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Research Paper

Study of correlation impact on credit default swap margin using a GARCH–DCC-copula framework

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ABSTRACT

We establish generalized autoregressive conditional heteroscedasticity–dynamic conditional correlation (GARCH–DCC) and constant conditional correlation (CCC) copula model frameworks to study time-varying correlation among credit default swap (CDS) single names (SNs) and its impact on certain risk measures of CDS portfolios that consist of names from different sectors within the eurozone (EU) and North America (NA). Our purpose is to better understand the direction and magnitude of impacts on such risk measures due to correlation changes. This study covers 188 NA SNs and 145 EU SNs from January 2008 to August 2017. We find that correlations between CDS SNs go through different correlation regimes during this period. As a result, CDS portfolio risk measures in the form of value-at-risk or expected shortfall show sizable variation due to correlation regime shifts from historical means. Depending on the correlation level (high or low) and the portfolio type, risk measures could be either underestimated or overestimated. Both directional and balanced portfolios could experience a sizable underestimation of the margin depending on the direction in which the correlation deviates from historical means.

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Therefore, it may be prudent for financial institutions managing portfolio risk, such as central counterparties, to take correlation changes into account when calculating such risk measures for risk management or margining purposes.

Keywords: credit default swap (CDS); generalized autoregressive conditional heteroscedasticity–dynamic conditional correlation (GARCH–DCC); Student *t* DCC copula; time-varying correlation; initial margin (IM).

1 INTRODUCTION

Correlation[1] is a critical input for many of the common tasks of financial risk management. For example, overall portfolio return is significantly affected by the asset correlations within a portfolio, and hedging requires estimation of the correlation between the returns of the assets in the hedge. Given the importance of correlation in this context, the quest for reliable estimates of correlation between financial variables has provided the motivation for many academic articles, practitioner conferences and research papers. Like correlation, volatility has been widely and comprehensively studied, with sophisticated models such as varieties of multivariate generalized autoregressive conditional heteroscedasticity (GARCH) or stochastic volatility having been constructed over the past two decades (eg, varieties of GARCH or GARCH-like models built to account for volatility clustering and heteroscedasticity). Correlation, however, is not as well understood as volatility, largely due to its complexity.

As it relates to central counterparty (CCP) risk models, correlation among different assets is a relevant factor in determining adequate risk measures such as margin charges to clearing members relating to the portfolios presented for clearing. Based on publicly available information, CCPs generally maintain basic assumptions regarding the estimation of correlation, affecting the final amount of margin collected from clearing members.[2] In practice, current margin frameworks, irrespective of cleared asset class, mostly employ relatively simple methods such as rolling historical correlations or average historical correlations (eg, over the entire lookback period) to calculate long–short offsets or diversification benefits for members' portfolios. Understandably, one of the motivations is to keep the models parsimonious with simplified assumptions.

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[1] The scope of correlation discussed in this paper refers to the correlation, or an equivalent statistical parameter of dependence, between two or more financial instruments that is shown to be reliable over the lookback period and demonstrates resilience during stressed historical or hypothetical scenarios.

[2] Some CCPs have parametric value-at-risk (VaR) models that make explicit assumptions about correlation estimates, while others use simulation approaches that assume historical temporal correlation.

To mitigate correlation uncertainty risk, CCP risk modelers often introduce add-on charges or workarounds, while continuing to retain constant or quasi-constant correlation assumptions in their models. Certain CCP models introduce additional correlation uncertainty risk charges by considering extreme correlation scenarios; this may be relevant to compliance with European Market Infrastructure Regulation (see EMIR–RTS 2012, Article 28). Other CCPs utilize stress period correlation as a conservative measure. It is unclear whether these add-on charges are appropriate or sufficient without a corresponding study of the proper "correlation regime" in which margin is calculated at any given point in time. For example, during a stress period, correlations tend to either break down (ie, from $\rho \to 0$) or converge (ie, from $\rho \to 1$) and rapidly diverge from historical values or constant correlation assumptions. Such dramatic changes may be short term or temporal but could be indicative of significant risk shocks to financial systems (eg, during 2008–9 or 2011–12).[3]

There are several limitations that relate to the use of simplified correlation assumptions in CCP margin models:

(1) correlation may be assumed constant within a period, whether it is a rolling window or an entire lookback period;

(2) the period where correlation is calculated may be too long;

(3) even if a stress period is used to determine stress correlation for higher quantile charges, the time period might still be too long to sufficiently capture sudden, severe changes in correlation regimes and their corresponding impact on risk charges.

A correlation calculated in a such way could have limitations in multiple directions: an emerging large shock would only take meaningful effect once, after a period of time (eg, several months or even a year), the latent impact is fully absorbed in the lookback window. Once the correlation has been elevated considerably because of a large shock, a higher level of correlation is maintained even though the shock no longer characterizes the state of the financial system. These effects can be seemingly mitigated by adopting a shorter observation window; however, due to there being limited observations, confidence intervals on the calculated correlation could grow so wide that they become statistically meaningless.

The first motivation of this research is to apply and understand the effects of time-varying, conditional correlation for a representative margin model in a sufficient and

[3] Historically, several correlation divergences have been observed during, for example, the 2008 financial crisis or in the periods leading up to it (eg, correlation breakdown related to different collateralized debt obligation tranches in 2005).

prudent manner to isolate the effects of correlation risk.[4] The second motivation is to understand how sensitive various portfolios are to correlation shocks (eg, with a 20% correlation shock, how much will margin change?) and to identify the tools available for mitigating correlation risks. For example, to mitigate the model risk associated with risk factor dependence structure modeling, Ivanov (2017) suggests that risk measures obtained from different correlation regimes be incorporated into the portfolio margin requirements.

Driven by these two motivations, this research paper aims to

(1) establish a time-varying correlation measure using the GARCH–DCC[5] model framework, which integrates both GARCH volatility and the DCC correlation measure in a consistent manner; and

(2) build a margin model based on GARCH–DCC and GARCH–CCC,[6] and to apply these models to study time-varying correlation effects on the margin requirements of various credit default swap (CDS) portfolios.

GARCH–DCC is a GARCH model framework with a dynamic correlation estimator, whereas GARCH–CCC is a GARCH model framework with a constant correlation estimator. The portfolios are designed to be long, short or balanced with names from various sectors (investment grade or high yield) and jurisdictions (eurozone (EU) or North America (NA)). However, this paper does not aim to identify or explain the factors that drive correlation changes through different historical periods.[7] Rather, it takes dynamic correlation as given and utilizes a coherent model framework to calculate the corresponding margin impact on portfolios during correlation shifts.

Further, this research is not an attempt to advocate for or promote a specific margin model or methodology (ie, the GARCH–DCC framework) for use by any CCP; such research would involve a different analysis, inclusive of other considerations such as efficiency and anti-procyclicality effects, as suggested by Murphy *et al* (2014, 2016). However, this research could potentially help practitioners to gain insights into the procyclical nature of correlation and establish an effective anti-procyclicality mechanism for correlation, as has been done pragmatically for volatility in CCP margin

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[4] For historical simulation, even though historical temporal correlation is used, it is not proven that future correlation will mimic historical observations. In addition, there are limitations to stressing correlation for stress testing purposes.

[5] This stands for generalized autoregressive conditional heteroscedasticity–dynamic conditional correlation.

[6] This stands for generalized autoregressive conditional heteroscedasticity–constant conditional correlation.

[7] Factors include microeconomic factors that impact individual risk factor market moves and macroeconomic risk factors that might create a contagion effect across a sector or multiple sectors.

models.[8] For example, since establishing an optimal anti-procyclicality mechanism for thousands of risk factors is difficult, perhaps a weighted average-based DCC covariance could be used as an anti-procyclicality mechanism for both volatility and correlation altogether.

The rest of the paper is organized as follows. Section 2 provides a brief review of the literature. This is followed by an introduction to multivariate distribution using t copula, GARCH–CCC and GARCH–DCC model frameworks, and how these model frameworks are used to generate multivariate loss distributions for risk measures, in Section 3. Section 4 describes data, model fitting and estimation results. Portfolio construction is also discussed herein. With our models and portfolios ready, Section 5 is dedicated to an impact analysis, comparing portfolio margin (VaR or expected shortfall (ES)) results using GARCH–DCC and GARCH–CCC. Section 6 talks briefly about our model backtesting results, and Section 7 is our conclusion. An appendix to this paper is available online.

2 LITERATURE REVIEW

Engle and Kroner (1995) proposed the GARCH–BEKK model, which attempts to model the GARCH volatility and dynamic correlation simultaneously, ie, to model the dynamic variance–covariance in the same step. Such a model suffers from dimensionality issues when the number of risk factors in a portfolio increases, and consequently it has limited applications in practice. There have also been attempts to make the multivariate GARCH model parsimonious and practical to calibrate. To make this possible, the modeling process is performed in two separate steps: (1) modeling conditional volatility and then (2) modeling conditional correlation. Bollerslev (1990) was the first to introduce a suite of CCC models carrying the assumption that conditional correlation is constant over time with only conditional volatility being dynamic.

Engle (2002) proposed a class of models based on two-step calibration but with DCC. This is what is typically referred to as the GARCH–DCC model. Due to its relative simplicity and practicality, the DCC model has been used widely in the industry as well as in academic areas. The advantage of GARCH–DCC is that the dynamics of the correlation matrix are described by a small number of parameters. It also assumes the same correlation dynamic pattern for all risk factors in a portfolio, which makes it possible to apply to a large portfolio.

However, this key advantage of DCC may also be characterized as a weakness of the model, especially when the correlation dynamics of different risk factors are

[8] Some CCPs estimate volatility based on the maximums of fast- and slow-reacting models in order to mitigate some of the issues regarding lambda choice in exponentially weighted moving average (EWMA) approaches to reach the optimal anti-procyclical effect.

so different that it is insufficient to describe them using the same parameters. There have been several attempts to address this potential weakness, such as the Block DCC (BDCC) of Billio *et al* (2003) and the cluster-based parameterization DCC of Aielli and Caporin (2014).

Since our research focuses more on inherent risk sensitivity due to correlation changes, we purposely design our portfolios to be simple and homogeneous to a certain degree such that all risk factors or assets in a portfolio can be reasonably represented by the same correlation dynamics parameters. We find the Engle (2002) GARCH–DCC is suitable for our research purposes in this case.

