Mr. Brent J. Fields, Secretary
Securities and Exchange Commission
100 F Street, NE
Washington, DC 20549-1090
via SEC internet submission form

Re: File No. 4-725 - SEC Staff Roundtable on the Proxy Process

Dear Mr. Fields,

We are Shichao Ma and Yan Xiong, Ph.D. candidates from Department of Political Science, University of Rochester, and Rotman School of Management, University of Toronto, respectively. Particular focuses of our research and education have been finance and public policy, which we consider relevant to addressing developments in the proxy voting process.

We believe the input of the academic community can play a valuable role in providing the SEC with a rounded picture of how to address public policy issues generally and, in this instance, those within the proxy voting process. As those who have conducted recent research in this area, we welcome the opportunity to contribute to the Securities and Exchange Commission Roundtable.

Among the topics proposed for discussion, we note that the SEC is seeking responses on the role proxy advisors play in the proxy process. Motivated by the Commission's 2010 concept release on the U.S. Proxy System, we have been working on a research article titled *Information Bias in the Proxy Advisory Market* in the past year.

In our paper, we examined the influence of proxy advisors on their clients, and investigated various factors that may affect the quality of voting recommendations (especially unbiasedness) issued by proxy advisors. Our findings are particular relevant for the questions relating to investors' reliance on proxy advisors' recommendations; whether there is sufficient transparency about proxy advisor methodologies; and, whether conflicts of interests are adequately disclosed by proxy advisors.

Below, we set out a summary of our main findings:

- Conflicts of interest at proxy advisory firms will negatively influence the quality of voting recommendations as recommendations are biased and unhelpful to clients.

- Conversely, when investors are over-optimistic (or over-pessimistic) about the voting item, the voting recommendations issued by proxy advisory firms will be distorted to reflect this investor bias, which may serve to undermine the robustness of proxy advisors' methodologie. Like some mass media outlets, proxy advisory firms will produce reports for their clients containing information they want to hear – not what they need to hear in terms of governance risk – as a means of retaining business.

- On occasion, delegating voting choices to proxy advisors may be in the best interests of investors but only if the proxy advisory firm has no conflicts of interest and investors have reasonably correct, independent knowledge of the proposed voting item.

- Funds’ reliance on proxy advisory firms is not necessarily in the best interests of funds’ clients. Specifically, if the interests of funds and fund shareholders are not aligned, proxy advisors will cater for the interest of funds at the expense of funds’ clients (the ultimate beneficial owners)

Based on our research and findings, we believe the following public policy and regulatory alterations would serve to enhance the proxy voting process:

1. The introduction of a clear requirement for proxy advisory firms to disclose their potential conflicts of interest, particularly relating to consulting services.

2. Provide clarity that taking advice from proxy advisory firms does not necessarily result in funds fulfilling their obligations to vote in the best interests of their clients.

3. Provide guidance for investors on the limitations of proxy advisory firms: The information proxy advisory firms provide is inadequate in protecting them against their own misconceptions and biases.

We hope that you find our contribution helpful in discussing and determining the appropriate measures to improve the efficacy of the role of proxy advisors, and the proxy voting process overall. A copy of our paper, which details the extensive research underlying the findings outlined, is attached.

Sincerely,

Shichao Ma

Ph.D. candidate in Political Science
University of Rochester
Rochester, NY, USA
[redacted]

Yan Xiong

Ph.D. candidate in Finance
University of Toronto
Toronto, ON M5S 3E6, Canada
[redacted]

Information Bias in the Proxy Advisory Market

Shichao Ma and Yan Xiong*

May 2018

Abstract

We study an information-sale problem in which a monopolist proxy advisor sells voting recommendations to a firm's shareholders for corporate voting. The proxy advisor chooses the level of bias in the sold information and sets its price to maximize profits. We make a distinction between the information that is unbiased and the one that is desired by the shareholders. We show that the proxy advisor provides both unbiased and desirable voting advice when it has no conflicts of interest, and the shareholders have the correct beliefs and aim to maximize the firm value. However, when these conditions are not satisfied (as they typically will not be) the proxy advisor sends biased voting advice, and there is no inherent link between information bias and desirability. Our results point out that conflicted proxy advisor sends biased voting recommendations, whereas the unconflicted one may also send biased advice.

Keywords: Information sales, Bias, Proxy advisor, Conflicts of interest, Shareholder voting

JEL Classification: D82, G34, L15

*Ma: Department of Political Science, University of Rochester, s.ma@rochester.edu. Xiong: Rotman School of Management, University of Toronto, yan.xiong14@rotman.utoronto.ca. We thank Heski Bar-Isaac, Daniel Csaba (discussant), Ignatius J. Horstmann, Xianwen Shi, Liyan Yang, Jingyu Zhang (discussant), participants at Trans-Atlantic Doctoral Conference, Young Economist Symposium, the graduate student seminar at University of Rochester and University of Toronto for helpful comments.

When occasions present themselves, in which the interests of the people are at variance with their inclinations, it is the duty of the persons whom they have appointed to be the guardians of those interests, to withstand the temporary delusion, in order to give them time and opportunity for more cool and sedate reflection.

—Alexander Hamilton, *The Federalist Paper: No. 71*

1 Introduction

Proxy advisors provide investors with analysis of and recommendations for voting on matters presented for a shareholder vote. They wield increasing influence over institutional investors over the past three decades. Especially after 2003, when institutional investors are required by the SEC to vote on all matters on the corporate proxy and disclose their votes to beneficial owners of their holdings (Securities and Exchange Commission, 2003), institutional investors have substantially increased their use of proxy advisors due to the size and diversity of their investment portfolio. Currently, the largest proxy advisor is Institutional Shareholder Services (ISS). It helps over 1,700 institutional clients and covers about 40,000 meetings in 117 countries.

Given the nature of their business model, proxy advisors are information intermediaries, and thus we expect them to provide information to support informed corporate voting. As with other information intermediaries, however, we also concern about the quality of their information as their interests are not necessarily aligned with their clients'. In particular, we may wonder what proxy advisors' contribution to informed corporate voting is and at where their limitations are. Do proxy advisors provide institutional clients with accurate, unbiased information about the corporate proposals? Without an actual economic stake in the corporate, will their proxy services improve corporate values? How will the potential conflicts of interest inherent in some proxy advisors' business models affect the quality of the information? When clients have some particular preferences or beliefs, will proxy advisors cater to these clients, or stick to the principle as a clear-mind third party?

In this paper, we provide a framework to address these questions. Our model is built upon the strategic voting literature in political science (e.g., Austen-Smith and Banks, 1996; Feddersen and

Pesendorfer, 1996; Malenko and Malenko, 2017). In our model, a proxy advisor sells its binary voting recommendation to shareholders, who in turn use this piece of information to vote at a firm meeting on a proposal in order to maximize the firm value. Whether the proposal increases or decreases firm value, however, depends on the state of the world, which is uncertain to all agents ex ante. The proxy advisor has information advantage over shareholders. It owns private information about the value of the proposal and may use this private information to form a certain policy to issue its voting recommendation. In addition, the proxy advisor can also set the price of its recommendation. After observing the information price and the recommendation issue policy, each shareholder independently decides whether to buy information from the proxy advisor, or stay with their prior. All shareholders then vote simultaneously and the proposal is implemented if it is approved by the majority of shareholders.

Before studying information quality, we first define that a voting recommendation is *unbiased* if and only if the recommendation is consistent with the more likely state given the evidence. That is, an unbiased recommendation means that a positive (negative) recommendation is issued only if the proxy advisor's private information suggests the proposal is more likely to increase (decrease) firm value. However, the welfare implication of the unbiased voting recommendation remains nuanced: an unbiased voting advice may not be desired by a particular shareholder. We thus define that a voting recommendation is *desirable* for a shareholder if it gives the shareholder the highest utility. In other words, this is the voting recommendation the shareholder would have sent to herself if she had observed the proxy advisor's private information. Our results suggest that there is no inherent link between information bias and desirability.

As a meaningful benchmark, we present a setting in which the proxy advisor does not have conflicts of interest, the shareholders have the correct prior about the value of the proposal and they aim to maximize the firm value. In this ideal setting, the proxy advisor provides both unbiased and desirable voting recommendations to the shareholders, and the firm value is maximized as a consequence. This benchmark suggests that we can reasonably expect high-quality voting recommendations from the proxy advisor even though it is only a for-profit information intermediary in a one-shot game. It is also worth noting that this finding is unique to the proxy advisory market and is generally not the case for information intermediaries in other contexts.[1] In our context, the distinct incentives of the information seller and the buyers can be aligned because of the cooperative nature of voting. When more shareholders have high-quality information, the voting outcome is more likely to be correct and the information is not valued less because of information leakage in equilibrium. Consequently, the proxy advisor sends high-quality voting recommendations.

We next relax the assumptions of the ideal setting to study information bias and its welfare implications. Firstly, we consider the possibility that shareholders may have some particular preferences. That is, they may not aim to maximize the firm value. Scholars find institutional investors can be more sensitive to losses than gains (e.g., Haigh and List, 2005; Bodnaruk and Simonov, 2016) and thus, they are more reluctant to vote for the proposal. Alternatively, institutional investors tend to vote more often with a firm's management when they have business ties with that firm (Cvijanović et al., 2016). Under either case, will the proxy advisor cater to such preferences of shareholders? We find the answer is yes: when the shareholders tend to vote for the proposal, the proxy advisor will be more likely to send a positive recommendation, and vice versa. Doing so the proxy advisor maximizes its profits from selling information, which are determined by the information value perceived by shareholders. Therefore, such a recommendation is biased but desirable for shareholders. However, the firm value is impaired compared to the benchmark case because of this bias.

Secondly, what would be the voting recommendation if shareholders have incorrect prior information? Will the proxy advisor be the guardian of the shareholders' interest? Unfortunately, we find the proxy advisor has no incentive to correct the shareholders' "temporary delusion": the proxy advisor tends to approve the proposal when shareholders are over-optimistic, and disapprove when over-pessimistic. Such a voting recommendation is not only biased but also

[1] For example, Admati and Pfleiderer (1986) show that an information seller in a speculative market has an incentive to sell noisy information. This is because information may leak through market price and this leakage dilutes information value.

undesirable for the shareholders, and the firm value gets hurt compared to the benchmark case.

Finally, we study the case where the proxy advisor has conflicts of interest.[2] Intuitively, the conflicts of interest will distort issued recommendations to an inferior level. This intuitive result is recovered by our model as well. We find the voting recommendation issued by the proxy advisor is biased and not desired by the shareholders. Because of so, the firm value is impaired relative to that in the benchmark case. Our findings in this part are consistent with the negative consequences of ISS's conflicted business model documented in Li (2017).

In sum, we point out that information bias and information desirability are related, but fundamentally different concepts. The conflict of interest on the proxy advisor side is not the only source of information bias in the proxy advisory market. A proxy advisor free from conflicts of interest may also issue biased voting recommendations, which may or may not be desired by shareholders.

Related Literature Our study is related to two streams of literature. First of all, our paper is part of the growing literature on information provision in financial markets. In their seminal papers, Admati and Pfleiderer (1986, 1988) analyze the sale of information to investors who in turn trade on the information in financial markets. These models share an important characteristic. That is, an agent becoming more informed imposes a negative externality upon others due to the competition among agents (i.e., strategic substitutability in information acquisition as in Grossman and Stiglitz (1980)). Thus, in equilibrium, the information seller has an incentive to dilute the sold information. This insight has been confirmed in more general settings. Bergemann and Morris (2017) offer a unified perspective of information design problem and summarize that it is often optimal for the information designer to selectively obfuscate information.

We focus on the sale of information that is used for shareholder voting. Unlike the competition in speculative market, the nature of the shareholder voting is cooperative. Each agent becoming more informed helps to make a collectively more informative voting decision. Hence, the information seller in the voting context will not dilute the value of information as buyers do

[2] For instance, ISS also provides consulting services to the firm.

not value dilution. We thus contribute to this literature by showing it is possible that the for-profit information seller provides the most accurate information in financial markets. In other words, the distinct pursuits of the proxy advisor and shareholders can be aligned in the proxy advisory market.

