that the number of shareholders N is not too high. In this section, we examine the role of proxy advisors for a large enough N, that is, for the case of sufficiently dispersed ownership. Specifically, let us fix c and increase N. Proposition 1 implies that there exists a threshold $\bar{N}$ such that for any $N > \bar{N}$, the equilibrium of the benchmark model without the advisor features no information acquisition at all. Intuitively, if the number of shareholders is very large, the free-rider problem in information acquisition is so severe that no shareholder finds it optimal to become informed. Clearly, because the resulting decisions are as good as pure noise, the advisor's presence cannot decrease the quality of decision-making. What is more interesting, as the next result establishes, the advisor's presence *strictly* improves decision-making in this case:

Proposition 7 (dispersed ownership). *Let $\bar{N}$ be the unique N that solves $c = \bar{c}$. Then, for any $N > \bar{N}$, firm value in the benchmark case without the advisor is strictly lower than firm value when the advisor is present.*

The reason for this result is that the advisor partly internalizes the free-rider problem among shareholders in its pricing policy: as the number of shareholders increases, it lowers its fee to ensure that at least some shareholders buy its recommendations. For example, in the limit case $N \to \infty$, the fee becomes infinitesimal. Hence, at least some shareholders become informed, making the decision somewhat informative even if the free rider problem in information acquisition is very severe.[28] Proposition 7 implies that the proxy advisor's presence unambiguously improves voting outcomes when ownership is so dispersed that shareholders would vote uninformatively without the advisor. The advisor's presence can only decrease value in cases where private information production is relevant.

IV. Analysis of Regulation

A. *Litigation Pressure*

As discussed in the introduction, the influence of proxy advisors is frequently attributed to institutions' desire to reduce the risk of litigation for their voting practices. For example, the 2003 SEC rule states that an institution "could demonstrate that the vote was not a product

[28] As shown in the appendix, in the limit case $N \to \infty$, the equilibrium fraction of shareholders who buy the advisor's recommendation converges to zero, and thus, in the limit, per-share firm value with and without the advisor converge to the same value (zero). However, for any finite $N > \bar{N}$, firm value is strictly higher with the advisor.

of a conflict of interest if it voted client securities, in accordance with a pre-determined policy, based upon the recommendations of an independent third party." To incorporate these incentives into the model, we assume that if a shareholder subscribes to and follows the advisor's recommendation, he gets an additional payoff $\Delta > 0$, which can be interpreted as the present value of litigation costs that get saved by following the advisor.

We show that greater litigation pressure improves decision-making only if the quality of the advisor's recommendation is sufficiently high. Intuitively, an increase in litigation pressure Δ has two effects. On the one hand, it induces shareholders to vote informatively. On the other hand, it shifts the incentives from doing proprietary research to following the advisor's recommendations. The overall effect of higher Δ thus depends on the quality of recommendations π. If π is low, then, as Section III.D shows, there is overreliance on the advisor's recommendation and inefficient crowding out of private information production. In this case, higher litigation pressure leads to even more inefficient crowding out of private information, reducing firm value. In contrast, if π is high, there is underreliance on the advisor's recommendation, and in this case, greater litigation pressure improves decision-making by increasing the fraction of shareholders who follow the advisor instead of voting uninformatively. Formally, in the Internet Appendix, we show that a marginal increase in Δ decreases firm value if the equilibrium features incomplete crowding out of private information acquisition, but weakly increases firm value under complete crowding out.

B. Reducing Proxy Advisory Fees

It is frequently argued that proxy advisors, in particular ISS, have too much market power. Indeed, the industry is dominated by two players, ISS and Glass Lewis, with ISS controlling 61% of the market and Glass Lewis controlling 37% (CEC (2011)). As a result, proposals to restrict proxy advisors' market power have been widely discussed (e.g., Edelman (2013)). For example, according to the GAO (2007) report, many institutional investors believe that "increased competition could help reduce the cost [of]... proxy advisory services."

Our analysis implies that an exogenous reduction in the advisor's fees improves decision-making only if the advisor's information is precise enough. Formally, the Internet Appendix shows that a marginal reduction in f increases firm value if equilibrium features complete crowding out of private information acquisition, but decreases firm value if equilibrium features incomplete crowding out. Intuitively, suppose that the advisor's information is not very precise, so that there is overreliance on its recommendations but some private information acquisition still occurs. In this case, lowering the advisor's fees would induce even more investors to follow its recommendations instead of doing independent research, which would be detrimental for firm value. In contrast, if the advisor's information is sufficiently precise and there is complete crowding out of private information, reducing the advisor's fees and thereby encouraging more shareholders to buy its recommendations instead of voting uninformatively is beneficial.

While formally studying competition is beyond the scope of this model, the above argument suggests that the entry of a new firm into the proxy advisory industry need not necessarily lead to more informative voting outcomes. On the one hand, the entry of a new advisor adds new information and can also increase the incumbent's incentive to improve the quality of its recommendations. For example, Li (2016) finds that the entry of Glass Lewis alleviated the pro-management bias of ISS recommendations. On the other hand, keeping the quality of recommendations fixed, new entry also lowers the equilibrium fees, which can be harmful if there is overreliance on proxy advisory recommendations. Thus, the overall effect of entry depends on how competition affects both the price and the quality of recommendations and on the amount of new information the entrant adds.

C. Improving the Quality of Recommendations

Market participants have raised concerns about the quality of proxy advisors' recommendations, pointing out potential conflicts of interest, inaccuracies in proxy advisory reports, and a one-size-fits-all approach to governance. Accordingly, several proposals have been made to improve the quality of recommendations, such as setting "qualification standards for proxy analysts" or requiring proxy advisors "to have a process that demonstrates due care towards formulating accurate voting recommendations." Moreover, some proposed regulations would make proxy advisors "subject to the wide range of fiduciary duties and obligations ... such as the duties of loyalty and prudence," effectively exposing them to legal risk for issuing low-quality recommendations (CEC (2011)).

Our analysis shows that an exogenous increase in quality π does not necessarily lead to more informative voting outcomes.[29] Intuitively, higher recommendation quality can encourage even more shareholders to follow the advisor instead of doing independent research,

[29]In particular, note that the benchmark model in which the advisor does not exist is equivalent to the model in which the advisor exists and its recommendation is pure noise ($\pi = 0.5$), because in this case no shareholder acquires it. Thus, Propositions 3 and 5 imply that firm value under $\pi > 0.5$ is lower than firm value under $\pi = 0.5$ if π is low enough. See also Figure 3d.

which can be detrimental to firm value if the quality of recommendations is not high enough.

E. Disclosing the Quality of Recommendations

Another commonly discussed policy is to increase the transparency of proxy advisors' methodologies and procedures, to make it easier for investors to evaluate the quality of their recommendations. For example, the 2010 SEC concept release on the U.S. proxy system discusses "increased disclosure regarding the extent of research involved with a particular recommendation and the extent and/or effectiveness of its controls and procedures in ensuring the accuracy of issuer data."[30]

To evaluate the effects of such proposals, we consider the following modification of the basic setting. The actual precision of the advisor's signal can be high or low, $\pi \in \{\pi_l, \pi_h\}$, $\pi_l < \pi_h$, with probabilities μ_l and μ_h, $\mu_h + \mu_l = 1$. We compare the quality of decision-making in two regimes – when the precision π is publicly disclosed and when it remains unknown to the shareholders. In the first case, precision $\pi \in \{\pi_l, \pi_h\}$ is first realized and learned by all parties, and then the game proceeds exactly as in the basic model. In the second case, the timing of the game is the same as in the basic model, but both the advisor's decision about the fee and shareholders' decisions about which signal to acquire and how to vote are made without knowing whether $\pi = \pi_l$ or $\pi = \pi_h$. In this case, as we show in the proofs, the equilibrium coincides with the equilibrium of the basic model for $\pi = \mu_l \pi_l + \mu_h \pi_h$.

In the Internet Appendix, we develop sufficient conditions under which disclosure improves decision-making, but also demonstrate that such disclosure can sometimes be harmful. Intuitively, the benefit of disclosure is that it allows shareholders to tailor their information acquisition decisions to the quality of recommendations – shareholders do not acquire the advisor's recommendations if they learn that recommendations are of low quality, $\pi = \pi_l$, and do not acquire private information if they learn that recommendations are of high quality, $\pi = \pi_h$. If π_h is high enough, such tailored information acquisition is more efficient than decision-making under uncertainty about π. However, if π_h is not very high, such tailored information acquisition can decrease firm value: in this case, π_h is not high enough to improve decision-making but is sufficiently high to crowd out private information acquisition. Voting would be more informed if shareholders were unsure about recommendation quality

[30]With respect to conflicts of interest, the 2014 SEC Staff Legal Bulletin No. 20 requires that proxy advisors disclose potential conflicts of interest to their existing clients, but many market participants push for further regulation, which would require conflicts of interests to be disclosed to the broader public.

and thus relied on their private information more.

V. Endogenous Quality of the Advisor's Recommendation

Our basic model takes the quality of the advisor's information as given. In this section, we extend the model by assuming that the advisor decides on the precision of its signal before offering to sell it to shareholders. Specifically, the advisor can acquire signal of precision π at cost $C(\pi, t)$, which is twice continuously differentiable in π, satisfying $C\left(\frac{1}{2}, t\right) = 0$ and $\frac{\partial}{\partial \pi} C(\pi, t) > 0$ with $\lim_{\pi \to \frac{1}{2}} \frac{\partial}{\partial \pi} C(\pi, t) = 0$ and $\lim_{\pi \to 1} \frac{\partial}{\partial \pi} C(\pi, t) = \infty$ for any $t \in (0, \infty)$ and $\pi \in \left(\frac{1}{2}, 1\right)$. These assumptions are intuitive: The cost of a signal is increasing in its precision with the purely noisy signal ($\pi = \frac{1}{2}$) being costless and the perfectly precise signal ($\pi = 1$) being infinitely costly. Parameter t captures the marginal cost of making the advisor's signal more precise and satisfies $\frac{\partial^2}{\partial \pi \partial t} C(\pi, t) > 0$ for any $\pi \in \left(\frac{1}{2}, 1\right)$ and $t \in (0, \infty)$, with $\lim_{t \to 0} \frac{\partial}{\partial \pi} C(\pi, t) = 0$ and $\lim_{t \to \infty} \frac{\partial}{\partial \pi} C(\pi, t) = \infty$ for any $\pi \in \left(\frac{1}{2}, 1\right)$. An example of the cost function that satisfies these restrictions is $C(\pi, t) = t\left(\frac{\pi}{1-\pi} - 1\right)^{\alpha}$ for any $\alpha > 1$. Both function $C(\pi, t)$ and parameter t are common knowledge.