Since the financial crisis, several research papers have used the DCC framework to explore correlation dynamics of assets: see, for example, Kazi and Salloy (2014) for CDS correlation dynamics of G14 dealers, and Tamakoshi and Hamori (2014) for correlations of bank industry CDS indexes for the EU, the United Kingdom and the United States. Other research relating to DCC focuses on using dynamic correlation to generate risk measures such as VaR and ES via dynamic copulas such as the DCC copula. Righi and Ceretta (2012) proposed a copula–DCC–GARCH model with a Student t innovation. Manner and Reznikova (2011) gave a comparative review of time-varying copulas and concluded that the DCC copula is a well-performing, practical approach for VaR calculation. However, their review only focused on bivariate cases. There is some research that focuses more narrowly on risk measure sensitivity to correlation changes using relatively complicated portfolios. The most relevant is Isogai (2015), who studied the VaR impact due to DCC correlation changes in Japanese equity portfolios.

The contributions of this paper are as follows.

(1) This paper is the first to use a systematic approach that employs a consistent and coherent model framework (ie, GARCH–DCC-copula) to isolate the correlation effect and study its singular impact on portfolio margin.

(2) This paper is also the first to comprehensively study the magnitude and direction of correlation changes for all the major NA and EU CDS names through different historical correlation regimes using GARCH–DCC, as well as their effect on VaR and ES for portfolios.

(3) The methodology and framework established here could also be extended to other asset classes.

3 MODELING MULTIVARIATE CDS DISTRIBUTION

3.1 Univariate autoregressive GARCH model framework

The CDS spread log return over a time interval Δt (equal to one or five days) is defined as

$$X_{i,t} = \ln \frac{s_{i,t}}{s_{i,t-\Delta t}},$$

where s_t is the CDS spread observed at time t.

The autoregressive GARCH framework follows the general form:

$$\begin{aligned} X_{i,t} &= \sum_{l=1}^{L} a_{i,l} X_{i,t-l} + \varepsilon_{i,t}, \\ \varepsilon_{i,t} &= \sigma_{i,t}\epsilon_{i,t}, \\ \sigma_{i,t}^2 &= c_i + \sum_{m=1}^{q} a_{i,m}\varepsilon_{i,t-1}^2 + \sum_{n=1}^{p} \gamma_{i,n}\sigma_{i,t-1}^2, \quad i = 1, \dots, k. \end{aligned}$$

In consideration of model sufficiency and parsimony, our model implementation follows an AR(1)–GARCH(1,1) model framework for univariate modeling of CDS time series. The autoregressive AR(1) process is as follows:

$$\left.\begin{aligned} X_{i,t} &= a_i X_{i,t-1} + \varepsilon_{i,t}, \\ \varepsilon_{i,t} &= \sigma_{i,t}\epsilon_{i,t}. \end{aligned}\right\} \tag{3.1}$$

Here, a_i is the autocorrelation coefficient, $\sigma_{i,t}$ is the volatility and $\epsilon_{i,t}$ is a standardized residual with unit variance; i stands for any risk factor:

$$E[(\epsilon_{i,t})] = 0, \qquad E[(\epsilon_{i,t}^2)] = 1.$$

σ_t follows a GARCH(1,1) process:

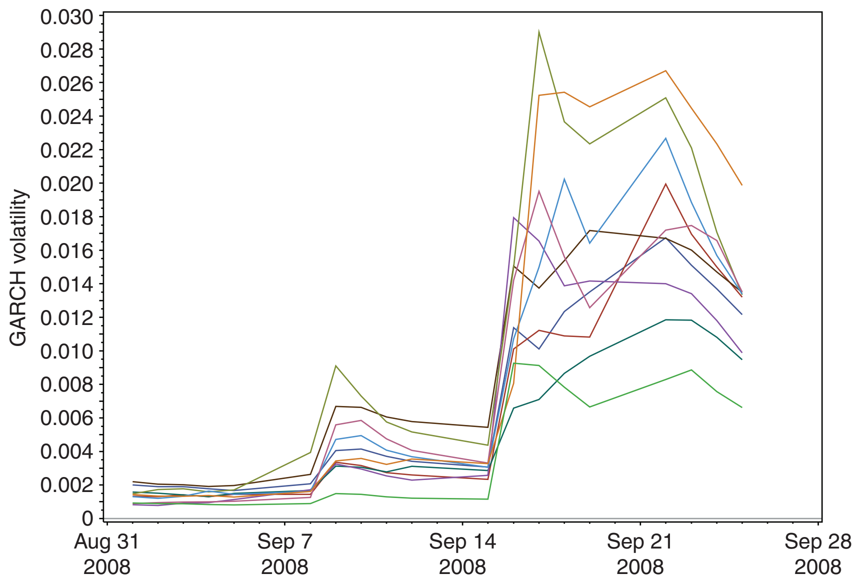
$$\sigma_{i,t}^2 = \omega_i + \alpha_i\varepsilon_{i,t-1}^2 + \gamma_i\sigma_{i,t-1}^2, \tag{3.2}$$

where ω is a dummy constant, and α_i and γ_i are two GARCH model parameters that follow Nelson and Cao (1992) restrictions, ie, $\alpha_i + \gamma_i < 1$. The conditional distribution of the standardized residuals (or innovations) is calculated as

$$\epsilon_{i,t} = \frac{\varepsilon_{i,t}}{\sigma_{i,t}}\bigg| \Psi_{i,t-1}. \tag{3.3}$$

ϵ_t is modeled by a symmetric Student t distribution, with degrees of freedom (DoF) estimated by maximum likelihood using historical time series (See Figure 1-2).

FIGURE 1 GARCH volatility for major EU financials during 2008 crisis.



3.2 Multivariate GARCH–CCC, GARCH–DCC and DCC-copula model framework

3.2.1 GARCH–CCC

Bollerslev (1990) proposes a multivariate GARCH model using time-varying conditional variances and covariance but with constant correlations. The conditional covariance matrix is given by

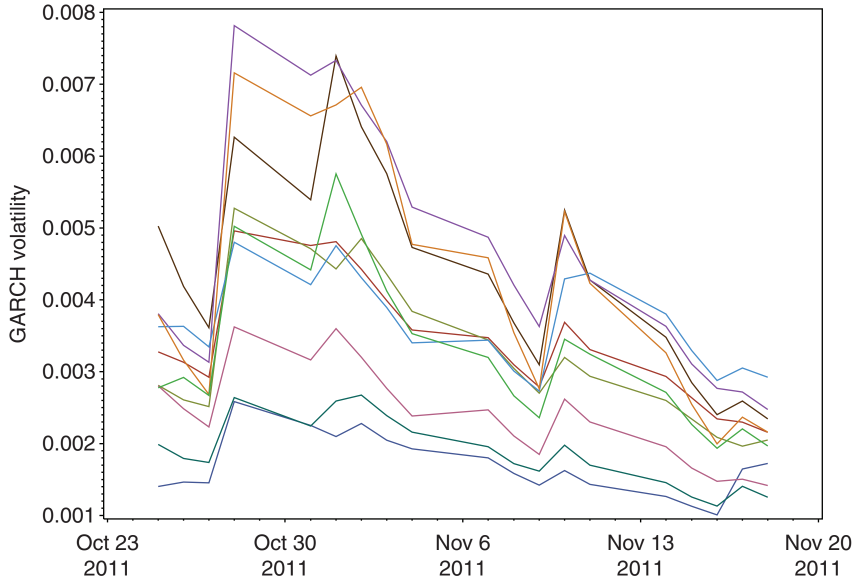
$$\boldsymbol{H}_t = \boldsymbol{D}_t \bar{\boldsymbol{R}} \boldsymbol{D}_t,$$

where $\boldsymbol{D}_t$ is an $n \times n$ stochastic diagonal matrix with elements $\sigma_{i,t}$, which follows a univariate GARCH process, and $\bar{\boldsymbol{R}}$ is an $n \times n$ time-invariant unconditional correlation matrix of the standardized error $\boldsymbol{\epsilon}_t$:

$$\begin{aligned}
\boldsymbol{D}_t &= \operatorname{diag}(\sigma_{1,t}, \sigma_{2,t}, \dots, \sigma_{n,t}),\\
\boldsymbol{\epsilon}_t &= \boldsymbol{D}_t^{-1} \boldsymbol{\varepsilon}_t,\\
\bar{\boldsymbol{Q}} &= \operatorname{cov}(\boldsymbol{\epsilon}\boldsymbol{\epsilon}_t^{\mathrm{T}}) = E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}_t^{\mathrm{T}}], \qquad (3.4)\\
\bar{\boldsymbol{R}} &= \operatorname{diag}(\bar{\boldsymbol{Q}})^{-1/2} \bar{\boldsymbol{Q}} \operatorname{diag}(\bar{\boldsymbol{Q}})^{-1/2}, \qquad (3.5)
\end{aligned}$$

where $\sigma_{i,t}^2$ follows the GARCH process defined in (3.2). $\boldsymbol{\varepsilon}_t$ is the de-autocorrelated residual defined in (3.1) above.

FIGURE 2 GARCH volatility for major NA financials during 2011 eurozone crisis.



The estimation of GARCH–CCC is computationally attractive because the correlation matrix is constant. However, this correlation estimator may be too restrictive based on empirical evidence. The model needs to be generalized by assuming the correlation matrix varies with time.

3.2.2 GARCH–DCC and DCC copula

The DCC model was introduced by Engle and Sheppard (2001). Its key design idea is that the dynamic covariance matrix $\boldsymbol{H}_t$ can be decomposed into conditional standard deviations $\boldsymbol{D}_t$ and a correlation matrix $\boldsymbol{R}_t$. Both $\boldsymbol{D}_t$ and $\boldsymbol{R}_t$ are time varying.

The conditional correlation estimator under multivariate DCC representation is

$$\boldsymbol{R}_t = \operatorname{diag}(\boldsymbol{Q}_t)^{-1/2}\boldsymbol{Q}_t \operatorname{diag}(\boldsymbol{Q}_t)^{-1/2}, \tag{3.6}$$

$$\boldsymbol{Q}_t = (1-\alpha-\beta)\bar{\boldsymbol{Q}} + \alpha\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}^{\mathrm{T}} + \beta\boldsymbol{Q}_{t-1}, \tag{3.7}$$

where $\boldsymbol{R}_t$ is the DCC correlation at time t. $\bar{\boldsymbol{Q}}$ is the unconditional covariance matrix, as defined in (3.4); α represents the dynamic term introduced by the interaction between the two innovations; and β represents the persistence term. To ensure matrix $\boldsymbol{R}_t$ is positive definite, the scalars α and β must satisfy

$$\alpha \geqslant 0, \qquad \beta \geqslant 0, \qquad \alpha + \beta < 1.$$

The DCC copula can be represented as

$$\boldsymbol{F}(\epsilon_{1t}, \epsilon_{2t}, \dots, \epsilon_{nt}) = \boldsymbol{C}(\boldsymbol{F}_1(\epsilon_{1t}), \boldsymbol{F}_2(\epsilon_{2t}), \dots, \boldsymbol{F}_n(\epsilon_{nt}); \boldsymbol{\psi}_t), \tag{3.8}$$

where $\boldsymbol{\psi}_t$ is the copula parameter including the dependence structure parameter $\boldsymbol{R}_t$ and the multivariate DoF or copula DoF $\boldsymbol{\nu}_c$.