The closest paper to ours is Malenko and Malenko (2017), which also study how proxy advisors affect corporate decision-making. However, they focus differently on the shareholders' trade-off between information purchase from the proxy advisor and private information acquisition. Their key insight is that the presence of the proxy advisor increases firm value only if the precision of its recommendation is sufficiently high. We instead focus on the information quality in the proxy advisory market and explore the potential sources of information bias, which to our knowledge have not been studied in the literature. Our work thus complements Malenko and Malenko (2017) and further adds to the understanding of how the proxy advisor influences corporate value.

A second stream of related research is on corporate voting and proxy advisors. Theoretical works include Maug (1999), Maug and Yilmaz (2002), Bond and Eraslan (2010), Brav and Mathews (2011), Levit and Malenko (2011), Eso et al. (2015), Bar-Isaac and Shapiro (2017), Cvijanovic et al. (2017), among others. We are different in that we focus on the information intermediary, which is a salient institutional feature of the corporate voting.

Recently, a growing empirical literature pays attention to proxy advisors. For example, researchers and regulators are concerned about the potential conflicts of interest inherent in some proxy advisors' business (Li, 2017), the influence of proxy advisors wield over their clients (Bethel and Gillan, 2002; Cai et al., 2009; Iliev and Lowry, 2014; Malenko and Shen, 2016), and the informational role of proxy advisors (Alexander et al., 2010; Ertimur et al., 2013; Aggarwal et al., 2015). Our investigation on information bias in the proxy advisory market is motivated by the institutional features of the corporate voting context and empirical evidence. And our framework helps understand the empirical findings and speaks to the policy concerns, which we will discuss in details in Section 5.

The rest of the paper is organized as follows. Section 2 describes the model setup. Section 3 characterizes the equilibrium in the model. Section 4 explores information bias in the proxy advisory market. Section 5 discusses the policy implications and Section 6 concludes. All proofs are relegated to the Appendix.

2 A Model of the Proxy Advisor and Information Bias

Suppose the state of the world, $\theta \in \{0, 1\}$, is unknown to all players ex ante. State 0 stands for bad economy while State 1 for good economy. At the beginning of the game, the state is randomly drawn from a Bernoulli distribution with parameter $\mu_0 \in (0, 1)$. That is, the probability of the economy being a good one is μ_0: $\Pr(\theta = 1) = \mu_0$.

Shareholders and the Proposal A firm is owned by N shareholders, where N is odd and greater or equal to three. We interpret the shareholders as institutional investors, which often have significant holdings in the firm. For simplicity, we assume each shareholder owns exactly one share.[3] The value of each share at the beginning of the game is normalized to zero. In addition, shareholders have no private information regarding the state other than their prior.[4]

There is a new proposal to be voted in a shareholder meeting. Shareholder i's voting decision is denoted by v_i. In particular, $v_i = 1$ if shareholder i votes for the proposal while $v_i = 0$ if she votes against it. The voting outcome is determined by the simple majority rule: the proposal is accepted if and only if $\sum_{i=1}^{N} v_i \geq (N + 1)/2$. In the remainder of the paper, we use $d \in \{0, 1\}$ to denote if the proposal is accepted ($d = 1$) or rejected ($d = 0$).

The value of the proposal is state dependent. Specifically, if the economy is good and the proposal is accepted ($\theta = 1$ and $d = 1$), the proposal generates profits and thus, the value of

[3] Bar-Isaac and Shapiro (2017) study blockholder voting by allowing for asymmetry among shareholders. We abstract from it because we focus on the information quality in the proxy advisory market.

[4] As Krouse et al. (2016) put it, "Vanguard has 15 people overseeing work on about 13,000 companies based around the world. BlackRock has about two dozen people who work on governance issues at some 14,000 companies held in its index funds and exchange-traded funds .... Boston-based State Street Global Advisors ... has fewer than 10 employees devoted to issues at around 9,000 companies ...." Therefore, it is reasonable to assume shareholders are not privately informed so that we can focus on the quality of the proxy advisor's information.

each share will increase to 1.[5] If the economy is bad but the proposal is accepted ($\theta = 0$ and $d = 1$), then the firm will lose money and its value per share will decrease to -1. If the proposal is rejected, then the firm value does not change.

A Proxy Advisor and the Voting Recommendation A monopolistic proxy advisor (PA) sells its binary voting recommendation, denoted by $r \in \{0, 1\}$, to shareholders. If $r = 1$, shareholders are suggested to vote in favor of the proposal, while if $r = 0$ they are suggested to vote against it. Note that in principle shareholders do not have to follow this voting advice.

The PA's voting recommendation is based on its own research, which we envision as a private signal of the state. For simplicity, we assume the PA receives a private signal $p \in [0, 1]$ that has the following conditional distribution:

$$h(p|\theta = 1) = 2p,$$
$$h(p|\theta = 0) = 2(1 - p),$$

where $h(p|\cdot)$ is the private signal p's probability density function conditional the state. Substantively, we interpret p as the PA's subjective assessment on the likelihood that the state is a good state based on its research.

After observing p, the PA must decide whether to send $r = 1$ or $r = 0$.[6] Following the Bayesian persuasion literature (e.g., Kamenica and Gentzkow, 2011), we assume the PA can *commit* to a particular information revelation scheme. As the voting recommendation is binary, we assume the PA uses a simple cut-off point scheme in sending the voting recommendation and we denote the cut-off point by $\varrho \in \{0, 1\}$. That is, $r = 1$ if and only if $p \geq \varrho$. Conditional on the

[5]Proposals relevant to stock value include director elections, appointment of outside auditors, issuance of new shares, creation of equity-based compensation plans, amendments to the corporate charter or bylaws, major mergers and acquisitions, ballot questions submitted in the form of advisory shareholder proposals, and so on (Yermack, 2010).

[6]In reality, proxy advisors offer both analysis (i.e., the original private signal p) and recommendations (i.e., the binary signal r) to shareholders. Following Malenko and Malenko (2017), we model information good here in the form of binary recommendations. In addition, the PA must take a stand on the proposal in practice.

state, the probabilities of the PA sending $r = 1$ and $r = 0$ are respectively

$$\pi_1 \equiv \Pr(r = 1|\theta = 1) = \int_{\varrho}^{1} h(p|\theta = 1)\,\mathrm{d}p = 1 - \varrho^2, \tag{1}$$

$$\pi_0 \equiv \Pr(r = 0|\theta = 0) = \int_{0}^{\varrho} h(p|\theta = 0)\,\mathrm{d}p = 2\varrho - \varrho^2. \tag{2}$$

The PA also determines the price of its voting recommendation, which is denoted by $f \geq 0$. Therefore, the PA's profit from proxy voting services is $f \cdot Q$, where Q is the number of shareholders who have purchased the voting recommendation.

As this paper studies information bias and its welfare implications in the proxy advisory market, we provide a formal definition of an unbiased voting recommendation.

Definition 1 (Unbiased Recommendation). *A voting recommendation is unbiased if and only if* $r = \mathbb{I}(\Pr(\theta = 1|p) \geq \Pr(\theta = 0|p))$, *where* $\mathbb{I}(\cdot)$ *is the indicator function.*

In plain words, a voting recommendation is unbiased if and only if the recommendation is consistent with the more likely state given the evidence. According to this definition, it is easy to verify that the PA's recommendation is unbiased if and only if $\varrho = 1 - \mu_0$. We thus define

$$\varrho^{\mathrm{U}} \equiv 1 - \mu_0 \tag{3}$$

as the unbiased cut-off point.

Note that the criterion of an unbiased recommendation does not rely on shareholders' preferences. We thus define the desirable recommendation for a particular shareholder as below.[7]

Definition 2 (Desirable Recommendation). *For a given shareholder i, a recommendation is desirable if it gives the shareholder the highest utility.*

The basic idea behind the desirable recommendation is as follows. Suppose that shareholders

[7] In principle, a voting recommendation can be desirable for some shareholders but not for others. Since we assume homogeneous shareholders, however, if a voting recommendation is desirable for shareholder i, then it is desirable for all other shareholders.

directly observe p and sends a binary voting recommendation to herself. The voting advice is the one that maximizes her utility and thus a desirable recommendation.

Potential Sources of Information Bias Motivated by Gentzkow et al. (2015), we consider the potential drivers of information bias from the demand side and the supply side. The demand-driven bias may arise from shareholders' preferences or beliefs. Specifically, in our setting shareholders may not necessarily aim to maximize firm value and their utility function is given by:

$$u_i(d,\theta) = \begin{cases} 1 & \text{if } \theta = 1, d = 1, \\ -\gamma & \text{if } \theta = 0, d = 1, \\ 0 & \text{if } d = 0. \end{cases}$$

Such preferences could arise from many sources. For example, mutual funds managers may be more sensitive to losses than gains due to career concerns (Haigh and List, 2005; Bodnaruk and Simonov, 2016), and such loss averse preference is captured by $\gamma > 1$. Alternatively, mutual funds tend to support management proposal if there are business ties with the firm (Cvijanović et al., 2016). This can be characterized by $\gamma < 1$. Only when $\gamma = 1$ do shareholders aim to maximize firm value.

We further assume shareholders may have incorrect prior beliefs: shareholders' subjective belief that the probability of the economy being $\theta = 1$ is μ, and in general $\mu \neq \mu_0$. That is, shareholders believe a good state is drawn with probability μ. We call that shareholders are *over-optimistic* if $\mu > \mu_0$, and conversely *over-pessimistic* if $\mu < \mu_0$.

To sum, the case where shareholders aim to maximize firm value and have the correct beliefs is simply nested by letting $\gamma = 1$ and $\mu = \mu_0$.

For the supply-driven bias, we consider the situation where the PA may have conflicts of interest. In particular, the PA obtains some extra payoff Φ only if it issues a positive recommendation to shareholders (i.e., $r = 1$). For example, the PA may provide consulting services to the firm, and the firm manager uses the PA's consulting services only if the PA recommends for approval on the management-initiated proposals.[8] When the PA has no conflicts of interest, we call the PA *unconflicted*.

Timing The timeline of the model is illustrated in Figure 1. There are four dates: $t = \{0, 1, 2, 3\}$. At $t = 0$, Nature draws the state of the economy based on the prior distribution μ_0. And the PA sets the voting recommendation price f and commits to the cut-off point ρ of sending $r = 1$. At $t = 1$, each shareholder decides whether to subscribe to the PA's recommendation or stay with the prior information. At $t = 2$, shareholders who have purchased the PA's voting advice at the previous stage receive their voting recommendation. All shareholders vote simultaneously based on their information. At $t = 3$, the proposal is implemented or not, depending on whether the majority of shareholders voted for it, and the payoffs are realized. We call $t = 0$ the *profit-maximization* stage, $t = 1$ the *information-purchase* stage, and $t = 2$ the *voting* stage.

