The Honorable Elisse B. Walter, Commissioner, U.S. Securities and Exchange Commission

Mr. Richard G. Ketchum, Chief Executive Officer, Financial Industry Regulatory Authority

Dr. Henry T. Hu, Director, U.S. Securities and Exchange Commission Division of Risk, Strategy, and Financial Innovation

Mr. Robert W. Cook, Director, U.S. Securities and Exchange Commission Division of Trading and Markets

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Appendix A
Agent-Based Modeling Examples

This appendix provides examples of how various government agencies and institutions are using agent-based modeling (ABM) and the problems solved by applying ABM techniques. This is not an exhaustive list, but a sample of existing efforts that include modeling domains such as markets, payment systems, homeland security, critical infrastructure, socio-cultural systems, public health, and the military. Rather than one-off studies, these efforts concentrate on constructing frameworks that can be continuously used, improved, and updated with new data.

In addition to these examples, an extensive repository of annotated pointers to published research, software, and research groups focused on ABM financial market research is maintained at: http://www.econ.iastate.edu/tesfatsi/afinance.htm. Research repositories for other highly active ABM research areas can be found at: http://www.econ.iastate.edu/tesfatsi/aapplic.htm

Argonne National Laboratory (ANL). Electricity Markets Complex Adaptive Systems (EMCAS) Model.
EMCAS is an agent-based computational framework that permits the exploratory study of a wide variety of electricity market designs via systematic computational experiments.
http://www.dis.anl.gov/projects/emcas.html

Bank of Finland. Payment and Settlement Simulator (BoF-PSS2).
The Bank of Finland Payment and Settlement System Simulator (BoF-PSS2) can be used for analysing liquidity needs, risk issues, settlement algorithms and changes in behavioural patterns, authority policies/regulations, settlement conventions and pricing/costs issues. The simulator is freely available for research purposes and has been introduced in dozens of countries.
http://www.bof.fi/en/rahoitusmarkkinat/kehityshankkeet/BoF-PSS2/index.htm

Carnegie-Mellon University Robotics Institute. Reusable Environment for Task-Structured Intelligent Networked Agents (RETSINA).
RETSINA is a multi-agent infrastructure that allows agent communication and coordination and has been applied to such diverse domains as financial portfolio management, auctions, logistics and military operations, and wireless communications.
http://www.cs.cmu.edu/~softagents/retsina_agent_arch.html

Defense Advanced Research Projects Agency (DARPA). Cougaar.
Cougaar is a Java-based architecture for the construction of highly scalable distributed agent-based applications. It is the product of a multi-year DARPA research project to develop an open-source agent-based architecture that supports applications ranging from small-scale systems to large-scale highly-survivable distributed systems."
http://www.cougaar.org/

Department of Homeland Security (DHS) and Department of Energy(DOE). The National Infrastructure Simulation and Analysis Center (NISAC).
NISAC conducts modeling, simulation, and analysis of the nation's critical infrastructure to assess critical infrastructure risk, vulnerability, interdependencies, and event consequences. Represented infrastructures include finance and banking, telecommunications, transportation, and electric power.
http://www.sandia.gov/nisac/

Federal Reserve Bank of New York
The Federal Reserve Bank of New York has used agent-based modeling to understand the features of payments flows and their dependency on the payment system rules and configuration.
See, *The Topology of Interbank Payment Flows* by K. Soramaki, M. L. Bech, J. Arnold, R. J. Glass, and W. Beyeler, Federal Reserve Bank of New York Staff Report No. 243 (2006).

Iowa State University. AMES Wholesale Power Market Test Bed
The Iowa State University *AMES Wholesale Power Market Test Bed* is an open source computational laboratory for testing the architecture, operation, and oversight of U.S. restructured electricity markets.
http://www.econ.iastate.edu/tesfatsi/AMESMarketHome.htm

National Aeronautics and Space Administration (NASA)
NASA uses agent-based modeling to understand the shuttle operations.
http://www.riacs.edu/research/projects/AgentBasedSimulation.jsp

National Institutes of Health (NIH)
NIH engaged the State University of New York at Buffalo to build upon traditional epidemiologic approaches using advanced simulation techniques to predict who would become infected, where they would be infected, when the infection would occur, and the outcome of the infection.
See, Chapter VI (Description of Agent-Based Models) in the *Draft Supplementary Risk Assessments and Site Suitability Analyses for the National Emerging Infectious Diseases Laborato*ry at Boston University prepared by the Division of Occupational Health and Safety of the National Institutes of Health (July 2007).

National Science Foundation (NSF). Cultural Anthropology Program
The Cultural Anthropology Program includes mathematical and computational models of sociocultural systems such as social network analysis, agent-based models, and integration of agent-based models with geographic information systems (GIS).
http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5388&org=BCS

U.S. Air Force. The Systems Effectiveness Analysis Simulation (SEAS) and the Unreal Tournament Semiautomated Force (UTSAF).
SEAS models collateral damage and UTSAF involves a multi-agent-based software bridge for interoperability between distributed military and commercial gaming simulation.
http://sim.sagepub.com/cgi/content/abstract/80/12/647

U. S. Navy. The Comprehensive Optimal Manpower and Personnel Analytic Simulation System (COMPASS).
COMPASS is designed to evaluate the feasibility of supply chain management, stochastic simulation, service-oriented architecture, and optimization by functionally representing the Navy's system of recruiting, selecting, and classifying candidates; training and advancing Sailors; distributing Sailor's to job assignments; and losing, separating, and re-enlisting Sailors.
http://www.stormingmedia.us/36/3638/A363805.html