To estimate the DCC copula, there are generally two key separate steps when using maximum loglikelihood. The loglikelihood is obtained from the following formula:

$$\ln(L(\theta)) = \sum_{t=1}^{T} \left(\ln\left[\Gamma\left(\frac{\boldsymbol{\nu}_c + \boldsymbol{n}}{2}\right)\right] - \ln\left[\Gamma\left(\frac{\boldsymbol{\nu}_c}{2}\right)\right] - \frac{\boldsymbol{n}}{2}\ln[\pi(\boldsymbol{\nu}_c - 2)] - \tfrac{1}{2}\ln[|\boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t|] - \frac{\boldsymbol{\nu}_c + \boldsymbol{n}}{2}\ln\left[1 + \frac{\boldsymbol{\epsilon}_t^{\mathrm{T}} R_t^{-1} \boldsymbol{\epsilon}_t}{\boldsymbol{\nu}_c - 2}\right]\right).$$

The parameter set θ is divided into two groups:

$$(\phi, \psi) = (\phi_1, \phi_2, \dots, \phi_n, \psi),$$

where $\phi_i = (\alpha_{1i}, \dots, \alpha_{ni}, \beta_{1i}, \dots, \beta_{ni})$ are the parameters of the univariate GARCH model for the risk factor, $i = 1, \dots, n$, and $= (\alpha, \beta, \boldsymbol{\nu}_c)$.

(1) Estimate the univariate GARCH to get ϕ[9] for calculating $\boldsymbol{D}_t$.

(2) Estimate ψ to simultaneously obtain the time-varying dependence structures $\boldsymbol{R}_t$ (ie, α and β) and $\boldsymbol{\nu}_c$ using the standardized residual from the first step.

The advantage of using a DCC copula is that the loglikelihood of the volatility and correlation can be maximized independently as long as consistence is ensured within these two steps (Engle 2002).[10]

3.2.3 *Correlation impact analysis: DCC versus CCC*

Based on (3.4)–(3.7), the two-step estimation process described in Section 3.2.2 and the assumptions of DCC (eg, no volatility spillover), if we use volatility, correlation and spot price to calculate VaR or ES using DCC and CCC at any time, one can observe that the only difference is correlation, ie, DCC versus CCC. These correlation estimators are then used to produce multivariate random draws to generate the copula. The effect on VaR or ES can then be attributed solely to the impact of the changing correlation.

[9] The autocorrelation coefficient and individual DoF are also estimated in the first step.

[10] The technical limitations of DCC have been discussed at length in the academic literature (see, for example, Caporin and McAleer 2013). One of these limitations is that the two step estimators in DCC may not be consistent because in (3.7) the matrix $\boldsymbol{Q}_t$ is not the expectation of the standardized residuals' cross-products. There have been various attempts at enhancement, but these are beyond the scope of this paper.

3.3 Loss distribution via GARCH–DCC and GARCH–CCC

3.3.1 Marginal distribution transformation

Student t copula implied variates[11] are transformed to univariate marginals based on

$$\epsilon_{i,t} = \sqrt{\frac{\nu_i - 2}{\nu_i}} t_{\nu_i}^{-1}\left(t_{\nu_c}\left(\sqrt{\frac{\boldsymbol{\nu}_c}{\gamma}} Z_{i,t}\right)\right),$$
$$Z_{i,t} \sim N(0, \boldsymbol{R}),$$
$$\gamma \sim \chi^2(\boldsymbol{\nu}_c),$$

where ν_i is DoF for the ith risk factor and $t_{\nu_i}^{-1}$ is the inverse Student t CDF. When mapping back to the marginal space from the multivariate space, the ranks of each simulation for all risk factors are preserved; this ensures rank correlation information is preserved in the marginal space.

3.3.2 Portfolio loss distribution

The scenarios are generated based on the marginal $\epsilon_{i,t}$ for a one-day return. The return is then scaled to the margin period of risk (MPOR) to estimate the portfolio loss during that period. Rab and Warnung (2011) suggested that the scaling factor for an AR(1) process follow

$$\delta(d) = \sqrt{d + 2\frac{\phi_1}{(\phi_1 - 1)^2}(d(1 - \phi_1) + \phi_1^d - 1)}.$$

Here, d is the scaling period, and ϕ_1 is first lag autocorrelation coefficient: α_i in our case.

Taking d as our five-day MPOR,[12] and ignoring higher orders,[13] we obtain the following five-day loss equation:

$$L_{i,\text{5-day}} = \sqrt{5 + 8\alpha_i + 6\alpha_i^2}\,\sigma_{i,t}\epsilon_{i,t},$$

where α_i is the autocorrelation coefficient. Simulated spreads are calculated based on these scenarios. CDS positions in a portfolio are repriced using the International Swaps and Derivatives Association CDS standard model. VaR and ES at 99% or other, higher quantiles are calculated out of all the simulated paths.

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[11] Refer to the online appendix for more details on generating t copula implied variates.
[12] Typical CDS liquidation horizon.
[13] CDS SN return autocorrelation is typically less than 0.2, and a higher order can be omitted without a material impact. It is assumed that autocorrelation remains the same during the liquidation period.

4 MODEL FITTING AND ESTIMATION RESULTS

4.1 CDS time series[14]

We consider daily observations of five-year CDS par spreads (or CDS spreads for simplicity) from January 1, 2008 to August 9, 2017, where the reference names (obligors) are from either NA or the EU and are publicly traded and relatively liquid.

Our sample covers the period before and during the subprime crisis (early 2008) as well as other key market stress periods (eg, the eurozone sovereign debt crisis, 2011–12; the taper tantrum, summer 2013; and US post-election market shock). Each SN whose data is available for the full sample period has a total of 2472 daily observations. These SNs are selected based on the data and compared with the CDS single-name voluntary clearing activity study performed by Burt Porter of the US Securities and Exchange Commission's Division of Economics and Risk Analysis (DERA) to make sure these names have sufficient liquidity (see Porter 2015). The data set spans a reasonably long period (2008–17) and covers all key market events since 2008 to provide a meaningful basis for the statistical analysis of CDS spreads, volatility and – most importantly to this research – various correlation regimes.

There are 188 NA SNs and 145 EU SNs across different sectors[15] (see Tables A1 and A2 in the online appendix).[16] The statistics of five-year CDS spreads and log returns are shown in Tables A3–A6.

As these tables show, the means are all very small, close to 0, which indicates a centered mean-reverting process over a long time period. In addition, all the log return time series exhibit a relatively small skewness but a relatively large kurtosis. These observations are consistent with prior literature about CDS return properties (Cont and Kan 2011a,b).

The behavior of the CDS spreads is described by various volatile periods. In particular, the CDS spreads are substantially larger and more volatile during the 2008–9 subprime crisis and the 2011–12 eurozone debt crisis. It is also interesting to note that some of the NA financials have higher spreads than their EU counterparts during the eurozone debt crisis.

4.2 Portfolio composition

Table 1 shows the types of portfolios used for this study. The main types are intuitively directional (net short or net long) and balanced portfolios, simply because

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[14] All data used in this paper is from Intercontinental Exchange (ICE) credit market infrastructure (CMA).

[15] Based on the Markit industry/sector definition, the majority of these names are also constituents of the CDX.NA.IG and iTraxx Main CDS indexes.

[16] All figures and tables that are not shown in the body of paper (identified by an "A" in their numbering) are in the online appendix (Section 9.2).

correlation-induced offsetting and diversification effects can be most easily seen in these portfolios. We define a portfolio as directional if it meets the following criteria:

$$\frac{|\text{net notional}|}{\text{gross notional}} > 80\%.$$

We define a portfolio as balanced if

$$\frac{|\text{net notional}|}{\text{gross notional}} < 20\%.$$

4.3 Univariate model estimates (AR–GARCH–Student *t*)

Other salient statistical properties of CDS log return time series include

- significant autocorrelation (especially positive serial correlation during stress periods),
- fat tail distribution of standardized residuals,
- volatility clustering,
- increasing comovements across names during stress periods and
- tail dependence.

We have independently verified these properties but do not provide all the details of our results due to length limitations; we refer the interested reader to a prior study by Cont and Kan (2011b).

We only provide results for AR(1) and DoF (see Figures 3–6) since they are directly related to the modeling framework we employ for this research.

Marginal DoF is estimated using maximum likelihood with constraint $\nu_i > 2$. This constraint is to ensure that the t distribution has a well-defined variance.

The autocorrelation and tail analysis (based on a Student t distribution) are consistent with prior literature (Cont and Kan 2011b). The first lag of autocorrelation AR(1) exhibits significant positive values, especially during stress periods.[17]

Tables 2 and 3 show AR and GARCH parameters for the selected NA and EU names. These parameters are calibrated based on (3.1) and (3.2). All of the model parameters are statistically significant based on the calibration results.

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[17] The AR(1) in this paper is calibrated based on the whole historical time series (ie, approximately ten years). The AR(1) results are also available for each name during stress periods (eg, during 2008–9). This paper chooses to use historic average values to simplify model assumptions because our research focus is correlation.

TABLE 1 Portfolio composition.

Portfolios	Description	Risk positions	No. of portfolios
Directional portfolios (short)	Net short notional/gross notional $>$ 80%	Short CDS protection	100+
Directional portfolios (long)	Net long notional/gross notional $>$ 80%	Long CDS protection	80+
Balanced portfolios	Net Notional ¡ 20% of gross notional	Both short or long CDS protections	100+
Financial portfolios	SNs from banking, financial services, insurance and others	Both short or long CDS protections	120+
Nonfinancial portfolios	SNs from other sectors defined by Markit	Both short or long CDS protections	80+
Blended portfolios	SNs from both financial and nonfinancial sectors	Both short or long CDS protections	10+

FIGURE 3 AR(1) distribution for NA names.