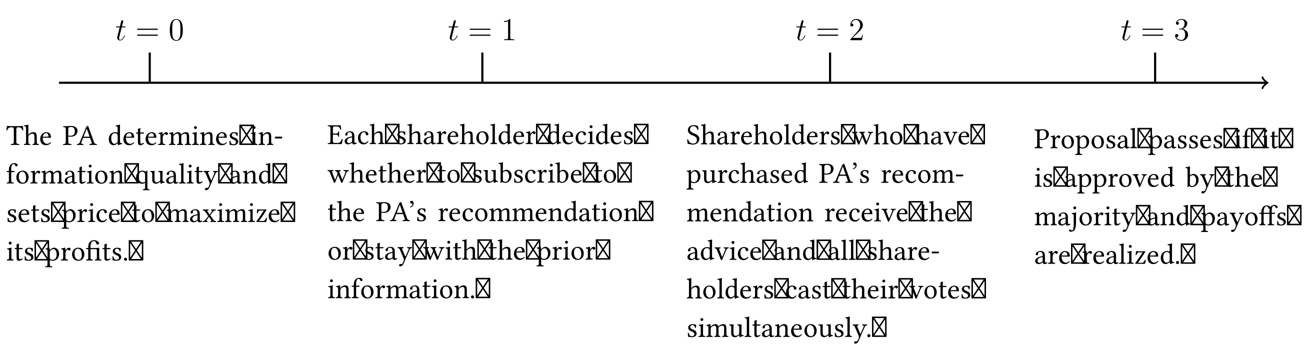

Figure 1: Timeline of events

Shareholders' and the PA's Strategies Shareholders' objective is to maximize their payoff by making the right voting decision. To achieve so, each shareholder simultaneously decides whether or nor to subscribe to the PA's recommendation after observing the price of the recommendation, f, and the cut-off point scheme, ρ. After the PA sending its voting recommendation, each shareholder updates her belief using Bayes rule and then votes accordingly. As shareholders

[8] The Government Accountability Office (GAO) report notes the most commonly cited conflicts of interest for proxy advisors take this form. Also, these potential conflicts are not limited to the United States (Organisation for Economic Co-operation and Development., 2004).

are symmetric, we assume they use a symmetric strategy throughout the paper. In particular, we let $q : [0, \infty) \times [0, 1] \to [0, 1]$, a function that maps the price of the PA's recommendation and its cut-off point scheme of sending $r = 1$ to $[0, 1]$, be the probability with which each shareholder purchases the PA's recommendation. We also denote $v : \{\emptyset, 0, 1\} \to [0, 1]$, a function that maps each shareholder's information set to the probability of voting in favor of the proposal, as each shareholder's voting strategy. Note in the domain of $v(\cdot)$, $\emptyset$ means the shareholder does not observe the PA's voting recommendation (i.e., the shareholder has not subscribed to the PA's recommendation), 0 means the shareholder observes a negative recommendation (i.e., $r = 0$), and 1 implies a positive recommendation (i.e., $r = 1$).

The PA's strategy can be summarized by a tuple (ϱ, f), where $\varrho \in [0, 1]$ and $f \in [0, \infty)$. Simply put, the PA chooses the cut-off point of sending $r = 1$ and the price of its recommendation to maximize the profits from proxy services and potential consulting services. As each shareholder purchases the PA's voting recommendation with probability q, the number of shareholders who have purchased the voting recommendation $Q = qN$. Therefore, the PA's objective function is $\Pi = qN \cdot f + \Pr(r = 1)\Phi$, where $\Phi \geq 0$ is the conflicted PA's extra payoff by making a positive recommendation. As N is a fixed number, we divide N on both sides and obtain the "per capita" form of PA's objective function

$$\pi = q\, f + \Pr(r = 1)\, \varphi, \tag{4}$$

where $\varphi = \Phi/N$. In the rest of the paper, we work with equation (4).

3 The Equilibrium

We focus on the symmetric perfect Bayesian equilibrium. That is, all shareholders follow the same information-purchase strategy, use the same voting strategy, and update their beliefs with Bayes rule whenever possible.[9] We define the equilibrium as follows.

Definition 3 (Equilibrium). *An equilibrium is characterized by the price of the PA's recommen-*

[9] As will be clear later, Bayes rule always applies along- and off-equilibrium path.

dation f, *the cut-off point* ϱ *of sending* $r = 1$, *a function* $v : \{\emptyset, 0, 1\} \to [0, 1]$ *that denotes the probability of shareholder* i *voting for the proposal given her information set, and a function* $q : [0, \infty) \times [0, 1] \to [0, 1]$ *that denotes the probability with which each shareholder purchases the PA's recommendation, such that,*

1. *at the voting stage, the voting strategy* $v(\cdot)$ *maximizes each shareholder's expected utility conditional on her information set and the voting strategies of all other shareholders;*

2. *at the information-purchase stage, the information-purchase strategy* $q(\cdot, \cdot)$ *maximizes each shareholder's expected utility; and*

3. *at the profit-maximization stage, the price of the voting recommendation* f *and the cut-off point* ϱ *of sending* $r = 1$ *maximize the PA's profits.*

We next solve the game by backward induction. First, we find the equilibrium at the voting stage. Next, we solve for the equilibrium strategy of shareholders' information-purchase decisions. Finally, we decide the PA's optimal selling strategy that maximizes its profits.

Voting Stage

A subgame equilibrium at the voting stage is called a *voting-stage equilibrium*. Without any restriction, some unrealistic voting-stage equilibria may occur in some off-equilibrium cases.[10] To rule out them, we assume each shareholder casts her vote sincerely when she cannot unilaterally change the voting outcome.

Assumption 1 (Sincere Voting). *When a shareholder can never change the voting outcome, the shareholder casts her vote in consistence with her belief.*

In plain words, for a given shareholder, if the voting outcome is already decided by other shareholders in all circumstances, then she votes in favor of the proposal if she thinks State 1 is more likely, and vice versa.

[10] For example, if all shareholders vote in favor of or against the proposal with probability one, then no shareholder can change the voting outcome unilaterally. Therefore, all shareholders vote against the proposal even if they all think they should vote for the proposal can still constitute a voting-stage equilibrium.

Next, to create a unique voting-stage equilibrium in the case of indifference, we assume the following.

Assumption 2 (Deferential Voting). *When indifferent, each shareholder who has purchased the voting recommendation follows the recommendation. When no shareholder has received any voting recommendation (i.e.,* $q = 0$*), they all vote in favor of the proposal when indifferent.*

Note that Assumption 2 does not specify the voting strategy for indifferent uninformed shareholders. Their voting strategy will be pinned down by equilibrium conditions. More importantly, Assumptions 1 and 2 never apply in equilibrium. In equilibrium, each shareholder always has a positive probability to change the voting result, a positive fraction of shareholders will purchase the voting recommendation, and shareholders who have purchased the voting recommendation are not indifferent between voting "for" and "against." Hence, Assumptions 1 and 2 are not introduced to select equilibrium. We adopt these two assumptions only to present the voting-stage equilibrium in a concise fashion.

Under Assumptions 1 and 2, we can derive the subgame equilibrium at the voting stage as follows.

Proposition 1 (Voting-Stage Equilibrium). *For any fixed* (q, ϱ)*, there is a unique voting-stage equilibrium. In particular,*

1. *when* $q = 0$*, all shareholders vote in favor of the proposal if and only if* $\mu \geq \frac{\gamma}{1+\gamma}$ *for all* ϱ*;*

2. *when* $q \in (0, 1]$*, all shareholders vote against the proposal if* $\varrho < \frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}$ *and for the proposal if* $\varrho > \frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$*;*

3. *when* $q \in (0, 1]$ *and* $\varrho \in \left[\frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}, \frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}\right]$*, a shareholder who has purchased the voting recommendation votes in consistence with the advice; and*

4. *when* $q \in (0, 1)$ *and* $\varrho \in \left[\frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}, \frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}\right]$*, an uninformed shareholder votes for the*

proposal with probability τ, *where*

$$\tau = \frac{(1-M-2q)+\sqrt{(1-M)^2+4q^2M}}{2(1-q)(1-M)} \in (0,1), \tag{5}$$

and $M = \sqrt[\frac{N-1}{2}]{\frac{\varrho}{1-\varrho}\frac{\gamma(1-\mu)(2-\varrho)-\mu\varrho}{\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)}}$.

Two points are worth noting on the voting-stage equilibrium. First, as shareholders believe a good state is drawn with probability μ, they will use μ to evaluate their voting strategies, and the true prior μ_0 does not enter the equilibrium voting strategy. Second, only the case where $q \in (0,1)$ and $\varrho \in \left[\frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}, \frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}\right]$ is on the equilibrium path. Otherwise, the voting advice does not change the voting behavior and thus has no value. On the equilibrium path, shareholders who have purchased the voting advice follows it and uninformed shareholders vote for the proposal with probability τ.

The intuition of the voting-stage equilibrium is as follows. Firstly, when $q = 0$, no shareholder has any information. Thus, Assumptions 1 and 2 kick in. Shareholders vote in favor of the proposal only if their prior belief about the state being good is high enough (i.e., $\mu \geq \frac{\gamma}{1+\gamma}$).

Secondly, when the PA's threshold is set too low (i.e., $\varrho < \frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}$), the PA sends positive recommendations too often and its voting advice contains little information. Such little information cannot beat shareholders' prior. Thus, if shareholders' prior belief about the state being good is low enough (i.e., $\mu < \frac{\gamma}{1+\gamma}$), all shareholders will follow their prior and vote against the proposal (note $\varrho < \frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}$ only if $\mu < \frac{\gamma}{1+\gamma}$). The intuition when the PA's threshold is set too high (i.e., $\varrho > \frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$) is analogous.

Thirdly, when the probability of purchasing the voting recommendation is positive ($q > 0$) and the threshold ϱ is neither too low nor too high, a shareholder who has purchased the PA's recommendation will follow it with probability one. The reason is simple. Suppose the shareholder always votes in the same way regardless of the voting recommendation, then she is better off by not paying the voting advice in the first place. On the other hand, if the shareholder uses a mixed voting strategy, then she must be indifferent between voting according to her advice or voting against it. So the shareholder has a profitable deviation by using a pure voting strategy and not paying for the voting advice in the first place. Taken together, one can conclude that the shareholder follows her purchased voting recommendation in equilibrium.

Fourthly, uninformed shareholders must use a mixed voting strategy in equilibrium. Otherwise, they would suffer the "Swing Voter's Curse" (Feddersen and Pesendorfer, 1996). To explain it, suppose all uninformed shareholders vote in favor of the proposal for sure, then given that informed shareholders follow their received voting recommendation, a randomly chosen shareholder will vote in favor of the proposal with probability one if the PA's recommendation is $r = 1$, and with probability $1 - q$ if the PA's recommendation is $r = 0$. Hence, an uninformed shareholder can possibly influence the voting result if and only if $r = 0$. However, in such a situation, the uninformed shareholder does not want to vote in favor of the proposal since she can infer the private information of the informed shareholders being $r = 0$. Therefore, in equilibrium, it is not possible for the uninformed type to always vote for the proposal. The same argument applies to the case when uninformed shareholders always vote against the proposal. Taken together, the uninformed shareholder must use a mixed voting strategy.

More importantly, uninformed shareholders can infer some information even though they do not obtain the voting recommendation directly from the PA. To understand why this is the case, consider an uninformed shareholder, indexed by i, who is considering with what probabilities to vote for and against the proposal. Conditional on other shareholders' voting strategies, her vote matters only if she is a pivotal shareholder, i.e., the number of "for" votes equals to the number of "against" votes. Since each shareholder has a positive probability of being pivotal in equilibrium, shareholder i casts her vote as if she is pivotal.[11] The event of being pivotal is informative. For example, suppose other uninformed shareholders are more likely to vote for than against the proposal, then shareholder i finds she is more likely to be pivotal when PA's voting recommendation is negative ($r = 0$). In other words, shareholder i can infer the voting recommendation is more likely to be $r = 0$ than $r = 1$ from the event of being pivotal. Thus,

[11] If she is not pivotal, her vote cannot change the result. Thus, she only considers the case when she can change the result.

she should lower her probability of voting "for" to reflect this information. Each uninformed shareholder does this calculation and adjusts her voting strategy accordingly. In equilibrium, uninformed shareholders' voting strategy is adjusted to the point where being pivotal implies the PA is equally likely to give either voting recommendation.

Information-Purchase Stage

With the equilibrium at the voting stage, we can compute the value of information to a shareholder for a given (q, ϱ), and further solve for the subgame equilibrium at the information-purchase stage.

Consider shareholder i who is contemplating whether to purchase the PA's voting recommendation given that she expects other shareholders to purchase the advice with probability q. By Proposition 1, she will follow the voting recommendation if she purchases the recommendation and will vote for the proposal with probability τ if she does not purchase it. Thus, the value of the voting recommendation consists of two parts: the benefit of improving the probability of a "for" vote from τ to one when $r = 1$ and the value of improving the probability of a "against" vote from $1 - \tau$ to one when $r = 0$.