The timing of the model is as follows. First, the advisor decides on precision π and pays cost $C(\pi, t)$. After that, all shareholders learn the advisor's choice of π, and the sequence of actions coincides with the basic model, illustrated in Figure 1. The next proposition establishes a result analogous to Proposition 5, the main result of the basic model:

Proposition 8. *Firm value in the presence of the advisor is strictly lower than in the benchmark case if and only if $t > \tilde{t}$, that is, the advisor's information acquisition technology is sufficiently inefficient.*

The argument is as follows. The basic model implies that the advisor's presence decreases firm value if the precision of its signal is below a certain cutoff $\tilde{\pi}$. When the advisor chooses the precision endogenously, parameter t of the cost function maps into the chosen precision $\pi^*(t)$ in a monotone way, from a purely noisy signal (if $t \to \infty$) to a perfectly precise signal (if $t \to 0$). When $t = \tilde{t}$, the endogenous precision $\pi^*\left(\tilde{t}\right)$ is exactly $\tilde{\pi}$.

While our main result remains unchanged, endogenous precision of the advisor's signal can be quite important for the policy implications, because regulation is likely to change the advisor's incentive to invest in information. For example, greater litigation pressure, analyzed in Section IV.A, can sometimes have another negative effect: by making the demand for the advisor's recommendations less sensitive to their informativeness, it can reduce the advisor's incentives to invest in high-quality research. To fully analyze the effects of suggested policy changes, one needs to consider this additional dimension through which they can affect the informativeness of voting.

VI. Discussion of Assumptions and Robustness

Our basic model is stylized and omits several features of the proxy advisory industry. In this section we discuss how it can be enriched to account for these features.

Correlated mistakes in private signals. The basic model assumes that private signals are independent conditional on the state, that is, $corr\,(s_i, s_j|\theta) = 0$. Thus, voting mistakes of shareholders that follow private signals are uncorrelated. It is, of course, possible that shareholders could make correlated mistakes, since their signals can be based on similar sources of information. A more general model would feature private signals with positive conditional correlation, that is, $corr\,(s_i, s_j|\theta) > 0$. However, as long as this correlation is imperfect, that is, $corr\,(s_i, s_j|\theta) < 1$, this model would feature exactly the same trade-offs and, we conjecture, the same qualitative results.

Possibility of getting the advisor's recommendation for free. In practice, recommendations of proxy advisors sometimes leak into the press, especially on high profile cases such as contested M&A cases and proxy fights. Hence, in principle, a shareholder can sometimes "buy" the advisor's recommendation without paying the subscription fee. Since our main result holds for any positive fee f, even infinitely small (see Proposition 3), many implications of the model with possible leakage will be similar to our basic model. It is also worth noting that in addition to getting the recommendation per se, an institution subscribing to the proxy advisor receives a detailed research report presenting the analysis underlying the final binary recommendation.[31] This possibility can be captured in an extension in which the advisor's research report consists of a continuous signal $r_1 \in (-\infty, \infty)$ and a binary recommendation $r_2 = I\{r_1 > 0\}$, where $I(\cdot)$ is an indicator function. While the binary recommendation can be obtained for free, a shareholder must pay the fee to get the continuous signal. Thus, the shareholder's value from subscribing to the advisor can be positive even if the binary recommendation is available for free.

[31] For example, the length of ISS's research reports on high-profile M&A cases and proxy contests is more than 20-30 pages, which, of course, provides more information than a binary recommendation. See https://www.issgovernance.com/solutions/governance-advisory-services/special-situations-research/.

Shareholders communicating or selling their information. Our setup assumes that shareholders do not communicate with each other prior to voting. In practice, the extent of such communication is limited: first, investors fear that communication with others can be considered as "forming a group"[32]; in addition, they are often reluctant to publicly disclose their intentions to vote against management, fearing it will be viewed as an activist campaign and lead to managerial retaliation. Studying communication between shareholders and examining its implications for the laws governing group formation could be an interesting direction for further research. Such a model would need to incorporate potential heterogeneity in investors' objectives — a feature that the current model abstracts from.

More generally, while our paper takes the presence of proxy advisors as given, understanding why these intermediaries exist is an important question on its own. Shareholders that do their own governance research could also sell vote recommendations to other investors. Why is this not happening? In addition to the arguments above, there are two plausible reasons for the existence of proxy advisors. One is an advantage in information production about governance matters, which was arguably the key reason for ISS emergence in 1985.[33] The other reason, which we address in Section IV.A, is regulatory guidance suggesting that following the "recommendations of an *independent* third party" could fulfill institutions' fiduciary duties to their clients (emphasis added).

Possibility of acquiring both signals in equilibrium. In equilibrium of our model, no shareholder acquires both the recommendation from the advisor and a private signal. In practice, some large institutional investors both subscribe to proxy advisors' services and do their own proprietary research. The likely reason is that a shareholder's cost of producing private information differs across proposals, depending on the type of the proposal and the shareholder's knowledge of the company. Because shareholders cannot buy the advisor's recommendations selectively, for a subset of proposals (proxy advisors sell their research on all firms and proposals as a bundle), we observe shareholders that both establish their own proxy research departments and subscribe to proxy advisors. To capture this feature, the model could be extended to two proposals, such that some shareholders would pay the fee for the bundle of two recommendations but would only follow the recommendation for one

[32] Forming a group requires filing a 13D and may trigger a poison pill. For example, according to the 2011 report by Dechert LLP, "shareholder concern about unintentionally forming a group has chilled communications among large holders of shares in U.S. public companies."

[33] According to Nell Minow, one of the founders of ISS, "All of a sudden, there were big, complicated issues that people wanted some guidance on," leading institutions to say "You know what I would like? I'd really like some advice on how to vote proxies." See the 2013 SEC Proxy Advisory Firms Roundtable transcript.

of the proposals and would acquire and follow their private signals for the other proposal. Such a model would feature the same forces as our basic model: the advisor's presence would crowd out private information acquisition on those proposals for which shareholders would do private research without the advisor.

Another reason why shareholders could find it optimal to acquire two signals is complementarity between the advisor's and private signal, which we discuss next.

Information structure and complementarity between signals. In our simple binary information structure, signals are substitutes: the value of the private signal s_i to an uninformed shareholder is higher than its value to a shareholder who buys the advisor's recommendation r. With different information structures, for example, if signals are continuous, knowledge of r may increase the value of s_i to a shareholder, that is, signals can be complements. A model with complementarity between r and s_i may feature some shareholders acquiring both signals and will have an additional force, which goes in the direction of the advisor "crowding in" private information acquisition and, if complementarity is very strong, can outweigh the "crowding out" force we study in the paper. However, apart from the substitutability vs. complementarity between signals, the binary information structure is not important for the results. In particular, if signals are continuous but are substitutes, the same type of equilibrium and same effects will emerge.[34]

In practice, both the substitution and the complementarity effect could be in play because proxy advisors perform two informational roles. First, they provide their clients with the actual voting recommendation, which is likely to have the crowding out effect since it is a substitute for the shareholder's own decision. Second, proxy advisors thoroughly read the long and often complicated proxy statements and aggregate the information in these proxy statements for their clients. This second informational role could arguably have both the substitution and the complementarity effect. On the one hand, it is likely to substitute private research in that shareholders may not read the proxy statements themselves and may miss some important information as a result. On the other hand, having a well-organized

[34]To see this, suppose, for example, that c is large enough so that some shareholders remain uninformed (analogously to restriction $c > \hat{c}$ in Assumption 2). If signals are substitutes, no shareholder will acquire both signals in equilibrium. This is because the value of an additional signal to a shareholder who already has another signal is lower than the value of the same signal to an uninformed shareholder. Hence, if some shareholders find it optimal to acquire both signals, it must be that uninformed shareholders find it optimal to acquire at least one signal, leading to a contradiction. Thus, shareholders will either stay uninformed, or acquire a private signal, or acquire the advisor's signal. At the voting stage, as long as distributions satisfy MLRP, shareholders would vote for the proposal if and only if their signal exceeds a certain cutoff. At the information acquisition stage, we would observe the crowding out effect highlighted in the paper.

summary of the proxy statement can help shareholders focus on interpreting this information and come up with the optimal voting decision,that is, such a summary can be thought of decreasing shareholders' costs of independent research.

VII. Empirical Implications

Our analysis shows that proxy advisors have a two-fold effect on the informativeness of shareholder votes, and thereby on firm value. The positive effect is that their presence improves voting decisions of those shareholders who would vote uninformatively otherwise, for example, of small shareholders who would always vote with management or vote randomly. The negative effect is that if many shareholders would invest in independent research without the proxy advisor (e.g., shareholders with relatively large stakes in the company), the advisor's presence crowds out this independent research and induces excessive conformity in shareholders' votes, leading them to make perfectly correlated mistakes. Which of the two effects dominates depends on firms' ownership structure: the positive effect is more likely to dominate if ownership is dispersed.[35]

Thus, an important implication of our paper is that, other things equal, the introduction of a proxy advisor's coverage or an exogenous shock increasing the advisor's influence increases value in firms with sufficiently dispersed ownership, but decreases value in firms with relatively concentrated ownership if recommendations are not sufficiently precise. To test this prediction in the time series, one could look at changes in firm value after proxy advisors initiate coverage for this firm, subject to the caveat that coverage initiation may not be fully exogenous. Alternatively, one could study the effect of regulations increasing proxy advisors' influence, such as the 2003 SEC rule discussed above and two 2004 no-action letters by the SEC, which clarified how asset managers could resolve their own conflicts of interest by relying on proxy advisors' recommendations.[36] For example, according to Sangiorgi and Spatt (2017), "these no-action letters have been very controversial because of the favorable impact upon the proxy-voting advisory firm business and the adverse societal consequences of the proxy-voting advisory firm reducing the extent of diverse information production."