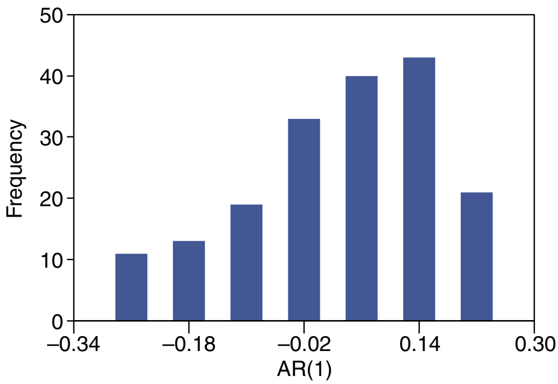
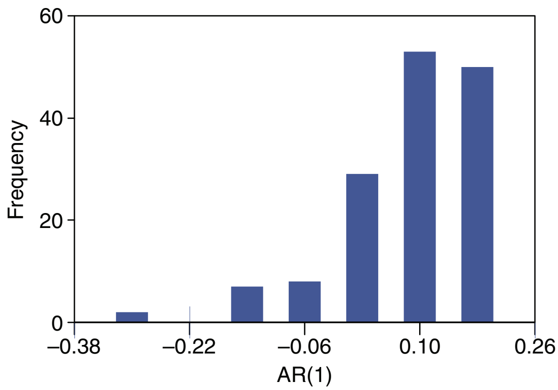


FIGURE 4 AR(1) distribution for EU names.



As summarized above, the majority of these names (NA or EU) have significant positive autocorrelation coefficients (from 0.1 to 0.2). The ARCH coefficient typically has about a 10–20% effect, while the GARCH coefficient that carries the persistence in variance typically has about a 80–90% effect.

FIGURE 5 DoF distribution for NA names.

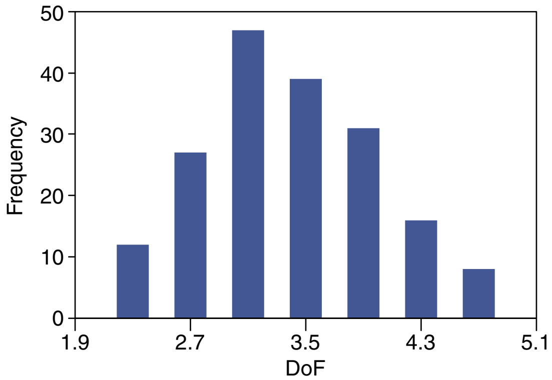
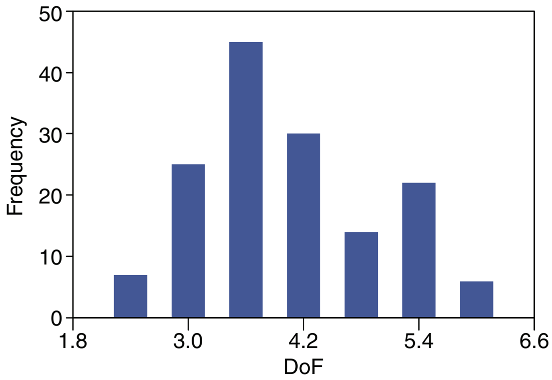


FIGURE 6 DoF distribution for EU names.



4.4 Multivariate model estimates (GARCH–DCC-t-copula)

Following univariate GARCH parameterization, the multivariate GARCH–CCC only adds unconditional correlations $\bar{\boldsymbol{R}}$ as additional parameters to risk factors in a portfolio. This $\bar{\boldsymbol{R}}$ is calculated using (3.5).

The multivariate DCC t copula is the second-step calibration described in Section 3.2.2. Tables 4 and 5 show the DCC-t-copula parameterization based on (3.5)–(3.7). The sample portfolios selected are NA and EU portfolios from different industry sectors. These portfolios consist primarily of the CDS SNs in Section 4.1.

TABLE 2 Univariate AR(1)–GARCH(1,1) estimate (sample: NA SN).

Name	Parameter	Estimate	Standard error	t value	Pr > $\|t\|$
Dow Chemical	α	0.12			
	v	3.29	0.24	13.36	<0.0001
	α_1	0.13	0.01	18.50	<0.0001
	γ_1	0.86	0.01	147.18	<0.0001
Campbell's	α	−0.03			
	v	3.25	0.26	12.74	<0.0001
	α_1	0.16	0.01	9.96	<0.0001
	γ_1	0.81	0.02	53.39	<0.0001
CBS	α	0.15			
	v	3.44	0.26	12.85	<0.0001
	α_1	0.18	0.01	20.03	<0.0001
	γ_1	0.81	0.01	66.74	<0.0001
Devon Energy	α	0.20			
	v	4.26	0.34	11.35	<0.0001
	α_1	0.08	0.01	10.71	<0.0001
	γ_1	0.91	0.01	249.56	<0.0001
Morgan Stanley	α	0.21			
	v	4.05	0.35	11.55	<0.0001
	α_1	0.23	0.01	27.44	<0.0001
	γ_1	0.75	0.01	52.33	<0.0001
Aetna	α	0.05			
	v	3.87	0.33	11.22	<0.0001
	α_1	0.11	0.01	15.02	<0.0001
	γ_1	0.88	0.01	114.69	<0.0001
Caterpillar	α	0.19			
	v	3.40	0.25	13.05	<0.0001
	α_1	0.21	0.01	14.79	<0.0001
	γ_1	0.78	0.01	62.01	<0.0001
Dell	α	0.17			
	v	3.20	0.24	13.22	<0.0001
	α_1	0.20	0.01	17.09	<0.0001
	γ_1	0.78	0.01	60.51	<0.0001
AT&T	α	0.13			
	v	4.32	0.42	10.29	<0.0001
	α_1	0.11	0.01	12.19	<0.0001
	γ_1	0.86	0.01	86.29	<0.0001
American Electric	α	−0.09			
	v	3.48	0.30	11.71	<0.0001
	α_1	0.11	0.01	11.74	<0.0001
	γ_1	0.86	0.01	75.20	<0.0001

TABLE 3 Univariate AR(1)–GARCH(1,1) estimate (sample: EU SN).

Name	Parameter	Estimate	Standard error	t value	Pr > $\|t\|$
Glencore AG	α	0.18			
	v	4.27	0.38	11.11	<0.0001
	α_1	0.16	0.01	16.78	<0.0001
	γ_1	0.82	0.01	95.44	<0.0001
Peugeot SA	α	0.18			
	v	5.89	0.67	8.76	<0.0001
	α_1	0.13	0.01	12.59	<0.0001
	γ_1	0.84	0.01	67.51	<0.0001
Tesco	α	0.15			
	v	3.49	0.3	12.86	<0.0001
	α_1	0.07	0.01	20.09	<0.0001
	γ_1	0.92	0.01	404.78	<0.0001
BP plc	α				
	v	3.26	0.25	12.80	<0.0001
	α_1	0.15	0.01	19.70	<0.0001
	γ_1	0.82	0.01	111.37	<0.0001
Barclays Bank	α	0.16			
	v	5.04	0.51	9.93	<0.0001
	α_1	0.12	0.01	12.13	<0.0001
	γ_1	0.85	0.01	70.86	<0.0001
Bayer AG	α	0.03			
	v	3.27	0.27	12.76	<0.0001
	α_1	0.13	0.01	15.19	<0.0001
	γ_1	0.86	0.01	117.73	<0.0001
Siemens	α	−0.04			
	v	3.38	0.27	12.55	<0.0001
	α_1	0.08	0.01	15.09	<0.0001
	γ_1	0.91	0.01	202.20	<0.0001
Ericsson	α	0.03			
	v	3.26	0.25	13.21	<0.0001
	α_1	0.16	0.01	16.86	<0.0001
	γ_1	0.75	0.01	59.92	<0.0001
OTE	α	0.20			
	v	3.22	0.24	13.47	<0.0001
	α_1	0.15	0.01	15.88	<0.0001
	γ_1	0.80	0.01	66.34	<0.0001
EDP	α	0.15			
	v	3.92	0.34	11.55	<0.0001
	α_1	0.14	0.01	13.88	<0.0001
	γ_1	0.84	0.01	83.27	<0.0001

TABLE 4 DCC-t-copula parameters (NA sector portfolios).

Parameters	α	β	ν_c
Portfolio 1 (NA FIN)	0.014955 (0.001091)	0.981496 (0.001569)	4.41 (0.10)
Portfolio 2 (NA FIN)	0.008120 (0.000702)	0.988003 (0.001232)	3.90 (0.07)
Portfolio 3 (NA BM)	0.010877 (0.002128)	0.979450 (0.005659)	4.20 (0.08)
Portfolio 4 (NA CG)	0.014933 (0.002757)	0.959899 (0.010941)	4.29 (0.09)
Portfolio 5 (NA CS)	0.043236 (0.005367)	0.727260 (0.055457)	4.25 (0.09)
Portfolio 6 (NA EG)	0.010048 (0.001107)	0.977925 (0.003230)	3.86 (0.07)
Portfolio 7 (NA IND)	0.009390 (0.000959)	0.987316 (0.001591)	4.13 (0.09)
Portfolio 8 (NA TEL)	0.007551 (0.001122)	0.988665 (0.002123)	4.12 (0.08)
Portfolio 9 (NA UTL)	0.009285 (0.001090)	0.982745 (0.00258)	4.46 (0.10)

The DCC t copula introduces three more parameters – α, β and $\boldsymbol{\nu}_c$ – for each portfolio. Together with unconditional correlations $\bar{\boldsymbol{R}}$, the parameters α, β and $\boldsymbol{\nu}_c$ describe the time-varying dependence structure including DCCs for all risk factors in a portfolio. Each risk factor also has three parameters from univariate GARCH(1,1) calibration, one parameter from autoregressive AR(1) calibration and one parameter from residual distribution calibration.[18] For example, a portfolio of ten names has fifty univariate parameters, forty-five unconditional correlation coefficients and three DCC-t-copula parameters.

From the calibration results, it can be seen that α, which introduces dynamic correlations via residual interactions, is typically a very small term compared with β, which carries the persistence of prior states. However, large residuals during volatile times could cause "jumps" in correlations. This is consistent with prior research using DCC on financial time series (Kazi and Salloy 2014). On the one hand, $\alpha > 0.02$ typically causes DCC to oscillate frequently around the unconditional mean, which indicates a rather swift change in correlation regime. On the other hand, $\alpha < 0.01$ typically causes persistence in a correlation regime for a longer period of

[18] We use standard GARCH and symmetric Student t distributions. More parameters are needed if a different variety of GARCH is employed or a nonstandard Student t or other distribution is used.