For the first part, suppose shareholder i purchases the PA's voting advice and receives $r = 1$, then i's utility from voting in favor of the proposal for sure versus voting "for" with probability τ is[12]

$$V(q, \varrho|r = 1) = (1 - \tau)[\Pr(\theta = 1|PIV_i, r = 1) - \Pr(\theta = 0|PIV_i, r = 1)] \Pr(PIV_i|r = 1), \quad (6)$$

where PIV_i means shareholder i is pivotal. Similarly, if shareholder i receives $r = 0$, then i's utility of voting "against" for sure versus voting "against" with probability $1 - \tau$ is

$$V(q, \varrho|r = 0) = -\tau[\Pr(\theta = 1|PIV_i, r = 0) - \Pr(\theta = 0|PIV_i, r = 0)] \Pr(PIV_i|r = 0). \quad (7)$$

[12] As in the voting stage, shareholders use their prior μ to assess the value of the voting advice. Hence, all probability calculations in this section should base on μ, rather than μ_0.

Therefore, the value of the PA's recommendation is

$$V(q,\varrho) = \Pr(r=1)V(q,\varrho|r=1) + \Pr(r=0)V(q,\varrho|r=0). \quad (8)$$

When deciding whether to purchase the voting advice, shareholder i compares the expected value from the voting recommendation, $V(q,\varrho)$, with its price f. She purchases the advice if and only if $V(q,\varrho) \geq f$. In equilibrium, shareholders do not purchase the voting recommendation with probability one no matter how cheap the voting recommendation is. The reason is simple. If otherwise, then all shareholders would vote in the same way and a single shareholder would never be pivotal. Under this circumstance, shareholder i can stop paying the voting advice and free ride without influencing the voting result, which is a profitable deviation.

When the price of the voting advice is too prohibitive, no shareholders buy the voting advice. When the price of the voting advice is sufficiently low, shareholders use a mixed information-purchase strategy (i.e., $q \in (0,1)$). For shareholders to use a mixed strategy, the value of recommendation must equal to its price: $V(q,\varrho) = f$. From this condition we can solve for q, which is a function of f and ϱ. The next proposition summarizes the equilibrium at this stage.

Proposition 2 (Information-Purchase Equilibrium). *Given f and ϱ, each shareholder acquires the PA's recommendation with probability q^*, which is given as follows:*

1. *If $f > \tilde{f}$, where $\tilde{f}$ is given by equation (B.23) in the Appendix, $q^* = 0$.*

2. *If $f = \tilde{f}$, $q^* = \tilde{q}$, where $\tilde{q}$ is given by equation (B.22) in the Appendix.*

3. *If $f < \tilde{f}$, there co-exist two equilibria, q^{I} and q^{II}:*

$$\begin{cases} q^{\mathrm{I}} = \sqrt{\frac{M-2G(M+1)+\sqrt{M(1-4G)(M-4G)}}{2M}}, \\ q^{\mathrm{II}} = \sqrt{\frac{M-2G(M+1)-\sqrt{M(1-4G)(M-4G)}}{2M}}, \end{cases} \quad (9)$$

where M is given in Proposition 1 and G is given by equation (B.19) in the Appendix.

The reason we may have two equilibria at this stage is as follows. When considering subscribing to the PA's voting advice, each shareholder faces two types of considerations. On the one hand, voting advice helps the shareholder to vote in a more informed way. On the other hand, each shareholder has an incentive to free ride. When q is small, the incentive of free riding is small while the marginal benefit of voting advice is large. The converse is true when q becomes large. Thus, $V(q, \varrho)$ is increasing in q when q is small and decreasing in q when q is large. And when $f < \tilde{f}$, we can find two q's that satisfy $V(q, \varrho) = f$ in general.

Perceived Benefit and the Desirable Recommendation

In order to measure the quality of decision-making with the proxy advisor, now we compute the expected value of the proposal perceived by shareholders (or simply perceived benefit), denoted by ψ^{P}.[13]

As we have two states and the PA can send two types of voting recommendations, we need to consider four possible cases when calculating the perceived benefit: (i) $\theta = 1$ and $r = 1$; (ii) $\theta = 1$ and $r = 0$; (iii) $\theta = 0$ and $r = 1$; and (iv) $\theta = 0$ and $r = 0$. Considering these four cases, the perceived benefit can be written as

$$\psi^{\mathrm{P}} = (\mu\pi_1 - \gamma(1-\mu)(1-\pi_0)) \Pr(d = 1|r = 1) + (\mu(1-\pi_1) - \gamma(1-\mu)\pi_0) \Pr(d = 1|r = 0). \quad (10)$$

Note that equation (10) depends on μ instead of μ_0. This is because shareholders use their subjective belief to evaluate the expected value of the proposal. We are (and shareholders should be) also interested in the perceived benefit calculated with the true prior μ_0. This quantity is denoted by ψ^0 and it means the *true* benefit of the proposal for shareholders. Specifically, ψ^0 is obtained by replacing μ with μ_0 in equation (10).

For generic q's and ϱ's, the next lemma shows the relationship between the perceived benefit ψ^{P} and (q, ϱ).

[13] As shareholders do not necessarily aim to maximize the firm value, the perceived benefit discussed in this subsection is in general not equal to the firm value. See Appendix A for further discussion.

Lemma 1 (Perceived Benefit). *Assume the voting-stage equilibrium and the information-purchase equilibrium. The proposal value perceived by shareholders $\psi^{\mathrm{P}}(q, \varrho)$ is strictly increasing with respect to q for all ϱ, and single-peaked around $\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ with respect to ϱ for all q.*

Lemma 1 verifies that $\varrho = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ maximizes the perceived benefit (and thus the perceived shareholder welfare) for any given q. Therefore, if ψ^{P} and ψ^{0} coincide, which occurs only if shareholders' prior belief is correct (i.e., $\mu = \mu_0$), $\varrho = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ also maximizes the the true expected value of the proposal for shareholders. However, in general $\varrho = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ is not necessarily desirable for shareholders, which is summarized in the next proposition.

Proposition 3 (Desirable Recommendations). *Let ϱ^{D} be a desirable voting recommendation scheme, then*

1. $\varrho^{\mathrm{D}} = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ *if* $\mu = \mu_0$;
2. $\varrho^{\mathrm{D}} > \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ *if* $\mu > \mu_0$; *and*
3. $\varrho^{\mathrm{D}} < \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ *if* $\mu < \mu_0$.

Now we are ready to compare an unbiased recommendation and a desirable recommendation. The following corollary summarizes the result.

Corollary 1. *A desirable recommendation is unbiased (i.e., $\varrho^{\mathrm{D}} = \varrho^{\mathrm{U}}$) if shareholders have the correct prior belief (i.e., $\mu = \mu_0$) and aim to maximize firm value (i.e., $\gamma = 1$).*

Therefore, information bias and information desirability are related but different concepts. For example, unbiased voting advice is not desirable for a loss averse shareholder. With loss aversion, shareholders are more conservative to vote in favor of the proposal since the proposal is less appealing in bad states. For this reason, shareholders are now willing to see a stricter standard of sending a positive recommendation.

Profit-Maximization Stage

To have a well-defined problem of pricing of information, we need to take a stand on which equilibrium is selected in the event of multiple equilibria at the information-purchase stage. The two equilibria, q^{I} and q^{II}, can be ranked with respect to (perceived) shareholder welfare (perceived benefit minus information price). Following Malenko and Malenko (2017), we assume that when multiple equilibria exist at the information-purchase stage, shareholders coordinate on the equilibrium in which perceived shareholder welfare is maximized.

Assumption 3 (Equilibrium Selection). *When $f < \tilde{f}$, shareholders coordinate on the equilibrium that maximizes the total shareholder welfare.*

In our setting, the information fees are the same in the two equilibria. Thus, the equilibrium with the higher perceived benefit is selected. As shown in the previous section, the perceived benefit is monotonically increasing in q and hence the equilibrium with a higher q is always associated with a higher expected perceived benefit. Or simply, q^{I} is selected.

We now study the PA's optimal selling strategy. Taking into account how the choice of f and ϱ affects the shareholders' information-purchase and voting decisions, the PA maximizes its profits. As $q(\cdot,\cdot)$ is given by q^{I}, the inverse demand function for the PA's voting recommendation, $f(q,\varrho)$, is well-defined. Therefore, for a given ϱ, the PA can decide information demand q directly by announcing the information price f. Thus, the PA's problem as in equation (4) can be put into

$$\max_{q,\varrho}\; qf(q,\varrho) + \Pr(r=1)\,\varphi. \tag{11}$$

The following proposition summarizes the solution to this maximization problem.

Proposition 4 (Optimal Selling Strategy). *In equilibrium, there exists a unique (ϱ^*, q^*) that maximizes the PA's profits, such that:*

$$q^* = \frac{\sqrt{4M^* - N\left(1-M^*\right)^2 + \left(1+M^*\right)\sqrt{4M^* + N^2\left(1-M^*\right)^2}}}{2\sqrt{2M^*N}},$$

where $M^* = \sqrt[N-1]{\gamma\frac{1-\mu}{\mu}}$, $\varrho^* = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ *if* $\varphi = 0$, *and* $\varrho^* < \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ *if* $\varphi > 0$. *The corresponding equilibrium information price is* $f^* = V(q^*, \varrho^*)$, *where* $V(\cdot,\cdot)$ *is defined in equation* (8).

One distinguishing feature of the equilibrium is that the optimal information demand q^* is independent of ϱ, and therefore does not depend on information quality. That is, when the monopolistic PA issues a poor voting recommendation, it will adjust its price in equilibrium instead of accepting a drop in sales.

4 Information Bias in the Proxy Advisory Market

In this section, we study information bias in the proxy advisory market. To begin with, we consider an ideal setting where in equilibrium the PA's recommendation is unbiased and desirable. This serves as a benchmark for us to explore the sources of information bias in the market and the welfare implications.

Based on Definitions 1, 2 and Proposition 4, when the PA has no conflicts of interest (i.e., $\varphi = 0$), and shareholders aim to maximize firm value (i.e., $\gamma = 1$) and have correct beliefs (i.e., $\mu_0 = \mu$), the equilibrium voting recommendation is unbiased and desirable. The following corollary summarizes the result.

Corollary 2 (Benchmark). *Assume* $\varphi = 0, \gamma = 1$ *and* $\mu_0 = \mu$. *In equilibrium the PA sends unbiased and desirable voting recommendations:* $\varrho^* = \varrho^{\mathrm{U}} = \varrho^{\mathrm{D}}$. *The firm value is maximized.*

In this ideal setting, the PA sends unbiased recommendation to shareholders to maximize its profits. That is, the PA recommends to approve the proposal only if it thinks the state is more likely to be a good one. Moreover, the voting recommendation is desirable. Suppose instead of selling the binary voting recommendation r, the unconflicted PA sells its private signal p directly, then ϱ^* is also the cut-off point that each informed shareholder votes in favor of the proposal after observing p. In other words, if a shareholder observes p and sends a binary voting recommendation to herself, she will choose ϱ^* as her cut-off point. Therefore, shareholders may lose nothing by delegating their voting choices to a for-profit PA. In the real world, shareholders can potentially benefit from such arrangements because of the efficiency gain from the division-of-labor. It is worth noting that the expected firm value is maximized in the ideal setting, and we call it the first-best case.

We next discuss the information bias by relaxing the assumptions in Corollary 2. We first analyze the economy with an unconflicted PA, and later turn to the economy where the PA has conflicts of interest. In the former situation, the information bias can only arise from the "demand-side" in the proxy advisory market, while in the latter case, the information bias comes from the "supply-side." Such differentiation informs us of the potential sources of information bias in the proxy advisory market and shows that the bias from different sources may have different welfare implications.

Unconflicted PA

For an unconflicted PA, its profits entirely come from proxy services. We first consider the case where shareholders do not maximize firm value (i.e., $\gamma \neq 1$). Based on Definition 1, in equilibrium the PA sends biased recommendation. However, these biased voting recommendations are desirable for shareholders. This is because the PA's profits depend on shareholders' perceived information value, which is determined by their preferences. Regardless of its own judgment, the PA sends voting recommendations that are viewed the most valuable by shareholders.