[35] Formally, Proposition 3 and Proposition B.1 in the Internet Appendix show that the advisor's presence or its stronger influence due to, for example, stronger litigation pressure, has a positive (negative) effect on firm value in equilibrium with complete (incomplete) crowding out of private information. In turn, the proof of Proposition 7 shows that complete crowding out is more likely when N is large, that is, ownership is dispersed.

[36] See the "Investment Advisers Act of 1940 - Rule 206(4)-6" letter to Egan Jones and the "Investment Advisors Act of 1940 - Rule 206(4)-6" letter to ISS.

Calluzzo and Dudley (2017) follow a different approach to testing the above prediction by looking at cross-sectional variation in the influence of proxy advisors: they develop a firm-level measure of ISS influence based on the propensity of the firm's shareholders to vote with ISS. They show that ISS influence is positively associated with firm value in firms with dispersed ownership, but is negatively, albeit often insignificantly, associated with firm value when ownership is more concentrated. The authors interpret this evidence as being consistent with the implications of our paper.

To test the crowding out effect more directly, one could explicitly examine shareholders' decisions to invest in independent research. One way to infer the extent of private information acquisition is to look at shareholders' votes: shareholders who acquire private information are more likely to deviate from proxy advisors' recommendations. For example, the evidence in Iliev and Lowry (2015), Ertimur, Ferri, and Oesch (2013), Larcker, McCall, and Ormazabal (2015), and Malenko and Shen (2016) suggests that shareholders are more likely to do independent research when they are large, have a large investment in the firm, and have low turnover. Another, more direct, way to measure private information acquisition is the approach of Iliev, Kalodimos, and Lowry (2018), who study the downloads of firms' proxy statements and proxy-related SEC filings by large mutual fund families using the IP address data. The authors find that an institution's tendency to vote against ISS is higher when it does such independent research more.

Another prediction of our analysis is that the quality of proxy advisors' recommendations (π) has a non-monotonic effect on firm value. Indeed, as Figure 3 demonstrates, when π is not very high, an increase in π allows the advisor to crowd out more private information acquisition, which decreases value. However, when π is sufficiently high, shareholders do not invest in private information production anyway, so a further increase in π has a positive effect on value. Regulation of the proxy advisory industry is a potential source of variation in the quality of recommendations. For example, one intention of the 2014 SEC Staff Legal Bulletin No. 20 was to reduce the conflicts of interest in proxy advisors' recommendations (which could be interpreted as an increase in π) by increasing the pressure on both asset managers and proxy advisors to be vigilant about such conflicts.

VIII. Conclusion

In this paper, we provide a simple framework for analyzing the impact of proxy advisors on shareholder voting. In our model, a monopolistic advisor (proxy advisory firm) offers to sell its information (vote recommendations) to voters (shareholders) for a fee, and voters decide whether to engage in private information production and/or buy the advisor's recommendation, and how to cast their votes. Our main results can be summarized as follows. First, the proxy advisor's presence increases firm value only if the quality of its recommendations is sufficiently high. Second, if it is not sufficiently high, there is overreliance on the advisor's recommendations relative to the degree that would maximize firm value. Finally, if the information of the advisor is very precise, there is under-reliance on its signal: because of market power, the advisor rations its information to maximize profits.

We also examine the effects of several proposals that have been put forward to regulate the proxy advisory industry. We show that increasing litigation pressure increases incentives of shareholders to vote informatively but shifts them from doing independent research to following the proxy advisor. As a consequence, increasing litigation pressure improves decision-making only if the advisor's recommendations are sufficiently precise. Likewise, reducing the advisor's fees improves decision-making if the advisor's recommendations are of high quality, but increases shareholders' overreliance on the advisor and lowers firm value if recommendations are of low-quality. Finally, higher recommendation quality and higher transparency about the quality do not unambiguously improve decision-making.

Several extensions of our model can be fruitful. First, it is natural to extend the model to allow for conflicts of interest among shareholders. Second, allowing for heterogeneity of shareholders in their voting power can lead to additional effects. Finally, it can be interesting to examine the optimal voting rules in this framework. Since extending the model in these directions is not straightforward, we leave them for future research.

References

Admati, Anat R., and Paul Pfleiderer (1986). A monopolistic market for information, *Journal of Economic Theory* 39, 400–438.

Admati, Anat R., and Paul Pfleiderer (1990). Direct and indirect sale of information, *Econometrica* 58, 901–928.

Alexander, Cindy R., Mark A. Chen, Duane J. Seppi, and Chester S. Spatt (2010). Interim news and the role of proxy voting advice, *Review of Financial Studies* 23, 4419–4454.

Alonso, Ricardo, and Odilon Câmara (2016). Persuading voters, *American Economic Review* 106, 3590–3605.

Angeletos, George-Marios, and Alessandro Pavan (2007). Efficient use of information and social value of information, *Econometrica* 75, 1103–1142.

Austen-Smith, David, and Jeffrey S. Banks (1996). Information aggregation, rationality, and the Condorcet Jury Theorem, *American Political Science Review* 90, 34–45.

Banerjee, Abhijit V. (1992). A simple model of herd behavior, *Quarterly Journal of Economics* 107, 797–817.

Bergemann, Dirk, Alessandro Bonatti, and Alex Smolin (2017). The design and price of information, *American Economic Review*, forthcoming.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch (1992). A theory of fads, fashion, custom and cultural change as informational cascades, *Journal of Political Economy* 100, 992–1026.

Bond, Philip, and Hulya Eraslan (2010). Strategic voting over strategic proposals, *Review of Economic Studies* 77, 459–490.

Bond, Philip, and Itay Goldstein (2015). Government intervention and information aggregation by prices, *Journal of Finance* 70, 2777–2812.

Boot, Arnoud W. A., and Anjan V. Thakor (2001). The many faces of information disclosure, *Review of Financial Studies* 14, 1021–1057.

Brav, Alon, and Richmond D. Mathews (2011). Empty voting and the efficiency of corporate governance, *Journal of Financial Economics* 99, 289–307.

Calluzzo, Paul, and Evan Dudley (2017). The real effects of proxy advisors on the firm, Working Paper.

Center on Executive Compensation (CEC) white paper "A call for change in the proxy advisory industry status quo," January 2011.

Cespa, Giovanni (2008). Information sales and insider trading with long-lived information, *Journal of Finance* 63, 639–672.

Chakraborty, Archishman, and Rick Harbaugh (2010). Persuasion by cheap talk, *American Economic Review* 100, 2361–2482.

Cuñat, Vicente, Mireia Gine, and Maria Guadalupe (2012). The vote is cast: the effect of corporate governance on shareholder value, *Journal of Finance* 67, 1943–1977.

Diamond, Douglas W. (1984). Financial intermediation and delegated monitoring, *Review of Economic Studies* 51, 393–414.

Diamond, Douglas W. (1985). Optimal release of information by firms, *Journal of Finance* 40, 1071–1094.

Edelman, Sagiv (2013). Proxy advisory firms: A guide for regulatory reform, *Emory Law Journal* 62, 1369–1409.

Ertimur, Yonca, Fabrizio Ferri, and David Oesch (2013). Shareholder votes and proxy advisors: evidence from say on pay, *Journal of Accounting Research* 51, 951–996.

Ertimur, Yonca, Fabrizio Ferri, and Stephen Stubben (2010). Board of directors' responsiveness to shareholders: Evidence from shareholder proposals, *Journal of Corporate Finance* 16, 53–72.

Feddersen, Timothy, and Wolfgang Pesendorfer (1998). Convicting the innocent: the inferiority of unanimous jury verdicts under strategic voting, *American Political Science Review* 92, 23–35.

Fishman, Michael J., and Kathleen M. Hagerty (1995). The incentive to sell financial market information, *Journal of Financial Intermediation* 4, 95–115.

Fos, Vyacheslav, and Wei Jiang (2015). Out-of-the-money CEOs: Private control premium and option exercises. *Review of Financial Studies* 29, 1549–1585.

Fos, Vyacheslav (2017). The disciplinary effects of proxy contests. *Management Science* 63, 655–671.

Gallagher, D. M. (2014). Outsized power & influence: The role of proxy advisers. Working Paper Series, Critical Legal Issues.

Garcia, Diego, and Francesco Sangiorgi (2011). Information sales and strategic trading, *Review of Financial Studies* 24, 3069–3104.

Gerardi, Dino, and Leeat Yariv (2008). Information acquisition in committees, *Games and Economic Behavior* 62, 436–459.

Gershkov, Alex, and Balázs Szentes (2009). Optimal voting schemes with costly information acquisition, *Journal of Economic Theory* 144, 36–68.

Goldstein, Itay, and Liyan Yang (2017). Information disclosure in financial markets, *Annual Review of Financial Economics* 9, 101–25.

Government Accountability Office (GAO) report "Corporate shareholder meetings: Issues relating to firms that advise institutional investors on proxy voting," June 2007.

Government Accountability Office (GAO) report "Proxy advisory firms' role in voting and corporate governance practices," November 2016.

Grossman, Gene M., and Elhanan Helpman (2001). *Special-Interest Politics*, MIT Press, Cambridge, MA.

Hellwig, Christian, and Laura Veldkamp (2009). Knowing what others know: Coordination motives in information acquisition, *Review of Economic Studies* 76, 223–251.