TABLE 5 DCC-t-copula parameters (EU sector portfolios).

Parameters	α	β	ν_c
Portfolio 1 (EU FIN)	0.062086 (0.006903)	0.707639 (0.049584)	4.42 (0.10)
Portfolio 2 (EU FIN)	0.007124 (0.000641)	0.989811 (0.001140)	4.96 (0.13)
Portfolio 3 (EU BM)	0.055899 (0.006602)	0.734504 (0.051313)	3.97 (0.08)
Portfolio 4 (EU CG)	0.030835 (0.002942)	0.917098 (0.010348)	4.34 (0.09)
Portfolio 5 (EU CS)	0.035714 (0.004623)	0.840220 (0.029176)	3.81 (0.07)
Portfolio 6 (EU EG)	0.051828 (0.005218)	0.730298 (0.039669)	3.88 (0.07)
Portfolio 7 (EU IND)	0.042043 (0.005881)	0.830309 (0.038569)	4.43 (0.10)
Portfolio 8 (EU TEL)	0.047778 (0.005184)	0.760685 (0.038763)	3.80 (0.08)
Portfolio 9 (EU UTL)	0.055837 (0.005964)	0.806025 (0.029850)	4.04 (0.08)

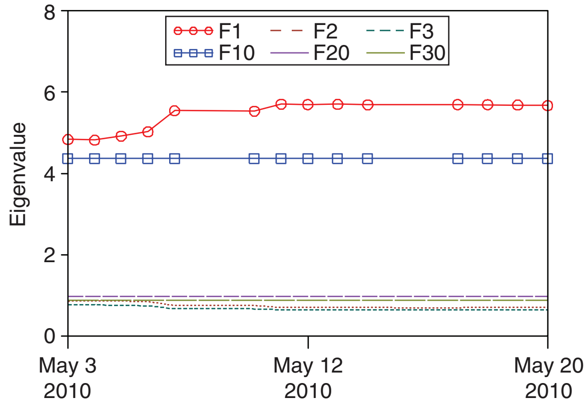
time. For EU names, the financials sector (excluding major EU financial institutions, eg, insurers, financial service firms, etc) is the only one that has $\alpha < 0.01$. The rest of the sectors follow faster correlation regime-change patterns. NA names present an interesting contrast when compared with EU names: all of the sectors except consumer services have $\alpha \lesssim 0.01$. All DCC parameters are statistically significant, and DCC parameterization provides a better model fit than its CCC counterpart when we compare statistical measures such as Akaike information criteria (AIC) or Schwarz criterion (SBC) scores.

The multivariate $\boldsymbol{\nu}_c$ parameters are typically aligned with the overall average of the individual DoFs of the marginals. Due to the complexity of estimating $\boldsymbol{\nu}_c$, we do not recommend calibrating this parameter frequently. To be practical and conservative, our research fixes this parameter at $\boldsymbol{\nu}_c = 3$ for all portfolios.

4.5 Principle component analysis (PCA) and correlation intensity

One of the main purposes of this research is to understand quantitatively how portfolio margin responds to correlations among risk factors in a portfolio. We could use VaR or ES to measure portfolio margin or tail risk; however, it is rather difficult to use a time-varying 10×10 or 50×50 correlation matrix R_t to quantify portfolio

FIGURE 7 Largest eigenvalue from DCC correlation matrix (circle) and largest eigenvalue from historical unconditional correlation (square): NA basic materials in eurozone crisis.



In the chart, F1 (circle) represents the largest eigenvalue for a sector portfolio. Similarly, F2 and F3 are the second- and third-largest eigenvalues for the portfolio. F1, F2 and F3 are calculated from the DCC correlation matrix and evolve through the selected time window. (These time windows are selected either because certain major market events occurred (eg, the eurozone crisis, Brexit, etc) or because correlations were elevated when compared with the historical mean.) F10 (square), F20 and F30 represent the top three eigenvalues from the historical unconditional correlation matrix.

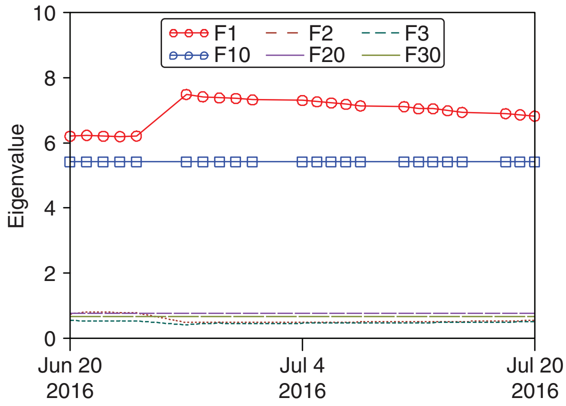
correlation changes. Therefore, we need a statistically significant and reliable proxy to represent the correlation intensity of a portfolio, as risk factors in the portfolio go through market events and individual idiosyncratic events.[19] The largest eigenvalue of the correlation matrix for a given portfolio can be used to represent the correlation intensity of the portfolio if it is proved to be statistically meaningful throughout the study period. Isogai (2015) used this approach to study the impact of correlation changes due to the Lehman default and the Great East Japan Earthquake on the Japanese equity market. This paper takes a similar approach, using the largest eigenvalue to measure portfolio correlation changes.

4.6 PCA for each sector portfolio

Figures 7 and 8 depict the top three eigenvalues for each selected NA or EU sector portfolio.

[19] The changes in a correlation matrix can be represented by two components: correlation intensity (eigenvalues) and direction (eigenvectors). Correlation intensity is used here to represent any dynamic magnitude changes, assuming that intensity has a larger influence than direction on portfolio risk.

FIGURE 8 Largest eigenvalue from DCC correlation matrix (circle) and largest eigenvalue from historical unconditional correlation (square): EU industrial in eurozone crisis.



See note to Figure 7.

The largest eigenvalue is dominant for all sample sector portfolios. It is intuitively clear that using the largest eigenvalue is a reasonable approximation for portfolio correlation intensity. Isogai (2015) uses the Marčenko–Pastur distribution (Marčenko and Pastur 1967) and the Tracy–Widom (TW) distribution at a 99% quantile to determine if the largest eigenvalue is significant. We also performed this significance test on all eigenvalues based on the TW distribution at a 99% quantile (Tracy and Widom 1993, 1994; see also Johnstone 2001, 2009; Bejan 2005) and found that almost all the eigenvalues are significant except for the last one or two. Next, we compared the changes of the largest eigenvalues with the matrix 2-norm of the difference between the correlation matrixes before and after the change. The relative change of the largest eigenvalues between the two correlation matrixes is defined as follows:

$$\delta_{\max - \lambda} = \frac{\lambda_{\max}^{1} - \lambda_{\max}^{2}}{\lambda_{\max}^{2}}.$$

The relative change of the matrix 2-norm is defined as follows:

$$\delta_{2-\text{norm}} = \frac{\|C - C_0\|_2}{\|C_0\|_2}.$$

Here, C and C_0 are the correlation matrixes after and before the correlation change, respectively.

The 2-norm of the difference between the correlation matrixes has been used previously to measure correlation change (Münnix *et al* 2010; Vershynin 2012). In

Table A8 in the online appendix, we can see that the largest eigenvalue change represents the overall 2-norm of the difference correlation matrix reasonably well (<5%). We see similar results for other sector portfolios as well. In addition, Conlon *et al* (2009) pointed out that the largest eigenvalue is even more dominant during stressed market conditions than during normal market conditions. From this point on, for simplicity, we will use the largest eigenvalue change of a correlation matrix to represent the portfolio correlation intensity change.

5 CORRELATION IMPACT STUDY

From Section 4.5, we conclude that it is a reasonable approximation to use the largest eigenvalue to quantify the changes in the portfolio correlations. Now we are ready to tackle our second motivation from the introduction. We follow the steps below to conduct our study.

(1) Identify different historical market periods (eg, the eurozone debt crisis, Brexit, etc) for different portfolios in different regions (NA or EU) by observing the correlation regime change via DCC correlations.

(2) Run GARCH–CCC first, then GARCH–DCC, throughout this period on each selected portfolio to calculate the VaR[20] and ES at a 99% quantile. The differences in VaR and ES from both methods are recorded for each day of the period.

(3) Calculate five-day averages of the VaR or ES differences obtained from step (2) to minimize statistical uncertainty since the start of the correlation "jump".[21]

(4) Run both models on different portfolios constructed based on directionality (long/short/balanced) and market sectors, as described in Section 4.2.

(5) Draw conclusions with respect to VaR or ES sensitivity to correlation changes, represented by the largest eigenvalue changes.

5.1 EU financial portfolio

The sample portfolio shown in Figure 9 consists of ten major EU financial names. It can be seen that correlations among the names in this portfolio shifted from a historical correlation regime to a high correlation regime during the initial stages

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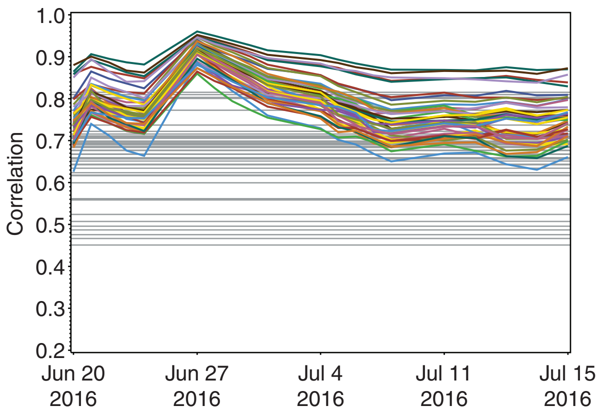
[20] The words "VaR" and "margin" are used interchangeably in this section.

[21] A correlation jump typically means more than a 10% shock within the liquidation horizon. If there is no correlation jump because the correlation regime change is gradual, we take the average of the correlation changes since the beginning of the period.

FIGURE 9 DCC correlations of ten major EU financials during Brexit.



The gray lines in the chart represent unconditional correlations among these names. The shocked lines represent DCC conditional correlations i within the time window around Brexit. The DCC correlations jumped on June 23, 2016; peaked on June 27, 2016; and then declined gradually but still maintained an elevated level.

of Brexit (June 23, 2016). The correlation jumped quickly within days as this market event began to unfold. We can also observe that the bandwidth of correlations quickly narrowed. The bandwidth gradually became wider after the market shock and maintained a higher level into the future. Figures 11–14 show the VaR impact (DCC versus CCC) for EU financial portfolios as a result of correlation changes.