Corollary 3 (Shareholder Preferences). *Assume $\varphi = 0$ and $\mu_0 = \mu$. If shareholders' objective is not to maximize firm value (i.e., $\gamma \neq 1$), the PA's voting recommendation is not unbiased but desirable: $\varrho^* \neq \varrho^{\mathrm{U}}$, and $\varrho^* = \varrho^{\mathrm{D}}$. The firm value is impaired.*

In reality, Corollary 3 implies a significant limit of the proxy advisory market: the PA only caters for the interests of its actual payers (i.e., institutional investors) and does not care about the preferences of their actual beneficiaries. For example, when a mutual fund manager is loss averse due to career concerns or reputation costs (Chevalier and Ellison, 1999; Brown et al., 2001) but the fund's clients are loss neutral, the PA will send a voting recommendation that is desirable for the fund manager. Hence, the fund purchasing voting recommendations from the PA cannot be viewed as a way to demonstrate that it votes in the best interest of its clients and its fiduciary duties are fulfilled.

We next consider the case where shareholders have incorrect beliefs (i.e., $\mu_0 \neq \mu$). As stated previously, ϱ^* is set to maximize the perceived benefit of the voting recommendation. This perceived benefit, however, is determined by shareholders' prior belief about the state while the true prior is irrelevant. The profit-maximization consideration forces the PA to set its cut-off point to *confirm* shareholders' belief. In other words, even if the PA knows the true prior and is highly confident its knowledge is correct, the PA has no incentives to correct shareholders' wrong belief. The next corollary summarizes the finding.

Corollary 4 (Over-Optimistic and Over-Pessimistic Shareholders). *Assume $\wp = 0$ and $\gamma = 1$. When shareholders are over-optimistic (i.e., $\mu > \mu_0$), $\varrho^* < \varrho^{\mathrm{D}}$. When shareholders are over-pessimistic (i.e., $\mu < \mu_0$), $\varrho^* > \varrho^{\mathrm{D}}$. In both cases, the voting recommendation is biased (i.e., $\varrho \neq \varrho^{\mathrm{U}}$) and the firm value is impaired.*

In plain words, when shareholders are misinformed, the PA will not help correct shareholders' inaccurate beliefs and guide to informed decision-making. Even worse, the cut-off point chosen by the PA only exacerbates the problem: when shareholders are over-optimistic, the PA is biased toward sending a positive recommendation; when shareholders are over-pessimistic, the PA sends a negative recommendation too often. In practice, this feature can either be a boon or a bane. On the one hand, a misinformed or a slanted PA (if such misinformation or slant can be captured by prior information) will not hurt shareholders. On the other, when shareholders are misinformed, the PA has no incentives to guide them.

To sum, on the flip side of Corollary 3 and 4, a potential driver of information bias is shareholders themselves. When shareholders do not maximize firm value or do not know the true prior, the PA's voting recommendations are biased. Regarding the welfare implications, the biased recommendations are desirable if the bias arises from shareholders' preferences, but not so if it comes from shareholders' incorrect beliefs.

Conflicted PA

When $\varphi > 0$, the PA has conflicts of interest. To focus on this case, we shut down the information bias originating from the demand-side by assuming $\mu = \mu_0$ and $\gamma = 1$. Then the next corollary follows Proposition 4 immediately.

Corollary 5 (Conflicted PA). *Assume $\gamma = 1$ and $\mu_0 = \mu$. If the PA has conflicts of interest (i.e., $\varphi > 0$), its voting recommendation is biased and undesirable: $\varrho^* < \varrho^{\mathrm{U}} = \varrho^{\mathrm{D}}$. The firm value is impaired.*

It is intuitive that the PA's proxy voting advice is distorted by the conflicts of interest in a way that more positive recommendations are provided. After all, such bias can bring the PA more revenue. Furthermore, the expected firm value is inferior to the first-best case. Consistently, Li (2017) demonstrates that the potential conflicted business of ISS has real, negative consequences.

5 Discussions

The widely use of proxy advisors in financial markets has drawn regulators' attention. In a 2010 Concept Release on the U.S. Proxy System, Securities and Exchange Commission (SEC) (2010) raised a number of potential issues. For example,

1. if and to what extent proxy advisors develop, disseminate, and implement their voting recommendations without adequate accountability for informational accuracy in the development and application of voting standards;

2. if and to what extent proxy advisors are controlling or significantly influencing shareholder voting without appropriate oversight, and without having an actual economic stake in the issuer; and

3. if and to what extent conflicts of interest on the part of proxy advisors could impair informed shareholder voting.

The SEC's first concern speaks to the information quality in the proxy advisory market. We do not examine information production of the proxy advisor. Instead, we investigate if given the private information, the proxy advisor has incentives to obfuscate it, which is usually the case for information sales in other contexts (e.g., Admati and Pfleiderer, 1986; Bergemann et al., 2016). Our result suggests that the proxy advisor will send unbiased voting recommendations to shareholders even though the proxy advisor is a for-profit third party operating in a one-shot game, which is different from other information intermediaries, e.g., sell-side analysts. This provides justification for the business model of these information intermediaries.

Following the same logic, as for the SEC's second issue regarding proxy advisors' influence over shareholders, our results suggest that, without further distortions, the influence should not be a concern, because the proxy advisor can truly fulfill their duties as an information intermediary. In other words, shareholders are equally well off in the following two cases: (i) the proxy advisor sells its voting recommendations and shareholders follow them; and (ii) the proxy advisor sells all its private information, shareholders themselves analyze the information and then cast votes. This is true even if there is no oversight, or the proxy advisor does not have an actual economic stake in the corporate. At the same time, our results caution that such ideal setting can be fragile given the various potential sources of information bias.

The third concern of the SEC is about the real effects of corporate voting. Intuitively, as confirmed in our model, the conflicts of interest from the supply side bias the voting recommendations and impair firm value. Further, our results point to the information bias arising from the demand side, which can impair informed voting as well. It is reasonable that the proxy advisor caters to its clients' particular preferences. However, when its clients have wrong prior beliefs about the voting item and it is confident about the accuracy of its information, the proxy advisor may fail to guard the interests of its clients. This suggests the limit of these information intermediaries.

Overall, the insights of our model are three-folded.

1. Information bias, information desirability, and conflicts of interest, though somewhat related, are fundamentally different concepts.

2. In general, the PA with conflicts of interest issues biased and undesirable voting recommendations.

3. Even an unconflicted PA may send biased voting recommendations to shareholders, and the recommendations may or may not be desirable for shareholders, depending on shareholders' preferences and beliefs.

6 Conclusion

In this paper, we provide a framework to examine information bias in the proxy advisory market, and its implications on the corporate value.

We show that the proxy advisor, without an actual stake or appropriate oversight, will send unbiased and desirable voting recommendations which help maximize firm value only if the proxy advisor has no conflicts of interest, shareholders have the correct prior belief, and they aim to maximize firm value. This offers a meaningful starting point for us to further investigate the potential information bias in the proxy advisory market. In general, a proxy advisor with conflicts of interest issues biased and undesirable voting recommendations to shareholders, which leads to inefficiency. Furthermore, the unconflicted proxy advisor may send biased voting recommendations to cater to its clients, and such recommendations may or may not be desired by shareholders.

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Appendix

A Perceived Benefit and Firm Value

As stated in the text, ψ^{P} is obtained by taking γ and μ into consideration and ψ^0 is calculated by replacing μ with μ_0 in ψ^{P}. The firm value, here denoted by ψ, is evaluated by using true prior μ_0 when computing probabilities and ignoring γ.

Shareholders in the game believe the expected benefit of the proposal is ψ^{P} while the true benefit of the proposal for shareholders is ψ^0. In the model the unconflicted PA maximizes ψ^{P} while a desirable recommendation should maximize ψ^0.

Before deriving the expression of these quantities, it is convenient to define the function $P(x, N, k)$ as the probability that the proposal gets k votes out of N when each shareholder independently votes for the proposal with probability x:

$$P(x, N, k) \equiv C_N^k x^k (1-x)^{N-k}, \tag{A.1}$$

where $C_N^k = \frac{N!}{k!(N-k)!}$ is the binomial coefficient.

Let ω_1 be the probability with which a randomly chosen shareholder votes for the proposal conditional on the PA sending $r = 1$ and ω_0 be the probability of voting in favor of the proposal when the PA sends $r = 0$. That is, $\omega_1 \equiv \Pr(v = 1|r = 1) = q + (1-q)\tau$ and $\omega_0 \equiv \Pr(v = 1|r = 0) = (1-q)\tau$. Therefore, we have the following:

$$\Pr(d = 1|r = 1) = \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k), \tag{A.2}$$

$$\Pr(d = 1|r = 0) = \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k). \tag{A.3}$$

Using equations (A.2) and (A.3), we can write ψ^{P}, ψ^0, and ψ respectively as follows:

$$\begin{aligned}
\psi^{\mathrm{P}} =& \mu(1-\varrho^2)\sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) + \mu\varrho^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k) \\
&- \gamma(1-\mu)(1-\varrho)^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) - \gamma(1-\mu)(2\varrho-\varrho^2) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k), \\
\psi^0 =& \mu_0(1-\varrho^2)\sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) + \mu_0\varrho^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k) \\
&- \gamma(1-\mu_0)(1-\varrho)^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) - \gamma(1-\mu_0)(2\varrho-\varrho^2) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k), \\
\psi =& \mu_0(1-\varrho^2)\sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) + \mu_0\varrho^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k) \\
&- (1-\mu_0)(1-\varrho)^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) - (1-\mu_0)(2\varrho-\varrho^2) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k).
\end{aligned}$$

B Proofs

Proof of Proposition 1

When $q = 0$, i.e., no shareholder purchases PA's recommendation, by Assumption 1, all shareholders cast their votes according to their prior. Thus, each shareholder votes "for" if and only if

$$\mu \geq \frac{\gamma}{1+\gamma}.$$

When $q > 0$, we fix the cut-off point ϱ of the PA sending $r = 1$. Suppose that shareholder i chooses to acquire the PA's recommendation and receives $r = 1$. Shareholder i can change the voting result if and only if she is a pivotal shareholder, i.e., the number of "for" votes among other shareholders is exactly $\frac{N-1}{2}$. Denote the event of being pivotal as PIV_i. Then for shareholder i, the payoff of voting "for" conditioning on the event of being pivotal is

$$\begin{aligned}
&\Pr(\theta=1|r=1,PIV_i)-\gamma\Pr(\theta=0|r=1,PIV_i) \qquad \text{(B.1)}\\
&=\frac{\begin{array}{c}\Pr(PIV_i|\theta=1,r=1)\Pr(r=1|\theta=1)\Pr(\theta=1)\\ -\gamma\Pr(PIV_i|\theta=0,r=1)\Pr(r=1|\theta=0)\Pr(\theta=0)\end{array}}{\Pr(r=1,PIV_i)}\\
&=\frac{\Pr(PIV_i|r=1)\left[\Pr(r=1|\theta=1)\Pr(\theta=1)-\gamma\Pr(r=1|\theta=0)\Pr(\theta=0)\right]}{\Pr(r=1,PIV_i)}\\
&=\frac{\Pr(r=1|\theta=1)\Pr(\theta=1)-\gamma\Pr(r=1|\theta=0)\Pr(\theta=0)}{\Pr(r=1)}\\
&=\frac{\pi_1\mu-\gamma(1-\pi_0)(1-\mu)}{\pi_1\mu+(1-\pi_0)(1-\mu)}\\
&=\frac{(1-\varrho)\left[\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)\right]}{\pi_1\mu+(1-\pi_0)(1-\mu)}.
\end{aligned}$$

Note the second line follows because the true state is unobservable to shareholder i, thus

$$\Pr(PIV_i|\theta=1,r=1)=\Pr(PIV_i|\theta=0,r=1)=\Pr(PIV_i|r=1).$$

For shareholder i to vote "for," it must be the case that equation (B.1) is nonnegative, which holds when

$$\varrho\geq\frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}. \qquad \text{(B.2)}$$

Therefore, if

$$\varrho<\frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu},$$

the shareholder who acquires a positive signal is not willing to vote in favor of the proposal. Since the shareholder with a positive recommendation is the most likely type to vote in favor of the proposal, no shareholders are willing to vote "for" when $\varrho<\frac{\gamma(1-\mu)-\mu}{\gamma(1-\mu)+\mu}$.