Iliev, Peter, and Michelle Lowry (2015). Are mutual funds active voters? *Review of Financial Studies* 28, 446–285.

Iliev, Peter, Jonathan Kalodimos, and Michelle Lowry (2018). Investors' attention to corporate governance, Working Paper.

Jackson, Matthew O., and Xu Tan (2013). Deliberation, disclosure of information, and voting, *Journal of Economic Theory* 148, 2–30.

Khanna, Naveen, and Richmond D. Mathews (2011). Can herding improve investment decisions? *RAND Journal of Economics* 42, 150–174.

Khanna, Naveen, and Mark Schroder (2015). "Rewarding disagreement" for optimal collective decisions: The role of skill and luck, Working Paper.

Larcker, David F., Allan L. McCall, and Gaizka Ormazabal (2015). Outsourcing shareholder voting to proxy advisory firms, *Journal of Law and Economics* 58, 173–204.

Levit, Doron, and Nadya Malenko (2011). Nonbinding voting for shareholder proposals, *Journal of Finance* 66, 1579–1614.

Li, Tao (2016). Outsourcing corporate governance: conflicts of interest within the proxy advisory industry, *Management Science*, forthcoming.

Ma, Shichao, and Yan Xiong (2018). Information bias in the proxy advisory market, Working Paper.

Malenko, Nadya, and Yao Shen (2016). The role of proxy advisory firms: Evidence from a regression-discontinuity design, *Review of Financial Studies* 29, 3394–3427.

Martinelli, César (2006). Would rational voters acquire costly information? *Journal of Economic Theory* 129, 225–251.

Maskin, Eric, and Jean Tirole (1988). A theory of dynamic oligopoly, II: Price competition, kinked demand curves, and edgeworth cycles, *Econometrica* 56, 571–599.

Maug, Ernst (1999). How effective is proxy voting? Information aggregation and conflict resolution in corporate voting contests, Working Paper.

Maug, Ernst, and Bilge Yilmaz (2002). Two-class voting: A mechanism for conflict resolution, *American Economic Review* 92, 1448–1471.

Maug, Ernst, and Kristian Rydqvist (2009). Do shareholders vote strategically? Voting behavior, proposal screening, and majority rules, *Review of Finance* 13, 47–79.

McCahery, Joseph A., Zacharias Sautner, and Laura T. Starks (2016). Behind the scenes: The corporate governance preferences of institutional investors, *Journal of Finance* 71, 2905–2932.

Morris, Stephen, and Hyun Song Shin (2002). Social value of public information, *American Economic Review* 92, 1521–1534.

Persico, Nicola (2004). Committee design with endogenous information, *Review of Economic Studies* 71, 165–191.

Persson, Torsten, and Guido Tabellini (2002). Political economics and public finance, *Handbook of Public Economics* 3, 1549–1659.

Piccolo, Alessio, and Joel Shapiro (2017). Credit ratings and market information, Working Paper.

Ramakrishnan, Ram T. S., and Anjan V. Thakor (1984). Information reliability and a theory of financial intermediation, *Review of Economic Studies* 51, 415–432.

Sangiorgi, Francesco, and Chester Spatt (2017). The economics of credit rating agencies, *Foundations and Trends in Finance* 12, 1–116.

Schnakenberg, Keith E. (2015). Expert advice to a voting body, *Journal of Economic Theory* 160, 102–113.

Van Wesep, Edward (2014). The idealized electoral college voting mechanism and shareholder power, *Journal of Financial Economics* 113, 90–108.

Appendix: Proofs

Proof of Proposition 1.

Fix probability q with which each shareholder i acquires a private signal s_i. In the Internet Appendix, we prove that for any q, the equilibrium $w_s(0) = 0$, $w_s(1) = 1$, and $w_0 = \frac{1}{2}$ exists (as argued before, this is the only possible equilibrium at the voting stage because otherwise information would have zero value and acquiring it would be suboptimal).

Next, consider shareholder i's value from becoming informed. Conditional on the shareholder's private signal being $s_i = 1$, whether he is informed or not only makes a difference if the number of "for" votes among other shareholders is exactly $\frac{N-1}{2}$. Let us denote this set of events by PIV_i. In this case, by acquiring the signal, the shareholder votes "for" for sure, instead of randomizing between voting "for" and "against," so his utility from being informed is $\frac{1}{2}\mathbb{E}[u(1,\theta)|s_i = 1, PIV_i]$. Similarly, conditional on his private signal being $s_i = 0$, the shareholder's utility from being informed is $-\frac{1}{2}\mathbb{E}[u(1,\theta)|s_i = 0, PIV_i]$. Overall, the shareholder's value of acquiring a private signal is

$$\begin{aligned} V_s(q) = {} & \Pr(s_i = 1)\Pr(PIV_i|s_i = 1)\,\tfrac{1}{2}\mathbb{E}[u(1,\theta)|s_i = 1, PIV_i] \\ & - \Pr(s_i = 0)\Pr(PIV_i|s_i = 0)\,\tfrac{1}{2}\mathbb{E}[u(1,\theta)|s_i = 0, PIV_i]. \end{aligned}$$

By the symmetry of the setup and strategies, $\mathbb{E}[u(1,\theta)|s_i = 1, PIV_i] = -\mathbb{E}[u(1,\theta)|s_i = 0, PIV_i]$

and $\Pr(PIV_i|s_i=1)=\Pr(PIV_i|s_i=0)$, so we get

$$V_s(q)=\tfrac{1}{2}\Pr(PIV_i|s_i=1)\,\mathbb{E}[u(1,\theta)|s_i=1,PIV_i]$$
$$=\tfrac{1}{2}\Pr(PIV_i|s_i=1)(\Pr[\theta=1|s_i=1,PIV_i]-\Pr[\theta=0|s_i=1,PIV_i])$$
$$=\Pr[\theta=1,PIV_i,s_i=1]-\Pr[\theta=0,PIV_i,s_i=1]=\tfrac{1}{2}p\Pr[PIV_i|\theta=1]-\tfrac{1}{2}(1-p)\Pr[PIV_i|\theta=0]$$

Conditional on $\theta=1$, other shareholders make their voting decisions independently and vote "for" with probability $qp+\frac{1}{2}(1-q)=\frac{1}{2}+q\left(p-\frac{1}{2}\right)$. Hence,

$$\Pr[PIV_i|\theta=1]=C_{N-1}^{\frac{N-1}{2}}\left(\frac{1}{2}+q(p-\frac{1}{2})\right)^{\frac{N-1}{2}}\left(\frac{1}{2}-q(p-\frac{1}{2})\right)^{\frac{N-1}{2}}.$$

Noting that $\Pr[PIV_i|\theta=1]=\Pr[PIV_i|\theta=0]$ gives (6). Note that $V_s(q)$ decreases in q. Since $P\left(x,N-1,\frac{N-1}{2}\right)$ decreases in N for any x, it follows that $V_s(q)$ decreases in N.

In deciding whether to acquire the private signal, shareholder i compares the expected value of his signal $V_s(q)$ with cost c. Since $V_s(q)$ is strictly decreasing in q, there are three possible cases. If $c<\underline{c}\equiv V_s(1)$, then each shareholder acquires information regardless of q. Hence, in the unique equilibrium all shareholders acquire private signals: $q^*=1$. If $c>\bar{c}\equiv V_s(0)$, then each shareholder is better off not acquiring information regardless of q. Hence, in the unique equilibrium all shareholders remain uninformed: $q^*=0$. Finally, if $c\in[\underline{c},\bar{c}]$, then q^* is given as the solution to $V_s(q^*)=c$. Plugging (6) and rearranging the terms, we get (7).

Finally, we derive the equilibrium firm value given q_0^* :

$$V_0=\Pr(\theta=1)\textstyle\sum_{k=\frac{N+1}{2}}^{N}P(q_0^*p+\frac{1-q_0^*}{2},N,k)-\Pr(\theta=0)\sum_{k=\frac{N+1}{2}}^{N}P(q_0^*(1-p)+\frac{1-q_0^*}{2},N,k)$$
$$=\tfrac{1}{2}\textstyle\sum_{k=\frac{N+1}{2}}^{N}P(\frac{1}{2}+\Lambda,N,k)-\frac{1}{2}\sum_{k=\frac{N+1}{2}}^{N}P(\frac{1}{2}+\Lambda,N,N-k)]=\sum_{k=\frac{N+1}{2}}^{N}P(\frac{1}{2}+\Lambda,N,k)-\frac{1}{2},$$

where we used $\sum_{k=0}^{N}P(q,N,k)=1$.

Proof of Proposition 2.

Let us prove that there is no equilibrium in which a shareholder acquires both signals with positive probability. By contradiction, suppose such an equilibrium exists and consider a shareholder with both signals, r and s_i. Consider a realization $r=1$ and $s_i=0$. There are three possibilities: $w_{rs}(1,0)=1$, $w_{rs}(1,0)=0$, and $w_{rs}(1,0)\in(0,1)$. First, if $w_{rs}(1,0)=1$, then it must be that $w_{rs}(1,1)=1$ because the shareholder's posterior that $\theta=1$ is strictly higher in this case. By symmetry, $w_{rs}(0,1)=1-w_{rs}(1,0)=0$. In turn, $w_{rs}(0,1)=0$ implies $w_{rs}(0,0)=0$, since the shareholder's posterior that $\theta=1$ is strictly lower in this case. It follows that $v_i=r$, and hence the shareholder would be better off if he acquired only the advisor's signal. Second, if $w_{rs}(1,0)=0$, then it must be that $w_{rs}(0,0)=0$. By symmetry, $w_{rs}(0,1)=1-w_{rs}(1,0)=1$, and hence $w_{rs}(1,1)=1$. It follows that $v_i=s_i$, and hence the shareholder would be better off if he only acquired the private signal. Finally, if $w_{rs}(1,0)\in(0,1)$, then by symmetry $w_{rs}(0,1)=1-w_{rs}(1,0)\in(0,1)$. Hence, when $r\neq s_i$, the shareholder is indifferent between voting $v_i=r$ and $v_i=s_i$. Hence, the shareholder would be better off if he only acquired one signal of the two.