5.2 NA financial portfolio

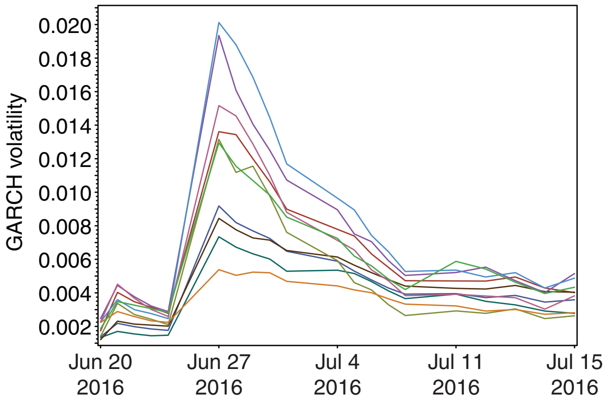
Figures 15–20 show similar results of correlation changes and their impacts on long, short and balanced NA financial portfolios.

5.3 VaR or ES impact due to breakdown of correlation regime

In some cases, the correlation of a portfolio may be significantly lower than its unconditional historical average. In other words, correlation could break down due to certain market events. As a result, using the historical average could significantly underestimate the margin or ES for balanced portfolios.

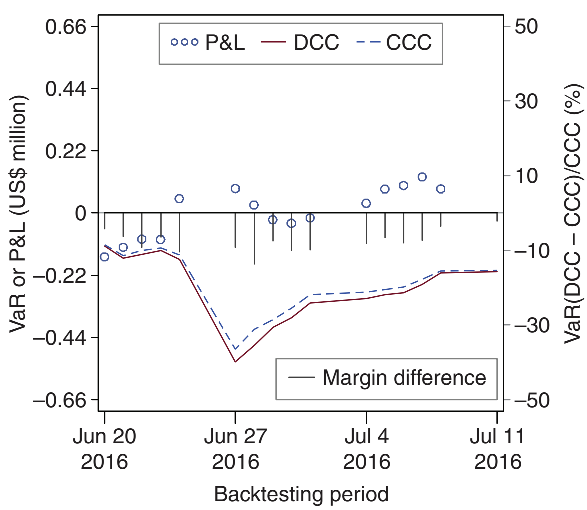
We select an NA energy portfolio to illustrate this point more clearly (see Figures 21–23 for correlation and volatility changes). We choose a period between June 15, 2014 and July 10, 2014, when oil prices started falling due to a significant increase in oil production in the United States and demand declined in emerging countries such as China.

FIGURE 10 GARCH volatility for EU financials within the time window around Brexit.



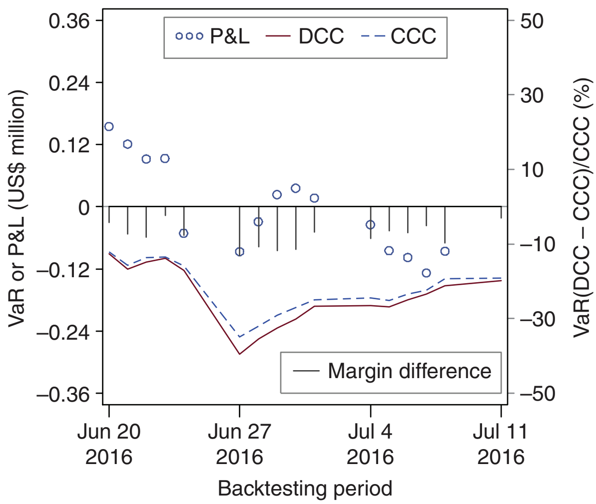
The volatility jumped on June 23, 2016; peaked on June 27, 2016; and then declined quickly compared with the DCC correlations.

FIGURE 11 GARCH–DCC (solid line) versus GARCH–CCC (dotted line) margin levels within the time window around Brexit (EU financial net short).



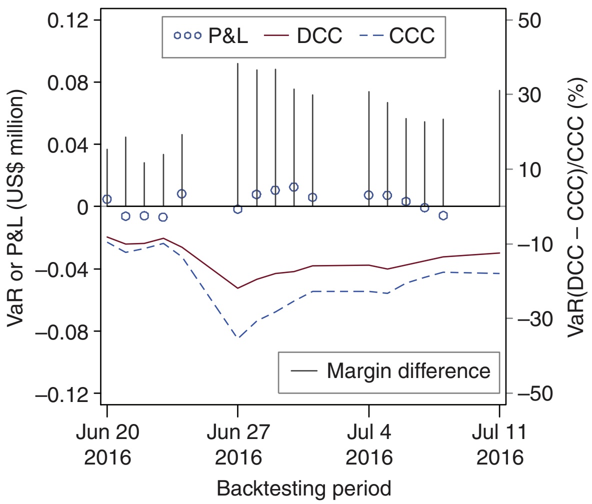
Circles represent the profit-and-loss of the portfolio at each time point. The bar (pointing downward in this chart) is the relative difference between DCC and CCC. The fact that the bar is pointing downward means DCC incurs more margin charge than CCC.

FIGURE 12 A net long portfolio (EU financial).



The directional long portfolio shows a similar result to the net short directional portfolio. The relative difference between DCC and CCC persists throughout the period.

FIGURE 13 Margin impact for a balanced portfolio (EU financial).



It can be seen that the impact due to the correlation regime change is much more sizable than for the directional portfolio (approximately 25–30%).

FIGURE 14 Another balanced portfolio that shows a similar result (EU financial).

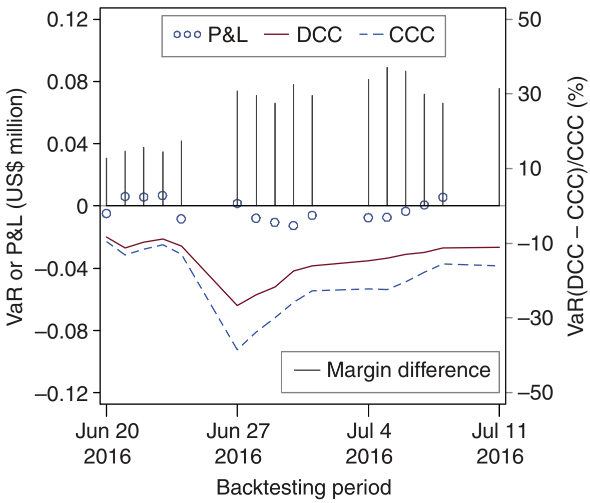
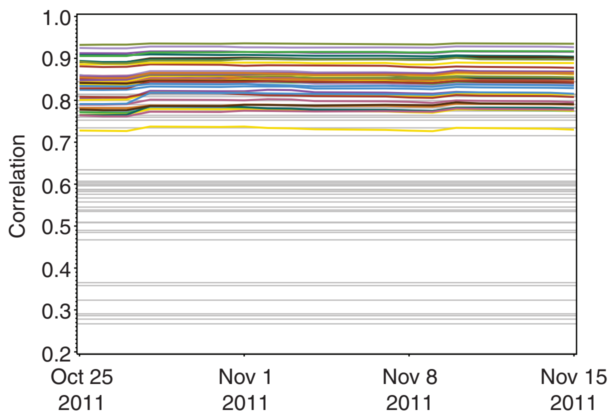


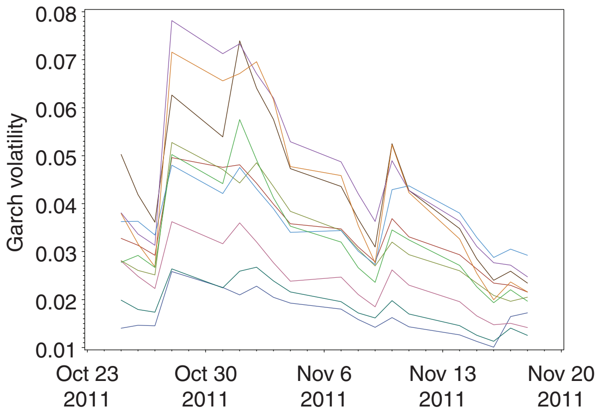
FIGURE 15 DCC correlations of ten NA financials during the eurozone debt crisis.



The correlation level remains elevated even after the volatility returns to a normal level (see Figure 16).

Based on Figures 24–26, VaR/ES could be sizably underestimated for balanced portfolios in a correlation breakdown scenario (the examples shown in previous sections mainly illustrate margin/ES underestimation for directional portfolios and

FIGURE 16 GARCH volatility for NA financials during the eurozone debt crisis.



The volatility jumped on October 25, 2011; then declined quickly; and then returned to its prior level within a month.

overestimation for balanced portfolios when they are in an elevated correlation regime).

5.4 Other sector portfolios

We conduct a similar analysis on other sectors as well. These lead to similar results, and similar conclusions can be drawn for portfolios – long, short or balanced – with names from investment grade or high yield, EU or NA.

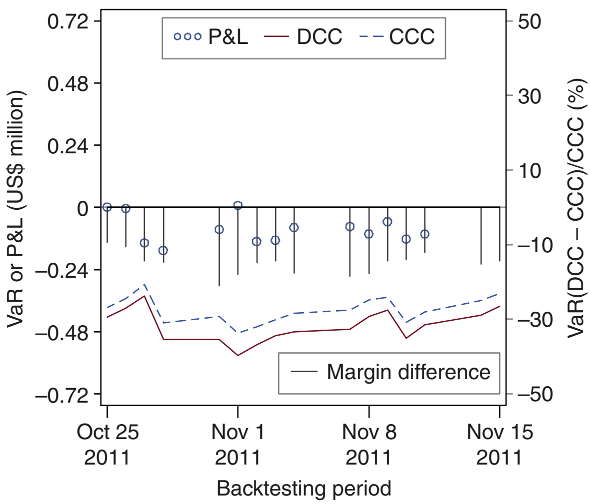
5.5 Margin impact calculations

This section quantifies the margin or ES impact due to correlation changes (ie, maximum eigenvalue changes) on different portfolios from previous sections. The calculation steps are as follows.

(1) Quantify margin impact by taking average margin differences between DCC and CCC during the five-day period after the market event occurs. If the correlation level is already in an elevated environment, the start of the five-day period is taken as the beginning of the period.