Similarly, if shareholder i receives $r=0$, the payoff of voting "for" conditioning on the event of being pivotal can be computed as follows

$$\Pr(\theta=1|r=0,PIV_i)-\gamma\Pr(\theta=0|r=0,PIV_i)$$
$$=\frac{(1-\pi_1)\mu-\gamma\pi_0(1-\mu)}{(1-\pi_1)\mu+\pi_0(1-\mu)}=\frac{\varrho\left[\mu\varrho-\gamma(1-\mu)(2-\varrho)\right]}{(1-\pi_1)\mu+\pi_0(1-\mu)}. \quad \text{(B.3)}$$

For the shareholder to vote against the proposal, equation (B.3) needs to be nonpositive, which holds when

$$\varrho\leq\frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}. \quad \text{(B.4)}$$

By the same logic, when $\varrho>\frac{2\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$, all shareholders will vote for the proposal.

When $q\in(0,1)$, for the uninformed shareholder, the payoff of voting "for" conditional on the event of being pivotal is

$$\Pr(\theta=1|PIV_i)-\gamma\Pr(\theta=0|PIV_i)=\frac{1}{\Pr(PIV_i)}\Big(\Pr(PIV_i|r=1)(1-\varrho)\times$$
$$(\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho))+\Pr(PIV_i|r=0)\varrho(\mu\varrho-\gamma(1-\mu)(2-\varrho))\Big). \quad \text{(B.5)}$$

We now prove by contradiction that equation (B.5) equals to zero in equilibrium; that is, the uninformed shareholder must use a mixed voting strategy. Suppose all uninformed shareholders vote "for" for sure. Then a randomly chosen shareholder will vote "for" with probability one if the PA's recommendation is $r=1$, and with probability q if the PA's recommendation is $r=0$. So an uninformed shareholder finds her being pivotal if and only if $r=0$. However, in such a situation, the uninformed shareholder can infer the private information of the informed shareholders and figure out State 1 is the more likely state, which makes her not want to vote for the proposal. Therefore, in equilibrium, the uninformed type cannot always vote for the proposal. The same argument applies to the case when the uninformed shareholder always votes against the proposal. Taken together, uninformed shareholders must use a mixed voting strategy in equilibrium.

Let $\tau\in(0,1)$ be the uninformed shareholders' probability of voting for the proposal. Given that the fraction of the shareholders who purchase PA's recommendation is q, for a generic shareholder, her probability of voting "for" conditioning on the event $r = 1$ is $\Pr(v_i = 1|r = 1) = q + (1-q)\tau$. Therefore,

$$\Pr(PIV_i|r=1) = C_{N-1}^{\frac{N-1}{2}} [q + (1-q)\tau]^{\frac{N-1}{2}} [(1-q)(1-\tau)]^{\frac{N-1}{2}}. \tag{B.6}$$

Similarly, for a generic shareholder, her probability of voting "for" conditioning on the event $r = 0$ is $\Pr(v_i = 1|r = 0) = (1-q)\tau$, and thus

$$\Pr(PIV_i|r=0) = C_{N-1}^{\frac{N-1}{2}} [(1-q)\tau]^{\frac{N-1}{2}} [1-(1-q)\tau]^{\frac{N-1}{2}}. \tag{B.7}$$

Plugging equations (B.6) and (B.7) into equation (B.5) and setting it to be zero, we obtain

$$\begin{aligned} 0 = (q+(1-q)\tau)^{\frac{N-1}{2}}((1-q)(1-\tau))^{\frac{N-1}{2}}(1-\varrho)[\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)] \\ +((1-q)\tau)^{\frac{N-1}{2}}(1-(1-q)\tau)^{\frac{N-1}{2}}\varrho(\mu\varrho-\gamma(1-\mu)(2-\varrho)). \end{aligned} \tag{B.8}$$

Based on equations (B.2) and (B.4), we have

$$(1-\varrho)[\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)] > 0$$

and

$$\varrho[\mu\varrho-\gamma(1-\mu)(2-\varrho)] < 0.$$

We define M as follows:

$$M \equiv \sqrt[\frac{N-1}{2}]{\frac{\varrho[\gamma(1-\mu)(2-\varrho)-\mu\varrho]}{(1-\varrho)[\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)]}} > 0. \tag{B.9}$$

Since $\varrho[\gamma(1-\mu)(2-\varrho)-\mu\varrho]-(1-\varrho)[\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)] = \gamma(1-\mu)-\mu$, $M \le 1$ when $\gamma \le \frac{\mu}{1-\mu}$, and $M \ge 1$ when $\gamma \ge \frac{\mu}{1-\mu}$. Rearranging equation (B.8) we obtain that the RHS of equation (B.8) is positively related to a quadratic function of τ: $h(\tau) \equiv (1-q)(1-M)\tau^2 -$

$(1-2q-M)\tau - q$. When $q<1$, solving $h(\tau)=0$ we obtain τ as follows, where the root that does not lie between 0 and 1 is omitted:

$$\tau = \begin{cases} \frac{(1-M-2q)+\sqrt{(1-M)^2+4q^2M}}{2(1-q)(1-M)} & \text{if } \gamma \neq \frac{\mu}{1-\mu}, \\ \frac{1}{2} & \text{if } \gamma = \frac{\mu}{1-\mu}, \end{cases} \tag{B.10}$$

which can be rewritten as in (5). □

Assuming on the equilibrium path, we have the following corollary that will be useful for future discussion.

Corollary A1. *For all $0<q<1$ and $M>0$, we have:*

1. *If $\mu > \frac{1}{2}$, $\frac{\partial \tau}{\partial \varrho} < (>)\, 0$ when $\varrho < (>)\, 1-\mu$ and $\frac{\partial \tau}{\partial q} < 0$; and*

2. *If $\mu < \frac{1}{2}$, $\frac{\partial \tau}{\partial \varrho} > (<)\, 0$ when $\varrho < (>)\, 1-\mu$ and $\frac{\partial \tau}{\partial q} > 0$.*

Proof. Taking derivative of τ with respect to q yields

$$\frac{\partial \tau}{\partial q} = -\frac{(M+1)\sqrt{(1-M)^2+4q^2M}-(1-M)^2-4qM}{2(1-M)(1-q)^2\sqrt{(1-M)^2+4q^2M}} = \begin{cases} <0, & \text{if } \gamma < \frac{\mu}{1-\mu}, \\ >0, & \text{if } \gamma > \frac{\mu}{1-\mu}. \end{cases} \tag{B.11}$$

This is because $(M+1)^2\left[(1-M)^2+4q^2M\right]-\left[(1-M)^2+4qM\right]^2=\left[(1-M)^2+4qM\right]4M(1-q)>0$. Taking derivative of τ with respect to M yields

$$\frac{\partial \tau}{\partial M} = -\frac{q\left[\sqrt{(1-M)^2+4q^2M}-(M+1)q\right]}{(1-M)^2(1-q)\sqrt{(1-M)^2+4q^2M}} < 0, \tag{B.12}$$

as $(1-M)^2+4q^2M-(M+1)^2q^2=(1-M)^2(1-q^2)>0$. If $\gamma < \frac{\mu}{1-\mu}$,

$$\frac{\partial M}{\partial \varrho} = \frac{4M}{N-1}\left(\frac{(\mu-(1-\mu)\gamma)(\gamma(1-\mu)(1-\varrho)-\mu\varrho)}{\varrho[\gamma(1-\mu)(2-\varrho)-\mu\varrho](1-\varrho)[\mu(1+\varrho)-\gamma(1-\mu)(1-\varrho)]}\right) > (<)\, 0 \tag{B.13}$$

when $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$. And if $\gamma > \frac{\mu}{1-\mu}$, $\frac{\partial M}{\partial \varrho} < (>)\, 0$ when $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$. Thus if $\gamma < \frac{\mu}{1-\mu}$, $\frac{\partial \tau}{\partial \varrho} < (>)\, 0$ when $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$, and if $\gamma > \frac{\mu}{1-\mu}$, $\frac{\partial \tau}{\partial \varrho} > (<)\, 0$ when $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$. Therefore, for all $0 < q < 1$ and $M > 0$,

$$\begin{cases} \frac{\partial \tau}{\partial \varrho} < (>)\, 0 \text{ when } \varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}, \text{ if } \gamma < \frac{\mu}{1-\mu}, \\ \frac{\partial \tau}{\partial \varrho} > (<)\, 0 \text{ when } \varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}, \text{ if } \gamma > \frac{\mu}{1-\mu}. \end{cases} \tag{B.14}$$

Also when $\varrho = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$, $\frac{\partial \tau}{\partial \varrho} = 0$. □

Proof of Proposition 2

At the information-purchase stage, a shareholder will use a mixed strategy on information acquisition ($q \in (0,1)$) only when the value of recommendation is equal to its price. To determine the fee, we need to first calculate the value of the recommendation. Suppose shareholder i purchases the PA's recommendation. If she receives $r = 1$, shareholder i's payoff of voting "for" for sure vs. voting "for" with probability τ is

$$\begin{aligned} & V(q, \varrho | r=1) \qquad \text{(B.15)} \\ &= (1-\tau)\left[\Pr(\theta=1|PIV_i, r=1) - \gamma \Pr(\theta=0|PIV_i, r=1)\right] \times \Pr(PIV_i|r=1) \\ &= (1-\tau)\frac{\pi_1\mu - \gamma(1-\pi_0)(1-\mu)}{\Pr(r=1)}\left[C_{N-1}^{\frac{N-1}{2}}\left[q+(1-q)\tau\right]^{\frac{N-1}{2}}\left[(1-q)(1-\tau)\right]^{\frac{N-1}{2}}\right]. \end{aligned}$$

Note that since the payoff of voting conditional on the non-pivotal case is zero, there is only one term in equation (B.15). Similarly, if she receives $r = 0$, shareholder i's payoff of voting "against" for sure vs. voting "agianst" with probability $1-\tau$ is

$$\begin{aligned} & V(q, \varrho | r=0) \qquad \text{(B.16)} \\ &= -\tau\left[\Pr(\theta=1|PIV_i, r=0) - \gamma \Pr(\theta=0|PIV_i, r=0)\right] \times \Pr(PIV_i|r=0) \\ &= -\tau\frac{(1-\pi_1)\mu - \gamma\pi_0(1-\mu)}{\Pr(r=0)}\left[C_{N-1}^{\frac{N-1}{2}}\left[(1-q)\tau\right]^{\frac{N-1}{2}}\left[1-(1-q)\tau\right]^{\frac{N-1}{2}}\right]. \end{aligned}$$

Therefore, the overall value of recommendation is

$$\begin{aligned} V(q,\varrho) &= \Pr(r=1)\,V(q,\varrho|r=1) + \Pr(r=0)\,V(q,\varrho|r=0) \\ &= (1-\tau)\left[\pi_1\mu - \gamma(1-\pi_0)(1-\mu)\right]\Pr(PIV_i|r=1) \\ &- \tau\left[(1-\pi_1)\mu - \gamma\pi_0(1-\mu)\right]\Pr(PIV_i|r=0) \\ &= \left[\pi_1\mu - \gamma(1-\pi_0)(1-\mu)\right]\Pr(PIV_i|r=1) \\ &= (1-\varrho)\left[\mu(1+\varrho) - \gamma(1-\mu)(1-\varrho)\right] C_{N-1}^{\frac{N-1}{2}}\left[q+(1-q)\tau\right]^{\frac{N-1}{2}}\left[(1-q)(1-\tau)\right]^{\frac{N-1}{2}}. \end{aligned} \tag{B.17}$$

Note the third equality uses

$$\begin{aligned} 0 &= \left[\pi_1\mu - \gamma(1-\pi_0)(1-\mu)\right]\Pr(PIV_i|r=1) \\ &+ \left[(1-\pi_1)\mu - \gamma\pi_0(1-\mu)\right]\Pr(PIV_i|r=0). \end{aligned} \tag{B.18}$$

The proof of equation (B.18) is as follows. Based on equations (B.1) and (B.3), the right-hand-side (RHS) of equation (B.18) can be re-written and simplified as below:

$$\begin{aligned} &\left[\Pr(\theta=1|r=1,PIV_i) - \gamma\Pr(\theta=0|r=1,PIV_i)\right]\Pr(r=1,PIV_i) \\ &+ \left[\Pr(\theta=1|r=0,PIV_i) - \gamma\Pr(\theta=0|r=0,PIV_i)\right]\Pr(r=0,PIV_i) \\ &= \left[\Pr(\theta=1,r=1,PIV_i) - \gamma\Pr(\theta=0,r=1,PIV_i)\right] \\ &+ \left[\Pr(\theta=1,r=0,PIV_i) - \gamma\Pr(\theta=0,r=0,PIV_i)\right] \\ &= \Pr(\theta=1,PIV_i) - \gamma\Pr(\theta=0,PIV_i) \\ &= \left[\Pr(\theta=1|PIV_i) - \gamma\Pr(\theta=0|PIV_i)\right]\Pr(PIV_i) \\ &= 0. \end{aligned}$$

The last equation follows equation (B.8).