The arguments in the text preceding Proposition 2 complete the proof. In the Internet Appendix, we derive the condition under which equilibrium $w_s(s_i)=s_i$, $w_r(r)=r$, and $w_0=\frac{1}{2}$ will exist for any possible sub-game. However, whenever this condition is violated, this sub-game features zero value of recommendation of the advisor, and hence is not reached on equilibrium path if $q_r > 0$.

Proof of Lemma 1. To prove the lemma, we derive the necessary and sufficient conditions for each type of equilibrium to exist.

1. Equilibrium with only private information acquisition. Consider the case of $q_r = 0$. In this case, a shareholder's choice between buying a private signal and staying uninformed is identical to the situation in which there is no advisor, covered in Proposition 1. Hence, $q_s = q_0^* \in (0,1)$. Pair $(q_r, q_s) = (0, q_0^*)$ is an equilibrium if and only if no shareholder would be better off deviating to buying recommendation from the advisor: $V_r(0, q_0^*) \leq f$. Since $\Omega_1(0, q_0^*) = \Omega_2(0, q_0^*) = \frac{c}{p-0.5}$ (the latter by indifference $V_s(0, q_0^*) = c$), $V_r(0, q_0^*) = \frac{\pi-0.5}{p-0.5}c$. Hence, $V_r(0, q_0^*) \leq f$ is equivalent to $f \geq \bar{f} = \frac{\pi-0.5}{p-0.5}c$.

2. Equilibrium with complete crowding out of private information acquisition. Consider the case of $q_s = 0$. Then it must be that $q_r \in (0,1)$. Indeed, it cannot be that $q_r = 0$, since if $q_r = 0$, then the value of acquiring a private signal is $V_s(0,0) = \bar{c} > c$ by Assumption 1, so a shareholder would be better off deviating to acquiring a private signal. It also cannot be that $q_r = 1$, since in that case no shareholder would be pivotal, so $V_r(1,0) = 0 < f$ for any $f > 0$. Thus, a shareholder would be better off deviating to staying uninformed. For $q_s = 0$ and $q_r \in (0,1)$ to constitute an equilibrium, it is necessary and sufficient that $V_s(q_r, 0) \leq c$ and $V_r(q_r, 0) = f$. When $q_s = 0$, the probabilities of being pivotal are:

$$\Omega_1(q_r, 0) = P\left(\frac{1+q_r}{2}, N-1, \frac{N-1}{2}\right) = P\left(\frac{1-q_r}{2}, N-1, \frac{N-1}{2}\right) = \Omega_2(q_r, 0) \equiv \Omega_r(q_r). \quad \text{(A1)}$$

Eq. $V_r(q_r, 0) = f$ yields $\Omega_r(q_r) = \frac{f}{\pi-0.5}$. Equating to (A1), we obtain that q_r is given by (13), which lies in $(0,1)$ if $f < C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(\pi - \frac{1}{2}\right)$. Otherwise, no solution exists. Plugging $\Omega_r(q_r) = \frac{f}{\pi-0.5}$ into $c \geq V_s(q_r, 0)$, we obtain $f \leq \frac{2\pi-1}{2p-1}c$. Note that

$$C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(\pi - \frac{1}{2}\right) > \frac{2\pi-1}{2p-1}c \Leftrightarrow \frac{1}{4} > \left(\frac{c}{\left(p-\frac{1}{2}\right) C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}},$$

which is satisfied by Assumption 1. Hence, the equilibrium with complete crowding out of private information exists if and only if $f \leq \bar{f}$.

3. Equilibrium with incomplete crowding out of private information acquisition. Consider the case of $q_s > 0$. If $q_r + q_s < 1$ in equilibrium, then a shareholder must be indifferent between acquiring r, acquiring s_i, and staying uninformed. Hence, q_s and q_r must satisfy $V_s(q_r, q_s) = c$ and $V_r(q_r, q_s) = f$, which yields a system of linear equations for Ω_1 and Ω_2:

$$\begin{cases} \pi\Omega_1 + (1-\pi)\Omega_2 = \frac{c}{p-0.5} \\ \pi\Omega_1 - (1-\pi)\Omega_2 = 2f \end{cases} \Leftrightarrow \Omega_1 = \frac{f + \frac{c}{2p-1}}{\pi} \text{ and } \Omega_2 = \frac{\frac{c}{2p-1} - f}{1-\pi}. \quad \text{(A2)}$$

In particular, such an equilibrium does not exist when $\pi = 1$. Suppose $\pi < 1$. Since the second equality implies $f \leq \frac{c}{2p-1}$, this system is equivalent to the following system of equations for q_r and

q_s:

$$\begin{aligned} \left(\tfrac{1}{2}q_r + \left(p-\tfrac{1}{2}\right)q_s\right)^2 &= \tfrac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}, \\ \left(\tfrac{1}{2}q_r - \left(p-\tfrac{1}{2}\right)q_s\right)^2 &= \tfrac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}. \end{aligned} \tag{A3}$$

It has a solution if and only if the right-hand sides of both equations are non-negative, that is, if $f \in \left[\underline{f}_1, 2^{1-N}\pi C_{N-1}^{\frac{N-1}{2}} - \frac{c}{2p-1}\right]$, where

$$\underline{f}_1 \equiv \frac{c}{2p-1} - 2^{1-N}\left(1-\pi\right)C_{N-1}^{\frac{N-1}{2}}, \tag{A4}$$

in which case there are two solutions:

1. Solution with $q_r \leq (2p-1)\,q_s$, denoted (q_r^a, q_s^a):

$$\begin{aligned} q_r^a &= \sqrt{\tfrac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} - \sqrt{\tfrac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}, \\ q_s^a &= \tfrac{1}{2p-1}\left(\sqrt{\tfrac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} + \sqrt{\tfrac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}\right). \end{aligned} \tag{A5}$$

2. Solution with $q_r \geq (2p-1)\,q_s$, denoted (q_r^b, q_s^b):

$$\begin{aligned} q_r^b &= \sqrt{\tfrac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} + \sqrt{\tfrac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}, \\ q_s^b &= \tfrac{1}{2p-1}\left(\sqrt{\tfrac{1}{4} - \left(\frac{f+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} - \sqrt{\tfrac{1}{4} - \left(\frac{\frac{c}{2p-1}-f}{(1-\pi)C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}\right). \end{aligned} \tag{A6}$$

Each solution is an equilibrium if and only if it satisfies $q_r > 0$, $q_s > 0$, and $q_r + q_s < 1$. Each solution satisfies $(q_r, q_s) > 0$ if and only if $\frac{f+\frac{c}{2p-1}}{\pi} < \frac{\frac{c}{2p-1}-f}{1-\pi} \Leftrightarrow f < \bar{f}$. Also, since $p \in \left(\frac{1}{2}, 1\right)$, it is easy to see that $q_r^b + q_s^b \leq q_r^a + q_s^a$.

If $q_r + q_s = 1$ in equilibrium, then a shareholder must be indifferent between acquiring r and s_i and weakly prefer this over staying uninformed. Hence, q_s and q_r must satisfy $V_s(q_r, q_s) - c = V_r(q_r, q_s) - f \geq 0$ and $q_s + q_r = 1$. The former implies

$$\left(p - \frac{1}{2}\right)\left(\pi\Omega_1 + (1-\pi)\,\Omega_2\right) - c = \frac{1}{2}\left(\pi\Omega_1 - (1-\pi)\,\Omega_2\right) - f \equiv \psi \geq 0. \tag{A7}$$

For any ψ, these two equations lead to a system identical to (A2)$\Leftrightarrow$(A3), but with $c+\psi$ and $f+\psi$ instead of c and f. It has a solution if and only if the right-hand sides of both equations are positive.

In that case, it has two solutions, analogous to (A5) and (A6), and given by (IA8) and (IA9) in the Internet Appendix.

To prove the lemma, we show the following sequence of three auxiliary claims, which are proved in the Internet Appendix.

1. Claim 1: If $f \geq \bar{f}$, then there is no equilibrium $(q_r, q_s) > 0$.
2. Claim 2: If $\frac{2p}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{\underline{f}_1+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \leq 1$, there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}_1, \bar{f}\right)$, where $\underline{f}_1$ is given by (A4).
3. Claim 3: If $\frac{2p}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{\underline{f}_1+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} > 1$, there exists $\underline{f}_2 \geq \underline{f}_1$ such that there is an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}_2, \bar{f}\right)$.

Combining Claims 2 and 3, we conclude that there exists an equilibrium $(q_r, q_s) > 0$ if and only if $f \in \left[\underline{f}, \bar{f}\right)$, where

$$\underline{f} \equiv \begin{cases} \underline{f}_1 & \text{if } \frac{2p}{2p-1}\sqrt{\frac{1}{4}-\left(\frac{\underline{f}_1+\frac{c}{2p-1}}{\pi C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}} \leq 1 \\ \underline{f}_2 & \text{otherwise,} \end{cases} \tag{A8}$$

where $\underline{f}_1$ is given by (A4) and $\underline{f}_2$ is defined in Claim 3, respectively. Combining this condition and the conditions of existence of equilibrium with only private information acquisition and equilibrium with complete crowding out of private information acquisition, we get the statement of the lemma.

Proof of Proposition 3.