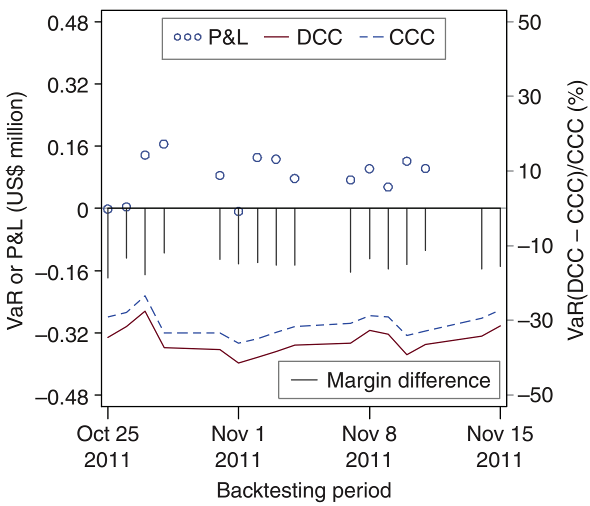
(2) If multiple periods are selected for a portfolio for margin impact calculation, the impacts from different periods are averaged (eg, a portfolio is selected for two periods, the eurozone crisis and the 2008 subprime crisis).

FIGURE 17 Margin impact for a net short portfolio (NA financial).



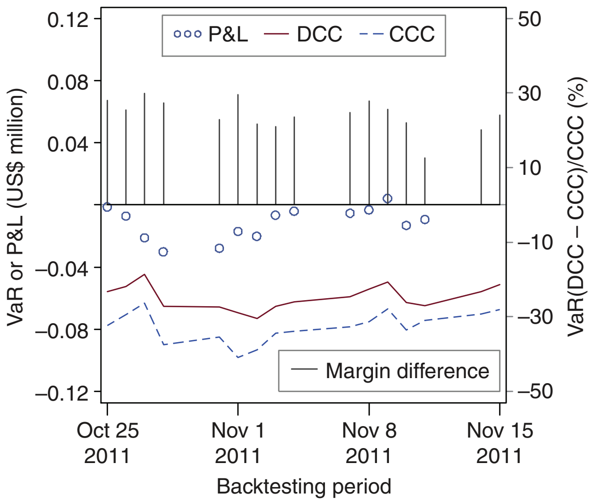
Circles represent the profit-and-loss of the portfolio at each time point. The bar (pointing downward in this chart) is the relative difference between DCC and CCC. This indicates that DCC incurs more of a margin charge than CCC.

FIGURE 18 Margin impact for a net long portfolio (NA financial).



This exhibits a similar result to the net short portfolio. In both cases, the differences between DCC and CCC are consistent throughout the period.

FIGURE 19 Margin impact for a balanced portfolio (NA financial).



This exhibits a similar result to the EU financial portfolio. The impact due to the correlation regime change is much more sizable than for the directional portfolio (approximately 25–30%).

FIGURE 20 Margin impact for another balanced portfolio that shows a similar result to Figure 19 (NA financial).

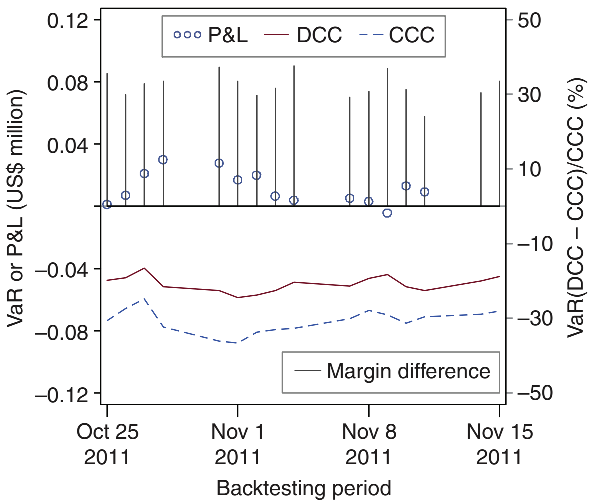
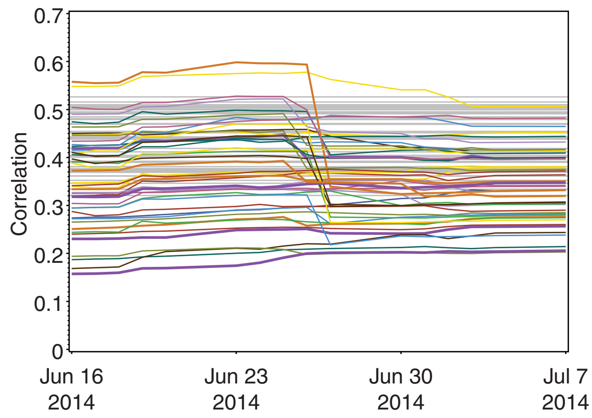
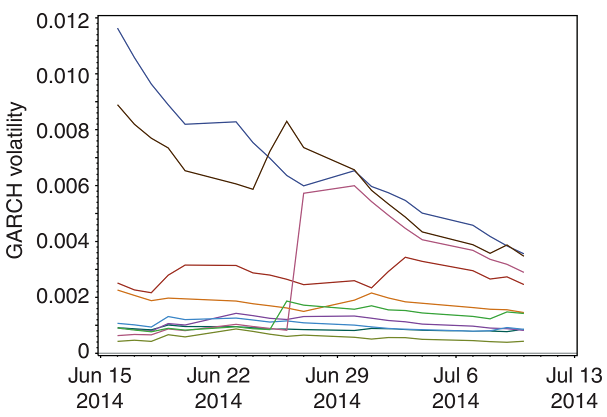


FIGURE 21 DCC correlations of NA energy names during the oil price decline.



The correlation among these names broke down, dropping sharply on June 27, 2014. Compared with historical average unconditional correlations, the DCC correlations remained at a reduced level for the next six months during the price decline, from over US$100 to US$40 per barrel.

FIGURE 22 GARCH volatility of NA energy names during the oil price decline.

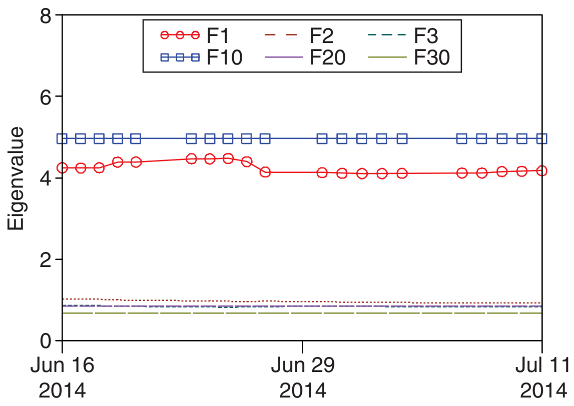


The volatility jumped on June 27, 2014, but not in a significant way: the exception to this is Devon Energy, whose volatility jumped almost six times.

(3) Calculate the impacts for different types of portfolios, directional (net short, net long) and balanced.

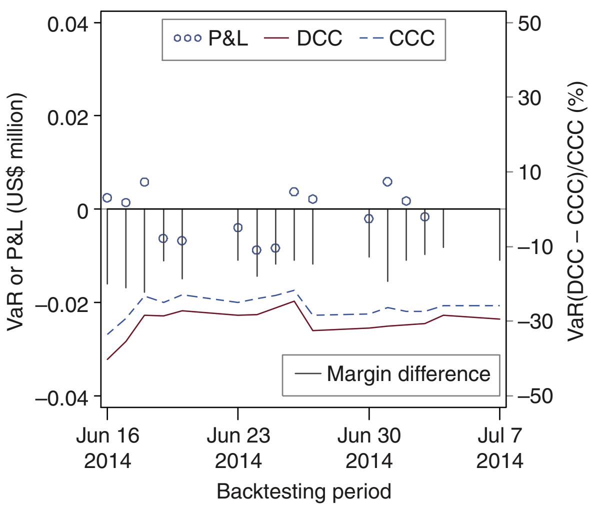
(4) Calculate the maximum eigenvalue of each correlation matrix out of the CCC

FIGURE 23 Largest eigenvalue from DCC correlation matrix (circle) and largest eigenvalue from historical unconditional correlation matrix (square): NA energy during the oil price decline in mid-2014.



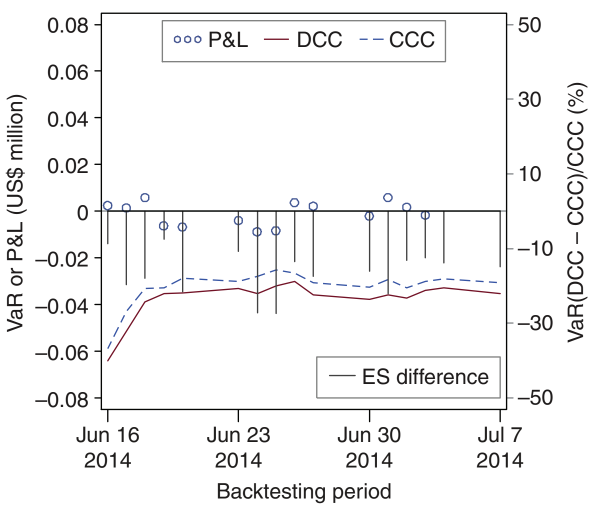
The largest eigenvalue from DCC shifted about 16% below the largest eigenvalue from CCC, indicating movement to a lower correlation regime.

FIGURE 24 Margin impact for the balanced energy portfolio.



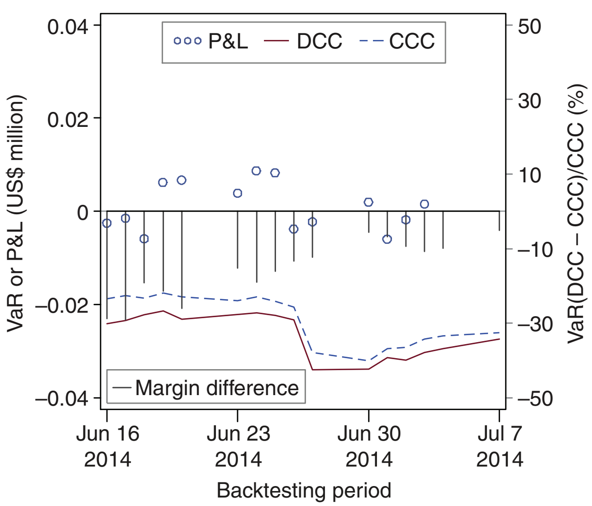
Compared with the elevated correlation regime, the lower correlation regime causes CCC to underestimate the margin. The DCC margin exceeds the CCC margin by about 20%.

FIGURE 25 ES impact for the balanced energy portfolio.