Next we need to solve q^* by comparing the information value V and the information fee f.

Setting $V(q,\varrho) = f$, we can solve q for given f and ϱ. First, we define

$$G \equiv \sqrt[\frac{N-1}{2}]{\frac{f}{C_{N-1}^{\frac{N-1}{2}}\left(1-\varrho\right)\left[\mu\left(1+\varrho\right)-\gamma\left(1-\mu\right)\left(1-\varrho\right)\right]}}. \tag{B.19}$$

Then, $V(q,\varrho) = f$ can be re-written as

$$\left[q+\left(1-q\right)\tau\right]\left(1-q\right)\left(1-\tau\right) = G. \tag{B.20}$$

Plugging equation (5) into equation (B.20), we have

$$F\left(q\right) \equiv \frac{Mq\left(\sqrt{\left(1-M\right)^2+4Mq^2}-q-Mq\right)}{\left(1-M\right)^2} = G. \tag{B.21}$$

Now we want to show that there exists a unique maximum point of $F(q)$ for $q \in (0,1)$. Define the left-hand-side (LHS) of equation (B.21) to be $F(q)$. Since $(1-M)^2 + 4Mq^2 - (q+Mq)^2 = (1-M)^2(1-q^2) \geq 0$, $F(q) \geq 0$. It is easy to check that $F(0) = 0$ and $F(1) = 0$. Taking derivative of $F(q)$ with respect to q and taking limits, we have $\lim_{q\to 0} \frac{dF}{dq} = \frac{M}{\sqrt{(1-M)^2}} > 0$ and $\lim_{q\to 1} \frac{dF}{dq} = \frac{-M}{1+M} < 0$. Taking second derivative of $F(q)$ with respect to q, we obtain

$$\begin{aligned}\frac{\mathrm{d}^2F}{\mathrm{d}q^2} &= 2M\frac{-1-M+\frac{16M^2q^3+6qM(1-M)^2}{\left[(1-M)^2+4Mq^2\right]^{\frac{3}{2}}}}{\left(1-M\right)^2} \\ &< 2M\frac{-1-M+\frac{16M^2+6M(1-M)^2}{\left[(1-M)^2+4M\right]^{\frac{3}{2}}}}{\left(1-M\right)^2} \\ &= -\frac{2M\left(1+M^2\right)}{\left(1+M\right)^3} < 0.\end{aligned}$$

The inequality follows because

$$\frac{\partial}{\partial q}\left(\frac{16M^2q^3+6qM\left(1-M\right)^2}{\left[\left(1-M\right)^2+4Mq^2\right]^{\frac{3}{2}}}\right) = \frac{6M\left(1-M\right)^4}{\left[\left(1-M\right)^2+4Mq^2\right]^{\frac{5}{2}}} > 0.$$

Taken together, $F(q)$ increases from 0 and then decreases to 0 as q moves from 0 to 1, and it reaches its maximum at an interior point. Define this interior maximum point to be

$$\tilde{q} = \underset{q}{\operatorname{argmax}} \left(\frac{Mq\left(\sqrt{(1-M)^2 + 4Mq^2} - q - Mq\right)}{(1-M)^2} \right). \tag{B.22}$$

Let $F(\tilde{q}) = G$ and solve the fee that satisfies the equation

$$\tilde{f} = F(\tilde{q})^{\frac{N-1}{2}} C_{N-1}^{\frac{N-1}{2}} \left[\pi_1 \mu - \gamma(1-\pi_0)(1-\mu)\right]. \tag{B.23}$$

Therefore, when $f > \tilde{f}$, $G > F(q)$ and thus $q^* = 0$. When $f = \tilde{f}$, there is a unique solution to equation (B.21), which is denoted by $\tilde{q}$ as in (B.22). When $f < \tilde{f}$, there are two solutions to equation (B.21) and they are given in equation (9). □

Proof of Lemma 1

In general, the perceived benefit is

$$\psi^{\mathrm{P}} = \mu(1-\varrho^2) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) + \mu\varrho^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k) - \gamma(1-\mu)(1-\varrho)^2 \sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) - \gamma(1-\mu)(2\varrho-\varrho^2) \sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k).$$

Using the fact that

$$\sum_{k=\frac{N+1}{2}}^{N} P(x, N, k) = \frac{B\left(x; \frac{N+1}{2}, \frac{N+1}{2}\right)}{B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)},$$

we have

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\psi^{\mathrm{P}} = [\mu(1-\varrho^2) - \gamma(1-\mu)(1-\varrho)^2] B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + [\mu\varrho^2 - \gamma(1-\mu)(2\varrho-\varrho^2)] B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right), \tag{B.24}$$

where $B(x;a,b)=\int_0^x t^{a-1}(1-t)^{b-1}\mathrm{d}t$ is the incomplete beta function and $B(a,b)$ is the beta function. Taking the derivative of the RHS of equation (B.24) with respect to ϱ, we have

$$2(\gamma(1-\varrho)(1-\mu)-\mu\varrho)\left(B\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)-B\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)\right) \\ +(1-q)\frac{\partial\tau}{\partial\varrho}\left[\begin{array}{c}[\mu(1-\varrho^2)-\gamma(1-\mu)(1-\varrho)^2]B_{\omega_1}\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)\\ +[\mu\varrho^2-\gamma(1-\mu)(2\varrho-\varrho^2)]B_{\omega_0}\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)\end{array}\right].$$

It is easy to verify that $B(x;a,b)$ is strictly increasing with respect to x by calculating

$$B_x\left(x;\frac{N+1}{2},\frac{N+1}{2}\right)=(x(1-x))^{\frac{N-1}{2}}>0. \tag{B.25}$$

Thus, $B\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)-B\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)>0$, $B_{\omega_1}\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)=(q+(1-q)\tau)^{\frac{N-1}{2}}((1-q)(1-\tau))^{\frac{N-1}{2}}$, and $B_{\omega_0}\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)=((1-q)\tau)^{\frac{N-1}{2}}(1-(1-q)(1-\tau))^{\frac{N-1}{2}}$. Using them, we have

$$\left[\begin{array}{c}[\mu(1-\varrho^2)-\gamma(1-\mu)(1-\varrho)^2]B_{\omega_1}\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)\\ +[\mu\varrho^2-\gamma(1-\mu)(2\varrho-\varrho^2)]B_{\omega_0}\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)\end{array}\right]= \\ \left[\begin{array}{c}[\mu(1-\varrho^2)-\gamma(1-\mu)(1-\varrho)^2]\left((q+(1-q)\tau)(1-q)(1-\tau)\right)^{\frac{N-1}{2}}\\ +[\mu\varrho^2-\gamma(1-\mu)(2\varrho-\varrho^2)]\left((1-q)\tau(1-(1-q)\tau)\right)^{\frac{N-1}{2}}\end{array}\right].$$

Using equation (B.18), one can conclude

$$[\mu(1-\varrho^2)-\gamma(1-\mu)(1-\varrho)^2]B_{\omega_1}\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)+ \\ [\mu\varrho^2-\gamma(1-\mu)(2\varrho-\varrho^2)]B_{\omega_0}\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)=0. \tag{B.26}$$

Therefore,

$$B\left(\frac{N+1}{2},\frac{N+1}{2}\right)\frac{\partial\psi^{\mathrm{P}}}{\partial\varrho}=2[\gamma(1-\mu)(1-\varrho)-\mu\varrho]B\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)>(<)\,0$$

if $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$.

Taking the derivative of the RHS of equation (B.24) with respect to q, we have

$$[\mu(1-\varrho^2) - \gamma(1-\mu)(1-\varrho)^2]B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + \left((1-q)\frac{\partial\tau}{\partial q} - \tau\right)\begin{bmatrix} [\mu(1-\varrho^2) - \gamma(1-\mu)(1-\varrho)^2]B_{\omega_1}\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) \\ +[\mu\varrho^2 - \gamma(1-\mu)(2\varrho-\varrho^2)]B_{\omega_0}\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right) \end{bmatrix}. \quad \text{(B.27)}$$

Similarly, we have

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\frac{\partial\psi^{\mathrm{P}}}{\partial q} = [\mu(1-\varrho^2) - \gamma(1-\mu)(1-\varrho)^2]B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) > 0$$

by using equation (B.26). □

Proof of Proposition 3

The first part of Proposition 3 follows Lemma 1 immediately. For the remaining two parts, we write ψ^0 as

$$\psi^0 = \mu_0(1-\varrho^2)\sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) + \mu_0\varrho^2\sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k) - \gamma(1-\mu_0)(1-\varrho)^2\sum_{k=\frac{N+1}{2}}^{N} P(\omega_1, N, k) - \gamma(1-\mu_0)(2\varrho-\varrho^2)\sum_{k=\frac{N+1}{2}}^{N} P(\omega_0, N, k).$$

Through the same process, we have

$$B\left(\frac{N+1}{2}, \frac{N+1}{2}\right)\psi^0 = [\mu_0(1-\varrho^2) - \gamma(1-\mu_0)(1-\varrho)^2]B\left(\omega_1; \frac{N+1}{2}, \frac{N+1}{2}\right) + [\mu_0\varrho^2 - \gamma(1-\mu_0)(2\varrho-\varrho^2)]B\left(\omega_0; \frac{N+1}{2}, \frac{N+1}{2}\right).$$

Taking the derivative of the RHS of the above equation with respect to ϱ and plugging $\varrho = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ in, we have

$$\frac{2\gamma(\mu-\mu_0)}{\gamma(1-\mu)+\mu}\left(B\left(\omega_1;\frac{N+1}{2},\frac{N+1}{2}\right)-B\left(\omega_0;\frac{N+1}{2},\frac{N+1}{2}\right)\right),$$

which is positive (negative) if $\mu > (<)\ \mu_0$. Therefore, $\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu} < (>)\ \varrho^{\mathrm{D}}$ if $\mu > (<)\ \mu_0$. □

The following lemma is used for the proof of Proposition 4.

Lemma A1. *When $\gamma < (>) \frac{\mu}{(1-\mu)}$, we have $M < (>) 1$, $\sqrt{(1-M)^2+4Mq^2} > (<) 2Mq$, and $2q > (<) \sqrt{(1-M)^2+4Mq^2}$.*

Proof. When $\gamma < \frac{\mu}{(1-\mu)}$, based on equation (B.9) we know that $M < 1$, thus $H \equiv (1-M)^2 + 4Mq^2 - (2Mq)^2 = (1-M)^2 + 4q^2M(1-M) > 0$. So $\sqrt{(1-M)^2+4Mq^2} - 2Mq > 0$. To show $2q > \sqrt{(1-M)^2+4Mq^2}$, it is equivalent to show $4q^2 > 1-M$. Plugging equation (B.29) into $4q^2 > 1-M$ and we obtain

$$\begin{aligned}
4q^2 > 1-M &\Leftrightarrow (1+M)\sqrt{4M+N^2(1-M)^2} > (-M^2N-4M+N)\\
&\Leftrightarrow \left((1+M)\sqrt{4M+N^2(1-M)^2}\right)^2 > (-M^2N-4M+N)^2\\
&\Leftrightarrow (-8N+4)M^3 - 8M^2 + (8N+4)M > 0.
\end{aligned}$$

It is easy to verify that the last expression of the above equation holds for all $M < 1$ and $N > 3$.