Consider an equilibrium defined by pair q_s and q_r. Let $U(q_r, q_s)$ denote the corresponding expected value of a proposal per share. By definition,

$$\begin{aligned} U(q_r, q_s) &= \mathbb{E}\left[u(1,\theta)\, d\right] = \tfrac{1}{2}\mathbb{E}\left[\textstyle\sum_{j=1}^{N} v_j > \frac{N-1}{2} | \theta = 1\right] - \tfrac{1}{2}\mathbb{E}\left[\textstyle\sum_{j=1}^{N} v_j > \frac{N-1}{2} | \theta = 0\right] \\ &= \tfrac{1}{2}\pi\left(\textstyle\sum_{k=\frac{N+1}{2}}^{N} P(p_a, N, k) - \sum_{k=\frac{N+1}{2}}^{N} P(1-p_a, N, k)\right) \\ &\quad + \tfrac{1}{2}(1-\pi)\left(\textstyle\sum_{k=\frac{N+1}{2}}^{N} P(p_d, N, k) - \sum_{k=\frac{N+1}{2}}^{N} P(1-p_d, N, k)\right), \end{aligned}$$

where

$$\begin{aligned} p_a &\equiv \Pr(v_i = \theta | r = \theta) = q_r + q_s p + \tfrac{1-q_r-q_s}{2} = \tfrac{1}{2} + \tfrac{1}{2}q_r + \left(p - \tfrac{1}{2}\right) q_s, \\ p_d &\equiv \Pr(v_i = \theta | r \neq \theta) = q_s p + \tfrac{1-q_r-q_s}{2} = \tfrac{1}{2} - \tfrac{1}{2}q_r + \left(p - \tfrac{1}{2}\right) q_s, \end{aligned} \tag{A9}$$

are the probabilities that a random shareholder votes correctly conditional on the proxy advisor's recommendation being correct and incorrect, respectively. Using $P(q, N, k) = P(1-q, N, N-k)$ and $\sum_{k=0}^{N} P(q, N, k) = 1$, the above expression simplifies to

$$U(q_r, q_s) = \sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(p_a, N, k) + (1-\pi) P(p_d, N, k)\right) - \frac{1}{2}. \tag{A10}$$

Proof of part 1. Note that the probability of a shareholder being pivotal in equilibrium with incomplete crowding out weakly exceeds that in the benchmark case:

$$\pi P\left(p_a, N-1, \frac{N-1}{2}\right) + (1-\pi) P\left(p_d, N-1, \frac{N-1}{2}\right) = \\ = \pi\Omega_1(q_r, q_s) + (1-\pi)\Omega_2(q_r, q_s) \geq \frac{2c}{2p-1}.$$

Indeed, it exactly equals $\frac{2c}{2p-1}$ if $q_s + q_r < 1$ based on (A2), and equals $\frac{2(c+\psi)}{2p-1} \geq \frac{2c}{2p-1}$ if $q_s + q_r = 1$, where $\psi \geq 0$ is given by (A7). Consider the following optimization problem:

$$\begin{array}{c} \max_{p_a, p_d} \sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(p_a, N, k) + (1-\pi) P(p_d, N, k)\right) - \frac{1}{2} \\ \text{s.to } \pi P\left(p_a, N-1, \frac{N-1}{2}\right) + (1-\pi) P\left(p_d, N-1, \frac{N-1}{2}\right) \geq \frac{2c}{2p-1} \end{array} \tag{A11}$$

This optimization problem chooses the probabilities of a correct vote, p_a and p_d, that maximize firm value subject to the "budget constraint" that the probability that a shareholder is pivotal, implied by p_a and p_d, cannot be below $\frac{2c}{2p-1}$ (i.e., that in the benchmark case). In what follows, we show that this optimization problem is solved by $p_a = p_d = \frac{1}{2} + q_0^*\left(p - \frac{1}{2}\right)$, that is, the same as in the benchmark case. Let $x_a \equiv P(p_a, N-1, \frac{N-1}{2})$ and $x_d \equiv P(p_d, N-1, \frac{N-1}{2})$. Let us define function $\varphi(x) \in (\frac{1}{2}, 1)$ as the higher root of $x = P(\varphi(x), N-1, \frac{N-1}{2}) = C_{N-1}^{\frac{N-1}{2}} (\varphi(x)(1-\varphi(x))^{\frac{N-1}{2}}$:

$$\varphi(x) \equiv \frac{1}{2} + \sqrt{\frac{1}{4} - \left(\frac{x}{C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}. \tag{A12}$$

Note that $p_a > \frac{1}{2}$ and hence $p_a = \varphi(x_a)$. If $p_d > \frac{1}{2}$, then $p_d = \varphi(x_d)$, and if $p_d < \frac{1}{2}$, then $p_d = 1 - \varphi(x_d)$. First, consider all equilibria with $p_d > \frac{1}{2}$. Then, we can rewrite (A11) as:

$$\begin{array}{c} \max_{x_a, x_d} \sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(\varphi(x_a), N, k) + (1-\pi) P(\varphi(x_d), N, k)\right) - \frac{1}{2} \\ \text{s.t. } \pi x_a + (1-\pi) x_d \geq \frac{2c}{2p-1}, \end{array} \tag{A13}$$

Auxiliary Lemma A1 at the end of the Appendix shows that function $f(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$ is strictly decreasing in x. Thus, the constraint in (A13) is binding. Auxiliary Lemma A1 also shows that $f(x)$ is strictly concave in x. Thus, by Jensen's inequality, for any x_a, x_d such that $\pi x_a + (1-\pi) x_d = \frac{2c}{2p-1}$, we have

$$\pi f(x_a) + (1-\pi) f(x_d) < f(\pi x_a + (1-\pi) x_d) = f\left(\frac{2c}{2p-1}\right) = \pi f\left(\frac{2c}{2p-1}\right) + (1-\pi) f\left(\frac{2c}{2p-1}\right)$$

Therefore, there is a unique solution to the maximization problem (A13), given by $x_a = x_d = \frac{2c}{2p-1}$, which gives firm value in the benchmark case. Hence, for any equilibrium with incomplete crowding out and $p_d > \frac{1}{2}$, firm value is strictly lower than in the benchmark case. Next, consider all equilibria with $p_d < \frac{1}{2}$. Note that $\sum_{k=\frac{N+1}{2}}^{N} P(1-q, N, k) = \sum_{k=\frac{N+1}{2}}^{N} P(q, N, N-k) = 1 - \sum_{k=\frac{N+1}{2}}^{N} P(q, N, k)$. In addition, $\sum_{k=\frac{N+1}{2}}^{N} P_q(q, N, k) = -\sum_{k=0}^{\frac{N-1}{2}} P_q(q, N, k) > 0$ for $q \geq \frac{1}{2}$ because $P_q(q, N, k) = P(q, N, k) \frac{k-Nq}{q(1-q)} < 0$ for any $k < \frac{N}{2}$ and $q \geq \frac{1}{2}$. Since $\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2}, N, k\right) = \frac{1}{2}$, it follows that $\sum_{k=\frac{N+1}{2}}^{N} P(1-q,N,k) < \frac{1}{2} < \sum_{k=\frac{N+1}{2}}^{N} P(q,N,k)$ for $q > \frac{1}{2}$. Therefore,

$$\sum_{k=\frac{N+1}{2}}^{N} \begin{pmatrix} \pi P(p_a,N,k) \\ +(1-\pi) P(p_d,N,k) \end{pmatrix} - \frac{1}{2} = \sum_{k=\frac{N+1}{2}}^{N} \begin{pmatrix} \pi P(\varphi(x_a),N,k) \\ +(1-\pi) P(1-\varphi(x_d),N,k) \end{pmatrix} - \frac{1}{2}$$
$$< \sum_{k=\frac{N+1}{2}}^{N} \left(\pi P(\varphi(x_a),N,k) + (1-\pi) P(\varphi(x_d),N,k)\right) - \frac{1}{2},$$

and the last expression, subject to the constraint in (A13), has already been shown to be below firm value in the benchmark case. Hence, the quality of decision-making in any equilibrium with incomplete crowding out is strictly lower than in the benchmark case.

Proof of part 2. Next, we prove the second part of the proposition. In the equilibrium with complete crowding out of private information, we have

$$\begin{aligned} p_a &= \frac{1}{2} + \frac{1}{2} q_r = \frac{1}{2} + \sqrt{\left(\frac{1}{4} - \frac{f}{(\pi-\frac{1}{2})C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}, \\ p_d &= \frac{1}{2} - \frac{1}{2} q_r = \frac{1}{2} - \sqrt{\left(\frac{1}{4} - \frac{f}{(\pi-\frac{1}{2})C_{N-1}^{\frac{N-1}{2}}}\right)^{\frac{2}{N-1}}}. \end{aligned} \tag{A14}$$

Since $p_d = 1 - p_a$, we can rewrite firm value as

$$U = \pi \sum_{k=\frac{N+1}{2}}^{N} P(p_a,N,k) + (1-\pi) \sum_{k=0}^{\frac{N-1}{2}} P(p_a,N,k) - \frac{1}{2} = \frac{1}{2} - \pi + (2\pi-1) \sum_{k=\frac{N+1}{2}}^{N} P(p_a,N,k). \tag{A15}$$

By (7) and (8), the expected value in the benchmark case without the advisor is given by $U = \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*,N,k) - \frac{1}{2}$, where $p_0^* = \frac{1}{2} + q_0^*\left(p - \frac{1}{2}\right)$. Firm value is higher with the advisor than without it if and only if

$$(2\pi - 1) \sum_{k=\frac{N+1}{2}}^{N} P(p_a,N,k) - \pi > \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*,N,k) - 1. \tag{A16}$$

In the Internet Appendix, we show that the left-hand side of (A16) is strictly increasing in π, that (A16) is violated for $\pi \to \frac{1}{2} + \frac{f}{c}\left(p - \frac{1}{2}\right)$ and is satisfied for $\pi \to 1$. By monotonicity, there exists a unique $\pi^*(f) \in (\frac{1}{2} + \frac{f}{c}(p - \frac{1}{2}), 1)$ such that the advisor's presence increases firm value if and only if $\pi \geq \pi^*(f)$.