Compared with the elevated correlation regime, the lower correlation regime causes CCC to underestimate the ES. The DCC ES exceeds the CCC ES by about 22%.

FIGURE 26 Margin impact for another balanced energy portfolio.



The DCC margin exceeds the CCC margin by about 20%.

TABLE 6 VaR and ES impacts versus maximum eigenvalue change for NA names.

		Net short		Net long		Balanced	
	$\delta_{\max-\lambda}$ (%)	δ_{VaR} (%)	δ_{ES} (%)	δ_{VaR} (%)	δ_{ES} (%)	δ_{VaR} (%)	δ_{ES} (%)
NA financial I	33	15	15	15	15	29	28
NA financial II	35	16	15	16	15	22	22
NA basic materials	20	12	15	12	14	23	23
NA consumer goods	20	13	15	11	11	20	21
NA consumer services	46	16	17	18	17	24	26
NA energy	27	15	16	12	14	28	28
NA industrial	30	13	14	13	13	18	17
NA telecommunication	35	13	16	17	15	16	16
NA utilities	15	12	11	12	11	27	28

$\delta_{\max-\lambda}$ is calculated as the difference between the maximum eigenvalue from the DCC correlation matrix and the maximum eigenvalue from the CCC unconditional correlation matrix. δ_{VaR} and δ_{ES} are the differences between the margin or ES calculation using DCC and CCC, respectively.

or the DCC; the differences between these two eigenvalues are calculated and averaged over the same five-day period.

(5) Establish sensitivity relationships between the margin impact and the corresponding maximum eigenvalue changes for each portfolio for each sector.

See Tables 6 and 7 for margin impact versus maximum eigenvalue change for NA and EU portfolios, respectively.

5.6 Margin impact analysis for NA and EU SN portfolios

From Tables 6 and 7, several key observations can be made.

(1) The margin impact results present quite consistent differences in terms of the VaR or ES impact between directional and balanced portfolios for both NA or EU portfolios.

(2) On average, the correlation change for EU portfolios is about 25% for all selected periods and all sectors. As a result, the average margin impact across all sectors is about 14% for VaR and 14% for ES related to net short portfolios, and 13% for VaR and 13% for ES related to net long portfolios. In contrast, for balanced portfolios with the same correlation change (ie, a 25% correlation change as represented by the maximum eigenvalue change), the average VaR and ES impacts across all sectors are 24% and 23%, respectively.

TABLE 7 VaR and ES impacts versus maximum eigenvalue change for EU names.

		Net short		Net long		Balanced	
	$\delta_{\max-\lambda}$ (%)	δ_{VaR} (%)	δ_{ES} (%)	δ_{VaR} (%)	δ_{ES} (%)	δ_{VaR} (%)	δ_{ES} (%)
EU financial I	28	10	13	11	10	34	34
EU financial II	19	12	12	12	11	25	26
EU basic materials	20	12	15	9	14	22	23
EU consumer goods	20	12	13	11	11	20	20
EU consumer services	25	14	15	12	12	21	19
EU energy	27	11	14	10	14	28	28
EU industrial	29	16	15	15	15	23	24
EU telecommunication	28	12	13	15	15	25	25
EU utilities	15	11	12	13	13	27	28

$\delta_{\max-\lambda}$ is calculated as the difference between the maximum eigenvalue from the DCC correlation matrix and the maximum eigenvalue from the CCC unconditional correlation matrix. δ_{VaR} and δ_{ES} are the differences between the margin or ES calculation using DCC and CCC, respectively.

(3) On average, the correlation change for NA portfolios is about 22% for all selected periods and all sectors. As a result, the average margin impact across all sectors is about 12% for VaR and 13% for ES related to net short portfolios, and 12% for VaR and 13% for ES related to net long portfolios. In contrast, for balanced portfolios with the same correlation change (ie, a 25% correlation change as represented by the maximum eigenvalue change), the average VaR and ES impacts across all sectors are 25% and 25%, respectively.

(4) Balanced portfolios are almost twice as sensitive as directional portfolios, net short or net long.

(5) For directional portfolios, utilities and energy sectors are most sensitive to the correlation changes.

(6) For balanced portfolios, utilities, energy and financials are the most sensitive to correlation changes.

(7) The ES impact tends to be larger than the corresponding VaR impact, but not in all cases.

6 BACKTESTING GARCH–DCC AND GARCH–CCC MARGIN

The backtesting of GARCH–DCC and GARCH–CCC models is not the focus of this study. However, it is performed to ensure the models built for our research are fit for purpose. We conducted VaR backtesting to evaluate the model performance of both GARCH–DCC and GARCH–CCC. The VaR value is calculated over selected stress periods at a 99% confidence level over the five-day liquidation horizon. The number of VaR exceedances is counted for every sample portfolio so that it may be compared with the theoretically expected number of exceedances. The widely adopted Kupiec likelihood ratio test is used to evaluate the frequency of exceedances statistically. The Kupiec test can be defined by the following null hypothesis:

$$\mathrm{H_0}: \quad \text{the expected proportion of violations is equal to } 0.01.$$

Tables 8 and 9 show the results of VaR backtesting. For the subprime crisis period, the null hypothesis cannot be rejected in all portfolios except for P5 (NA financial short) for DCC, and in all portfolios except for P5 and P8 (NA customer goods short).[22] For the eurozone debt crisis period, the null hypothesis cannot be rejected in all portfolios except for P8 for DCC, and in all portfolios except for P5, P8 and P10 (NA financial (insurers)). Even for the portfolios for which the null hypothesis is rejected, the exceedance counts are closer to the expected level in DCC than in CCC. Considering these two periods are the most stressed periods in recent history in terms of the CDS market, both GARCH–DCC and GARCH–CCC perform reasonably well. In addition, these comparative results suggest that DCC performs better than CCC in terms of VaR backtesting, which is largely consistent with prior research on GARCH–DCC versus GARCH–CCC.

7 CONCLUSIONS

This research aims to use a GARCH volatility and correlation framework via a DCC copula to study correlation and margin impact on different portfolios that consist of CDS SNs with differing characteristics. Our purpose is to better understand the direction and magnitude of margin impact due to correlation changes in different types of portfolios in different correlation regimes. We find that correlations between CDS SNs in the form of DCC could change and go through different correlation regimes. Sometimes these correlations could change dramatically within a short period of time. Further, the study concludes that VaR or ES calculated using the multivariate GARCH DCC model could be sizably different from that calculated with GARCH CCC, which assumes constant correlation. The more elevated the correlation compared with the historical average, the greater the impact on VaR or ES. We use maximum eigenvalue to quantify correlation change for a portfolio over a historical period. From our correlation change impact study for these historical periods, we find that when correlation changes by 24%, VaR changes from

[22] At a 5% significance level.

TABLE 8 VaR backtesting for selected portfolios during subprime crisis (January 1, 2008–December 31, 2008).

		DCC				CCC			
	Portfolio description	**Exceedance (ratio, %)**		**Kupiec**	***p*-value**	**Exceedance (ratio, %)**		**Kupiec**	***p*-value**
P1	EU financial long	1	0.4	1.1765	0.2781	2	0.8	0.1084	0.7419
P2	EU financial short	3	1.2	0.0949	0.1619	4	1.6	0.7691	0.3805
P3	EU financial balanced	2	0.8	0.1084	0.7419	1	0.4	1.1765	0.2781
P4	NA financial long	4	1.6	0.7691	0.3805	6	2.4	3.5554	0.0594
P5	NA financial short	7	2.8	5.4970	0.0190	10	4.0	12.955	0.0003
P6	NA financial balanced	4	1.6	0.7691	0.3805	3	1.2	0.0949	0.7580
P7	NA customer goods long	3	1.2	0.0949	0.7580	4	1.6	0.7691	0.3805
P8	NA customer goods short	6	2.4	3.5554	0.0594	8	3.2	7.7336	0.0054
P9	NA customer goods balanced	5	2.0	1.9568	0.1619	5	2.0	1.9568	0.1619

TABLE 9 VaR backtesting for selected portfolios during eurozone crisis (January 1, 2011–December 31, 2011).

		DCC				CCC			
	Portfolio description	**Exceedance (ratio, %)**		**Kupiec**	***p*-value**	**Exceedance (ratio, %)**		**Kupiec**	***p*-value**
P1	EU financial long	1	0.4	1.1765	0.2781	2	0.8	0.1084	0.7419
P2	EU financial short	1	0.4	1.1765	0.2781	3	1.2	0.0949	0.7580
P3	EU financial balanced	1	0.4	1.1765	0.2781	1	0.4	1.1765	0.2781
P4	NA financial long	1	0.4	1.1765	0.2781	5	2.0	1.9568	0.1619
P5	NA financial short	5	2.0	1.9568	0.1619	7	2.8	5.4970	0.0190
P6	NA financial balanced	4	1.6	0.7691	0.3805	1	0.4	1.1765	0.2781
P7	NA customer goods long	2	0.8	0.1084	0.7419	4	1.6	0.7691	0.3805
P8	NA customer goods short	7	2.8	5.4970	0.0190	9	3.6	10.229	0.0014
P9	NA customer goods balanced	5	2.0	1.9568	0.1619	3	1.2	0.0949	0.7580
P10	NA financial (insurers) long	6	2.4	3.5554	0.0594	9	3.6	10.229	0.0014
P11	NA financial (insurers) short	1	0.4	1.1765	0.2781	2	0.8	0.1084	0.7419
P12	NA financial (insurers) balanced	1	0.4	1.1765	0.2781	0	0.4	3.0212	0.0822

14% for directional portfolios and 25% for balanced portfolios on average across all sectors and NA/EU areas. Balanced portfolios are almost twice as sensitive to correlation changes as directional portfolios when the correlation level is medium-high or high (eg, above 60%); this manifests itself in the form of margin underestimation or overestimation, depending on the correlation regime a portfolio is in at the time. More specifically, balanced portfolios could experience sizable margin underestimation during correlation "breakdown" periods. Therefore, for balanced portfolios, more timely correlation calculation could be helpful, or a predefined correlation floor could help to prevent underestimation and corresponding VaR breaches. However, if a low correlation regime persists for a sufficiently long period of time, extra considerations may be needed for directional portfolios, since an upward correlation shock could also lead to a sizable margin underestimation. Overall, our research indicates that it may be prudent to account for correlation dynamics when calculating margin for CCPs.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper. The Securities and Exchange Commission (SEC) disclaims responsibility for any private publication or statement of any SEC employee or commissioner. This paper expresses the authors' views and does not necessarily reflect those of the Commission, the commissioners or members of the staff.

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