When $\gamma > \frac{\mu}{(1-\mu)}$, based on equation (B.9) we know that $M > 1$. Plugging equation (B.29) into H we obtain that

$$H = \frac{(M-1)\left[4M+N-M^2N+(1+M)\sqrt{4M+N^2-2MN^2+M^2N^2}\right]}{2N} \propto^{+} M-1.$$

Note that $H \propto^{+} M-1$ since $(4M+N-M^2N)^2 - (1+M)^2(4M+N^2-2MN^2+M^2N^2) = 4(M-1)M(M-1+2N(M+1)) > 0$ when $M > 1$. Thus $\sqrt{(1-M)^2+4Mq^2} - 2Mq < 0$ when $\gamma > \frac{\mu}{(1-\mu)}$. Finally, $2q < \sqrt{(1-M)^2+4Mq^2}$ holds because $2q < 2q\sqrt{M} < \sqrt{(1-M)^2+4Mq^2}$

when $M > 1$. □

Proof of Proposition 4

At the profit-maximization stage, the PA chooses the fee f and the cut-off point ϱ to maximize its profits. Given the selected equilibrium, the PA's problem is equivalent to choosing the informed fraction q and the cut-off point ϱ: $\max_{q,\varrho} \pi = qV$.

For the optimal q, we take the derivative of π with respect to q and set it to zero (V is given by equation (B.17)):

$$0 = \left[q + (1-q)\tau\right](1-q)(1-\tau) \\ + q\frac{N-1}{2}\left(1-\tau+(1-q)\frac{\partial\tau}{\partial q}\right)((1-q)(1-2\tau)-q).$$

Inserting equations (5) and (B.11) into $\frac{\partial\pi}{\partial q}$, we obtain that the RHS of $\frac{\partial\pi}{\partial q}$ is positively related to $h_1(q)$, where

$$h_1(q) \equiv \frac{qM\Gamma(q)}{2(1-M)^2\sqrt{(1-M)^2+4q^2M}}, \tag{B.28}$$

and $\Gamma(q) \equiv (1+N)(1-M)^2 + 8MNq^2 - 2Nq(M+1)\sqrt{(1-M)^2+4q^2M}$. Note that given $q \in (0,1)$, the sign of $h_1(q)$ depends on the sign of $\Gamma(q)$. To determine the sign of $\Gamma(q)$, we first have

$$\lim_{q\to 0}\Gamma(q) = (1-M)^2(N+1) > 0,$$

and

$$\lim_{q\to 1}\Gamma(q) = -(1-M)^2(N-1) < 0.$$

By the intermediate value theorem, there exists at least one solution to $h_1(q) = 0$ when $q \in (0,1)$. Taking derivative of $\Gamma(q)$ with respect to q yields a negative second-order condition

$$\Gamma'(q) \propto^{+} 8qM\sqrt{(1-M)^2+4q^2M} - \left[1+M^3+M(M+1)\left(8q^2-1\right)\right] < 0.$$

The inequality follows from the fact that for any $q \in (0, 1)$ and $M > 0$,

$$\left(8qM\sqrt{(1-M)^2 + 4q^2M}\right)^2 - \left[1 + M^3 + M(M+1)\left(8q^2 - 1\right)\right]^2$$
$$= -64M^2(1-M)^2 q^4 - 16M(1-M)^4 q^2 - (1-M)^4(1+M)^2 < 0.$$

Let the solution to $\frac{\partial \pi}{\partial q} = 0$ be q^*. We thus know that when $q < q^*$, $\Gamma(q) > 0$ and when $q > q^*$, $\Gamma(q) < 0$. Therefore given any ϱ the solution to $h_1(q) = 0$ (and thus $\frac{\partial \pi}{\partial q} = 0$) is unique, and maximizes π. Solving equation (B.28) yields (the inproper roots are neglected)

$$q(\varrho) = \frac{\sqrt{4M - N(1-M)^2 + (1+M)\sqrt{4M + N^2(1-M)^2}}}{2\sqrt{2MN}}. \tag{B.29}$$

When $\varphi = 0$

When $\varphi = 0$, for the optimal ϱ, we utilize the fact that the monotonicity of π is the same as that of $\log(\pi)$ and take the derivative of $\log(\pi)$ with respect to ϱ. Assuming $\gamma \neq \frac{\mu}{1-\mu}$ and using the fact that in equilibrium $f = V$, we obtain

$$\frac{\partial \log(\pi)}{\partial \varrho} = \frac{2[\gamma(1-\varrho)(1-\mu) - \mu\varrho]}{(1-\varrho)[\mu(1+\varrho) - \gamma(1-\varrho)(1-\mu)]} + \frac{N-1}{2}\frac{1}{G}\frac{\partial G}{\partial \varrho},$$

where G is defined by equation (B.19). Using the expression in equation (B.21), we obtain

$$\frac{\partial \log(\pi)}{\partial \varrho} = \frac{2[\gamma(1-\varrho)(1-\mu) - \mu\varrho]}{(1-\varrho)[\mu(1+\varrho) - \gamma(1-\varrho)(1-\mu)]} + \frac{N-1}{2}\frac{1}{G}\frac{\partial G}{\partial M}\frac{\partial M}{\partial \varrho}, \tag{B.30}$$

and

$$\frac{\partial G}{\partial M} = \frac{q[2M^2q^2 + 6Mq^2 + (1-M)^2 - q(3M+1)\sqrt{(1-M)^2 + 4Mq^2}]}{(1-M)^3\sqrt{(1-M)^2 + 4Mq^2}}. \tag{B.31}$$

Note that the first term of equation (B.30) is positive (negative) when $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$. To verify the sign of $\frac{\partial G}{\partial M}$, notice that

$$\left(2M^2q^2 + 6Mq^2 + (1-M)^2\right)^2 - \left(q(3M+1)\sqrt{(1-M)^2+4Mq^2}\right)^2 = \\ (1-q)(1+q)(1-M)^2[(1-M)^2 + 4Mq^2 - 4M^2q^2].$$

By Lemma A1, the sign of $(1-M)^2 + 4Mq^2 - 4M^2q^2$ is the same as $1 - M$. Therefore, the numerator and the denominator of equation (B.31) share the same sign. Hence, $\frac{\partial G}{\partial M} > 0$ for all $M \neq 1$.

When $\gamma < \frac{\mu}{1-\mu}$, $\frac{\partial M}{\partial \varrho}$ is also positive (negative) when $\varrho < (>) \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ (see Corollary A1). Therefore, π is single-peaked around $\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ with respect to ϱ when $\gamma < \frac{\mu}{1-\mu}$.

For the case of $\gamma > \frac{\mu}{1-\mu}$, we plug equation (B.13) into equation (B.30) and obtain

$$\begin{aligned}\frac{\partial \log(\pi)}{\partial \varrho} &= \frac{2[\gamma(1-\varrho)(1-\mu) - \mu\varrho]}{(1-\varrho)[\mu(1+\varrho) - \gamma(1-\varrho)(1-\mu)]} + \\ &\quad \frac{2M}{G}\frac{\partial G}{\partial M}\left(\frac{(\mu - (1-\mu)\gamma)(\gamma(1-\mu)(1-\varrho) - \mu\varrho)}{\varrho[\gamma(1-\mu)(2-\varrho) - \mu\varrho](1-\varrho)[\mu(1+\varrho) - \gamma(1-\mu)(1-\varrho)]}\right) \\ &= \frac{2[\gamma(1-\varrho)(1-\mu) - \mu\varrho]}{(1-\varrho)[\mu(1+\varrho) - \gamma(1-\varrho)(1-\mu)]} \times \left(1 + \frac{M}{G}\frac{\partial G}{\partial M}\frac{\mu - (1-\mu)\gamma}{\varrho[\gamma(1-\mu)(2-\varrho) - \mu\varrho]}\right).\end{aligned}$$

Therefore, to show π is single-peaked around $\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ with respect to ϱ, we need to show

$$\frac{M}{G}\frac{\partial G}{\partial M}\frac{(1-\mu)\gamma - \mu}{\varrho[\gamma(1-\mu)(2-\varrho) - \mu\varrho]} < 1,$$

which can be simplified to

$$\frac{M}{G}\frac{\partial G}{\partial M}\left(1 - \frac{1}{M^{\frac{N-1}{2}}}\right) < 1$$

by using the fact that

$$(1-\mu)\gamma - \mu = \varrho[\gamma(1-\mu)(2-\varrho) - \mu\varrho] - (1-\varrho)[\mu(1+\varrho) - \gamma(1-\mu)(1-\varrho)].$$

Since $M > 1$ when $\gamma > \frac{\mu}{1-\mu}$, it is sufficient to show $\frac{G}{M} > \frac{\partial G}{\partial M}$. Inserting equations (B.21) and (B.31) into $\frac{G}{M} > \frac{\partial G}{\partial M}$, we have

$$\begin{aligned}
\frac{G}{M} > \frac{\partial G}{\partial M} &\Leftrightarrow \left[\begin{array}{c} \frac{q\left(\sqrt{(1-M)^2+4Mq^2}-q-Mq\right)}{(1-M)^2} > \\ \frac{q[2M^2q^2+6Mq^2+(1-M)^2-q(3M+1)\sqrt{(1-M)^2+4Mq^2}]}{(1-M)^3\sqrt{(1-M)^2+4Mq^2}} \end{array} \right] \\
&\Leftrightarrow \left[\begin{array}{c} \sqrt{(1-M)^2+4Mq^2}-q-Mq > \\ \frac{q(3M+1)\sqrt{(1-M)^2+4Mq^2}-\left(2M^2q^2+6Mq^2+(1-M)^2\right)}{(M-1)\sqrt{(1-M)^2+4Mq^2}} \end{array} \right] \\
&\Leftrightarrow \left[\begin{array}{c} M((1-M)^2+4Mq^2)-q(M^2-1)\sqrt{(1-M)^2+4Mq^2} > \\ q(3M+1)\sqrt{(1-M)^2+4Mq^2}-(2M^2q^2+2Mq^2) \end{array} \right] \\
&\Leftrightarrow \left[\begin{array}{c} (1-M)^2+4Mq^2+(2Mq^2+2q^2) > \\ q(M+3)\sqrt{(1-M)^2+4Mq^2} \end{array} \right] \\
&\Leftrightarrow (1-q)(1+q)\,(M-1)^3\,(4q^2+M-1) > 0.
\end{aligned}$$

The last line of the above equation is obvious when $M > 1$ (which is valid when $\gamma > \frac{\mu}{1-\mu}$). Therefore, π is single-peaked around $\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ with respect to ϱ when $\gamma > \frac{\mu}{1-\mu}$.

When $\gamma = \frac{\mu}{1-\mu}$, $M = 1$ and $\tau = \frac{1}{2}$. Thus, equation (B.30) is reduced to

$$\frac{\partial \log(\pi)}{\partial \varrho} = \frac{2[\gamma(1-\varrho)(1-\mu)-\mu\varrho]}{(1-\varrho)[\mu(1+\varrho)-\gamma(1-\varrho)(1-\mu)]}.$$

Hence, π is also single-peaked around $\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ with respect to ϱ when $\gamma = \frac{\mu}{1-\mu}$. In sum, the optimal value for ϱ is $\varrho^* = \frac{\gamma(1-\mu)}{\gamma(1-\mu)+\mu}$ when $\varphi = 0$.

When $\varphi > 0$

When $\varphi > 0$, the PA's problem can be rewritten as

$$\max_{q,\varrho} \pi = qV + [\pi_1\mu_0 + (1-\pi_0)(1-\mu_0)]\varphi.$$

We take derivative of π with respect to ϱ:

$$\frac{\partial \pi}{\partial \varrho} = \frac{\partial (qV)}{\partial \varrho} + 2(\mu_0 + \varrho - 1 - 2\mu_0 \varrho)\varphi. \tag{B.32}$$

Based on the proof of the case where $\varphi > 0$, $\partial(qV)/\partial\varrho = 0$ if $\varrho = \gamma(1-\mu)/(\gamma(1-\mu)+\mu)$. Plugging $\varrho = \gamma(1-\mu)/(\gamma(1-\mu)+\mu)$ into equation (B.32) yields $\partial\pi/\partial\varrho < 0$. Thus, when the PA is conflicted, the equilibrium $\varrho^* < \gamma(1-\mu)/(\gamma(1-\mu)+\mu)$. □

Corollaries 1 to 5 immediately follow Propositions 3 and 4 and thus we omit their proofs.