Proof of Proposition 4. The first three statements of the proposition follow directly from Lemma 1 and from Lemma A3 in the Internet Appendix. Note also that given $c > \hat{c}$ in Assumption 2, we have $\underline{f} = \underline{f}_1$, where $\underline{f}_1$ is given by (A4). For any $\pi < 1$, the interval $[\underline{f}_1, \bar{f})$ is non-empty because $\underline{f}_1 < \bar{f} \Leftrightarrow c < \left(p - \frac{1}{2}\right) C_{N-1}^{\frac{N-1}{2}} 2^{1-N} = \bar{c}$, which is satisfied by Assumption 1. For $\pi = 1$, $\underline{f}_1 = \bar{f}$ and hence the interval is empty.

We next prove the last statement of the proposition. First, consider $f < \underline{f}$. From (13) q_r is strictly decreasing in f, and from (A15) firm value is strictly increasing in p_a (and hence, in q_r, as

$p_a = \frac{1}{2} + \frac{1}{2}q_r$). Hence, firm value is strictly decreasing in f for $f < \underline{f}$. Second, consider $f \in [\underline{f}, \bar{f})$. In this range, firm value equals $\pi f(x_a) + (1-\pi) f(x_d)$, where $f(x) = \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$, $x_a = \frac{f + \frac{c}{2p-1}}{\pi}$ and $x_d \equiv \frac{\frac{c}{2p-1} - f}{1-\pi}$. Differentiating firm value in fee f yields $f'(x_a) - f'(x_b) = -\int_{x_a}^{x_b} f''(x)\,dx > 0$ by $x_a < x_d$ (follows from $f < \overline{f}$) and $f''(\cdot) < 0$ (follows from Auxiliary Lemma A1). Hence, firm value is strictly increasing in f for $f \in [\underline{f}, \bar{f})$. Finally, if $f \geq \overline{f}$, then firm value equals V_0, so it is unaffected by f.

Proof of Proposition 5. Consider the first statement of the proposition. The first part of Proposition 3 implies that if equilibrium features incomplete crowding out, then firm value is strictly lower than in the benchmark case. Hence, firm value can only be higher with the advisor if the advisor sets fee in a way that crowds out private information acquisition. In case of complete crowding out, there is a one-to-one correspondence between the fee f set by the advisor and the fraction $q_r^H(f)$ buying its recommendation, where $q_r^H(f)$ is given by (13). Moreover, recall that the value of the advisor's signal to a shareholder is given by $V_r(q_r, 0) = (\pi - \frac{1}{2})P(\frac{1+q_r}{2}, N-1, \frac{N-1}{2})$ and must be equal to f. Thus, in this case, the advisor's problem is equivalent to maximizing $q_r V_r(q_r, 0)$ over q_r. Hence, instead of choosing fee f and maximizing $f q_r^H(f)$, the advisor can choose q_r and maximize $\eta(q_r) = P(\frac{1+q_r}{2}, N-1, \frac{N-1}{2}) q_r = C_{N-1}^{\frac{N-1}{2}} \left(\frac{(1+q)(1-q)}{4}\right)^{\frac{N-1}{2}} q$. Note that

$$\frac{d\eta}{dq} = const \times \frac{d}{dq}\left[q\left(1-q^2\right)^{\frac{N-1}{2}}\right] = const \times \left(1-q^2\right)^{\frac{N-3}{2}}\left(1-Nq^2\right).$$

Hence, $\eta(q)$ is inverted U-shaped in q with a maximum at $q_m = \frac{1}{\sqrt{N}}$. The optimal fraction $q_m = \frac{1}{\sqrt{N}}$ translates into the optimal fee given by

$$f_m \equiv (\pi - \frac{1}{2})P(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N-1, \frac{N-1}{2}), \tag{A17}$$

The fact that $\eta(q)$ is inverse U-shaped in q implies that under complete crowding out, the advisor's revenue is maximized at $f = f_m$ (the solution to the unconstrained maximization problem) and is monotonically decreasing as f gets farther from f_m in both directions. Hence, the optimal pricing strategy of the advisor is to set f_m if $f_m < \underline{f}$, and the optimal pricing strategy if $f_m \geq \underline{f}$ is to either (1) set $f \approx \underline{f}_1$ (specifically, the highest fee below $\underline{f}_1$ in the set of feasible prices), where $\underline{f}_1$ is given by (A4), that is, to set the highest possible fee that would allow to completely crowd out private information acquisition, or (2) choose the fee that maximizes the advisor's revenue under incomplete crowding out. In the second case, firm value is lower than in the benchmark case according to Proposition 3. In the first case, firm value is infinitely close to firm value under complete crowding out and $f = \underline{f}_1$. We next consider two cases separately: $\pi < 1$ and $\pi = 1$.

1. If $\pi < 1$, then, as shown in the proof of Proposition 4, the interval $[\underline{f}_1, \bar{f})$ is non-empty and hence, for $f = \underline{f}_1$, the equilibrium with complete crowding out co-exists with the equilibrium with incomplete crowding out. In the Internet Appendix, we show that the equilibrium with complete crowding out for $f = \underline{f}_1$ has strictly lower firm value than the equilibrium with incomplete crowding out for $f = \underline{f}_1$, which (by Proposition 3) is in turn lower than firm value in the benchmark case.

2. If $\pi = 1$, then $\underline{f}_1 = \bar{f}$, and hence there is no fee that would generate an equilibrium with incomplete crowding out. Hence, if $f_m \geq \underline{f}$, the advisor's only option is to engage in limit pricing,

that is, set fee $f \approx \underline{f}_1$(specifically, the highest fee below $\underline{f}_1$ in the set of feasible prices). Under limit pricing, using (13) and plugging in $\pi = 1$ and $f \approx \underline{f}_1 = \frac{c}{2p-1}$, we get $q_r \approx 2\Lambda$ and using (A10) and (A14), firm value in this case is infinitely close to $\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2}+\Lambda, N, k\right) - \frac{1}{2} = V_0$, that is, firm value in the benchmark case.

Combining the two cases, if $f_m \geq \underline{f}$, then firm value with the advisor is never strictly higher than in the benchmark case (it is either strictly lower if $\pi < 1$ or exactly the same if $\pi = 1$). Therefore, the only case where firm value can be strictly higher than in the benchmark case is when the advisor faces an unconstrained maximization problem, that is, when $f_m < \underline{f} = \underline{f}_1$, so that the advisor chooses fee f_m. The constraint $f_m < \underline{f}_1$ can be simplified to

$$\pi > \hat{\pi} \equiv \frac{1}{2}\left(1 + \frac{C_{N-1}^{\frac{N-1}{2}} 2^{1-N} - \frac{2c}{2p-1}}{C_{N-1}^{\frac{N-1}{2}} 2^{1-N}\left(1 - \left(\frac{N-1}{N}\right)^{\frac{N-1}{2}}\right)}\right)$$

If each shareholder acquires the advisor's signal with probability q_r and remains uninformed otherwise, expected firm value is given by

$$\begin{aligned} V^*(\pi, q_r) &= \Pr(\theta = 1)\sum_{k=\frac{N+1}{2}}^{N}\left[\pi P\left(q_r + \tfrac{1-q_r}{2}, N, k\right) + (1-\pi) P\left(\tfrac{1-q_r}{2}, N, k\right)\right] \\ &\quad - \Pr(\theta = 0)\sum_{k=\frac{N+1}{2}}^{N}\left[\pi P\left(\tfrac{1-q_r}{2}, N, k\right) + (1-\pi) P\left(q_r + \tfrac{1-q_r}{2}, N, k\right)\right] \\ &= (\pi - \tfrac{1}{2})\sum_{k=\frac{N+1}{2}}^{N}\left[P\left(\tfrac{1+q_r}{2}, N, k\right) - P\left(\tfrac{1-q_r}{2}, N, k\right)\right] \\ &= (2\pi - 1)\left[\sum_{k=\frac{N+1}{2}}^{N} P\left(\tfrac{1+q_r}{2}, N, k\right) - \tfrac{1}{2}\right]. \end{aligned} \tag{A18}$$

Plugging in $q_r = \frac{1}{\sqrt{N}}$ in (A18), we get firm value under unconstrained maximization,

$$V^*(\pi) = (2\pi - 1)\left[\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k\right) - \frac{1}{2}\right], \tag{A19}$$

and comparing it with V_0, we get

$$(2\pi - 1)\left[\sum_{k=\frac{N+1}{2}}^{N} P\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}, N, k\right) - \frac{1}{2}\right] > V_0 = \sum_{k=\frac{N+1}{2}}^{N} P(p_0^*, N, k) - \frac{1}{2} \Leftrightarrow \pi > \tilde{\pi}. \tag{A20}$$

In the Internet Appendix, we compare $\hat{\pi}$ and $\tilde{\pi}$ and show that $\hat{\pi} \leq \tilde{\pi} \Leftrightarrow g\left(\frac{1}{2} + \frac{1}{2\sqrt{N}}\right) \leq g\left(\frac{1}{2} + \Lambda\right)$ is satisfied if and only if $\frac{1}{2} + \frac{1}{2\sqrt{N}} \geq \frac{1}{2} + \Lambda \Leftrightarrow \Lambda \leq \frac{1}{2\sqrt{N}}$. Note also that $\Lambda \leq \frac{1}{2\sqrt{N}} \Leftrightarrow \tilde{\pi} \leq 1$, as follows from (A20). Hence, if $\Lambda \leq \frac{1}{2\sqrt{N}}$, then $\hat{\pi} \leq \tilde{\pi}$ and $\tilde{\pi} \leq 1$, so in this case, the advisor strictly improves the quality of decision-making compared to the benchmark case only if $\pi \in (\tilde{\pi}, 1]$. If $\Lambda > \frac{1}{2\sqrt{N}}$, then $\hat{\pi} > \tilde{\pi}$ and $\tilde{\pi} \geq 1$, so $f_m < \underline{f}_1$ requires $\pi > 1$, which is impossible. In this case, for any $\pi < 1$, the advisor strictly decreases the quality of decision-making and for $\pi = 1$ does not change it. Hence, in both cases, the advisor strictly improves decision-making compared to the benchmark case only if $\pi \in (\tilde{\pi}, 1]$.

The fact that $\tilde{\pi} \geq 1 \Leftrightarrow \Lambda \geq \frac{1}{2\sqrt{N}}$ also proves the second part of the proposition: if $\Lambda \geq \frac{1}{2\sqrt{N}} \Leftrightarrow (2p-1)\, q_0^* \geq \frac{1}{\sqrt{N}}$, the advisor strictly decreases firm value for $\pi < 1$ and does not change it for $\pi = 1$, that is, firm value is weakly lower with the advisor for any precision of its recommendations. (In this case, we have $\hat{\pi} \geq \tilde{\pi}$, so $\hat{\pi} \geq 1$, implying that $f_m \geq \underline{f}_1$, that is, the advisor either engages in limit pricing or accommodates private information acquisition.)

It remains to prove that the condition $\pi > \tilde{\pi}$ is also sufficient for the advisor to strictly increase firm value. As shown above, $\pi > \tilde{\pi}$ requires $\tilde{\pi} < 1$ and hence $\Lambda < \frac{1}{2\sqrt{N}}$, in which case $\hat{\pi} < \tilde{\pi}$. Hence, $\pi > \tilde{\pi}$ implies $\pi > \hat{\pi}$, which is equivalent to $f_m < \underline{f} = \underline{f}_1$. It follows that for such π, the advisor finds it optimal to set fee f_m and, as (A20) shows, firm value in this case is indeed strictly higher than in the benchmark case.

Note also that the result of the proposition does not depend on the equilibrium selection criterion. Currently, Assumption 2 assumes that in the region $f \in [\underline{f}, \bar{f}]$, where multiple equilibria exist, the equilibrium with incomplete crowding out given by (A5) is selected. Consider other possible equilibrium selection criteria. First, suppose that in the region $f \in [\underline{f}, \bar{f}]$, the second equilibrium with incomplete crowding out, given by (A6), is selected. Since Proposition 3 implies that firm value in equilibrium with incomplete crowding out is strictly lower than in the benchmark case, the only case where firm value could increase is when equilibrium features complete crowding out of private information. Hence, the above arguments apply without change to this equilibrium selection as well, leading to the same condition $\pi > \tilde{\pi}$ being necessary and sufficient for firm value to be strictly higher with the advisor. Similarly, suppose that in the region $f \in [\underline{f}, \bar{f}]$, the equilibrium with complete crowding out is selected. The arguments above apply to this case without change as well, leading to the same necessary and sufficient condition $\pi > \tilde{\pi}$.

Proof of Proposition 6. First, consider $q_s > 0$. Then, the planner's problem is $\max_{q_r, q_s} U(q_r, q_s)$ subject to $V_s(q_r, q_s) \geq c$. This problem is equivalent to problem (A11). As shown in the proof of Proposition 3, its solution coincides with the equilibrium of the benchmark case without the proxy advisor, that is, $(q_r, q_s) = (0, q_0^*)$. Second, consider $q_s = 0$. Then, the planner's problem is $\max_{q_r} U(q_r, 0)$, which is solved by $q_r = 1$. Thus, the solution to the planner's problem is either $(0, q_0^*)$ or $(1, 0)$, whichever leads to a higher $U(q_r, q_s)$. Since the probabilities of the correct decision under $(0, q_0^*)$ and $(1, 0)$ are π^{**} and π, respectively, we obtain the statement of the proposition.

Proof of Proposition 7. In the benchmark case, Proposition 1 implies that firm value equals zero: if $N > \bar{N}$, then $q^* = 0$, implying $V_0 = 0$. Consider the model with the advisor. It is sufficient to show that $(q_r, q_s) = (0, 0)$ is not an equilibrium. By contradiction, suppose that it is, and consider the proxy advisor's revenue if he sets fee f_m, given by (A17). This fee results in $q_r > 0$ and hence strictly positive revenues of the advisor, implying that $(0, 0)$ is not an equilibrium. Hence, the equilibrium firm value with the advisor is always strictly positive.

Note also that in the limit of $N \to \infty$, the equilibrium fraction of shareholders that buys the advisor's recommendation approaches zero. This is because when $N \to \infty$, $f_m \to 0$ and $\underline{f}_1 \to \frac{c}{2p-1}$. Since $\underline{f} \geq \underline{f}_1$, we have $f_m < \underline{f}$ in the limit of $N \to \infty$, and hence the proxy advisor sets fee f_m, which corresponds to $q_r = \frac{1}{\sqrt{N}}$. Because $\lim_{N \to \infty} \frac{1}{\sqrt{N}} = 0$, firm value converges to zero as well when $N \to \infty$. This argument also implies that if N is above a certain threshold, the equilibrium features complete crowding out of private information.

Proof of Proposition 8. After the seller has chosen π, the subgame becomes identical to the basic model, so its equilibrium is given by Proposition 4. Denote the expected revenues by the seller for a given choice of π by $R(\pi) = N \max_f f q_r(f, \pi)$, where $q_r(f, \pi)$ is given by (14). The optimal choice of precision, $\pi^*(t)$, solves

$$\pi^*(t) \in \arg\max_{\pi} \{R(\pi) - C(\pi, t)\}.$$

The proof is based on proving three statements: (1) $\lim_{t\to\infty} \pi^*(t) = \frac{1}{2}$; (2) $\lim_{t\to 0} \pi^*(t) = 1$; (3) $\pi^*(t)$ is decreasing in t. We prove each of them below.

1. The properties of $C(\pi, t)$ imply that $\lim_{t\to\infty} C(\pi, t) = \infty$ for any $\pi > \frac{1}{2}$. On the other hand, (10) implies that f is bounded from above by $\frac{\pi}{2}$ and hence $R(\pi)$ is bounded from above by $\frac{N\pi}{2}$. This implies that if $\lim_{t\to\infty} C(\pi, t) > \frac{1}{2}$ and hence $\pi^*(t)$ is bounded away from $\frac{1}{2}$ when t is sufficiently large, then the advisor's revenue $R(\pi^*(t)) - C(\pi^*(t), t)$ is negative for a sufficiently large t, which cannot be optimal — the advisor can always set $\pi = \frac{1}{2}$ and get zero revenue. Hence, indeed, $\lim_{t\to\infty} C(\pi, t) = \frac{1}{2}$.

2. As an auxiliary result, we prove in the Internet Appendix that $R'(\pi)$ is bounded away from zero for π in the neighborhood of 1. This property implies that there exists $\delta > 0$ and $r > 0$ such that $R'(\pi) > r$ for any $\pi > 1 - \delta$. We now prove that $\lim_{t\to 0} \pi^*(t) = 1$. Suppose instead that $\lim_{t\to 0} \pi^*(t) < 1$. Then, there exists $\varepsilon < \delta$ and $\underline{t}$ such that $\pi^*(t) < 1 - \varepsilon$ for any $t < \underline{t}$. Since $\lim_{t\to 0} \frac{\partial}{\partial\pi} C(\pi, t)|_{\pi=1-\varepsilon} = 0$ and $\frac{\partial^2}{\partial\pi^2} C(\pi, t) > 0$, there exists $\hat{t} < \underline{t}$ such that for $t < \hat{t}$, $\frac{\partial}{\partial\pi} C(\pi, t)|_{\pi=\pi^*(t)} < \frac{\partial}{\partial\pi} C(\pi, t)|_{\pi=1-\varepsilon} < r$. But then, $\frac{\partial}{\partial\pi}[R(\pi) - C(\pi, t)]|_{\pi=\pi^*(t)} > 0$. Hence, the advisor could marginally increase π and achieve a higher profit for t, which implies that $\pi^*(t)$ cannot be optimal. This contradiction proves that $\lim_{t\to 0} \pi^*(t) < 1$.

3. Finally, we prove that $\pi^*(t)$ is decreasing in t. Consider any $t_2 > t_1$. Denoting $\pi^*(t_i) = \pi_i$, $i \in \{1, 2\}$, we have

$$R(\pi_2) - C(\pi_2, t_2) \geq R(\pi_1) - C(\pi_1, t_2),$$
$$R(\pi_1) - C(\pi_1, t_1) \geq R(\pi_2) - C(\pi_2, t_1),$$

implying

$$C(\pi_2, t_1) - C(\pi_1, t_1) \geq C(\pi_2, t_2) - C(\pi_1, t_2) \Leftrightarrow \int_{\pi_1}^{\pi_2} \frac{\partial}{\partial\pi} C(\pi, t_1)\, d\pi \geq \int_{\pi_1}^{\pi_2} \frac{\partial}{\partial\pi} C(\pi, t_2)\, d\pi. \quad \text{(A21)}$$

Since $\frac{\partial^2}{\partial\pi\partial t} C(\pi, t) > 0$, then $\int_{\pi_1}^{\pi_2} \frac{\partial}{\partial\pi} C(\pi, t)\, d\pi$ is strictly increasing in t whenever $\pi_2 > \pi_1$. Hence, (A21) can only be satisfied if $\pi_2 \leq \pi_1$, which proves that $\pi^*(t)$ is decreasing.

Overall, we have proved that $\pi^*(t)$ is decreasing in t, taking values from arbitrarily close to one (for $t \to 0$) to arbitrarily close to $\frac{1}{2}$ (for $t \to \infty$). Define $\tilde{t} \equiv \max\{t : \pi^*(t) = \min(\tilde{\pi}, 1)\}$, where $\tilde{\pi}$ is given by (15). Then, Proposition 5 and the monotonicity of $\pi^*(t)$ imply that the equilibrium firm value is strictly lower than the benchmark case firm value if and only if $t > \tilde{t}$.

Auxiliary Lemma A1. *Function* $f(x) \equiv \sum_{k=\frac{N+1}{2}}^{N} P(\varphi(x), N, k)$, *where* $\varphi(x)$ *is defined by (A12), is strictly decreasing and strictly concave.*
The proof is relegated to the Internet Appendix.